Master studije
GEOINFORMATIKA

Mašinsko učenje - Linearni klasifikatori. Percepton.

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7. decembar 2021.

Građevinski fakultet Univerziteta u Beogradu

Podsetnik

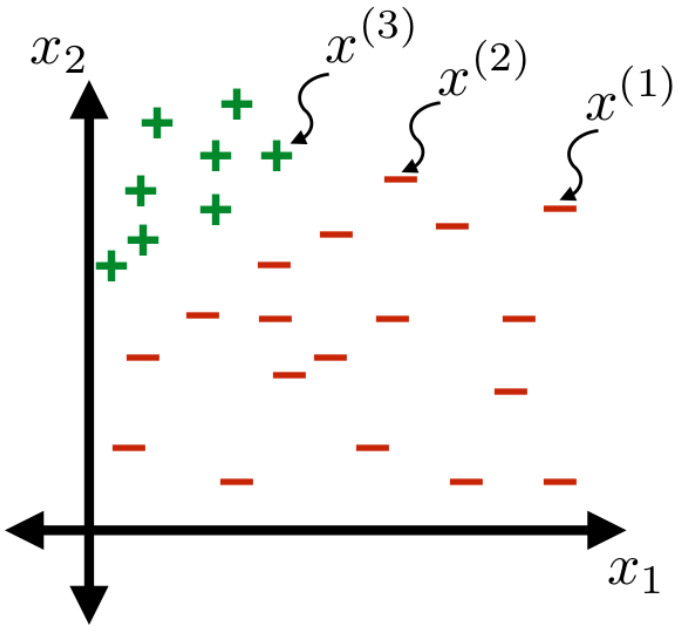
- Uopšteno linearno modelovanje.
- Objasnili smo šta je funkcija veze.
- Opisali logit (sigmoid) funkciju.
- Opisali Njutnovu metodu.
- Primer klasifikacije spam poruka.

Cilj za danas

- Linearni klasifikatori.
- Perceptron.
- Podsećanje na logističku regresiju/linearnu logističku klasifikaciji.

Linearni klasifikator

- Rekli smo da je tipično kodiranje sa 1 i 0. Tipično je i $+1$ i -1.
- Postavka problema:
  - Ulazni podacu su m svojstava: $x^{(i)} = (x_1^{(i)}, \ldots, x_d^{(i)}) \in \mathcal{R}^d$, gde $i \in \{1, \ldots, n\}$.
  - Ishod je binaran i kodiran sa $+1$ i -1.
- Primer. Srčani udar sa svojstvima x_1 (krvni pritisak) i x_2 (starost pacijenta). Obeležje (labela) je $+1$ ako je pacijent imao srčani udari, -1 ukoliko nije imao srčani udar.
- Pretpostavimo da imamo situaciju na slici! Neka je skup za obučavanje skup $D_n = \{(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})\}$.
- Definišimo hipotezu $h : \mathcal{R}^d \to \{+1, -1\}$



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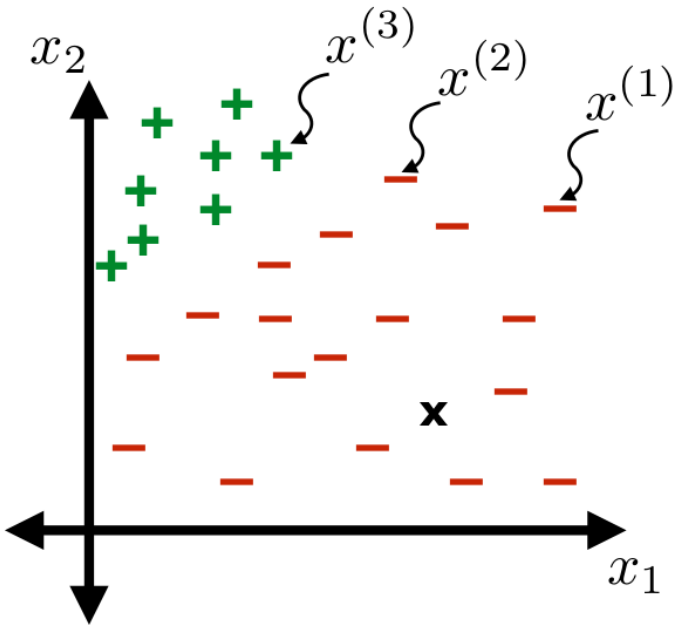

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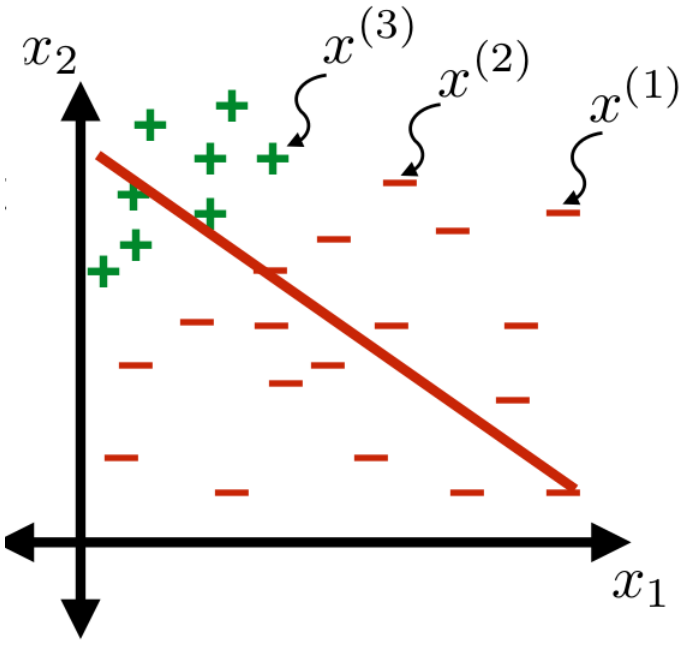
- Ostatak prezentacije izveden je iz predavanja prof. Tamara Broderick (https://tamarabroderick.com/files/ml_6036_2020_lectures/broderick_lecture_02.pdf)



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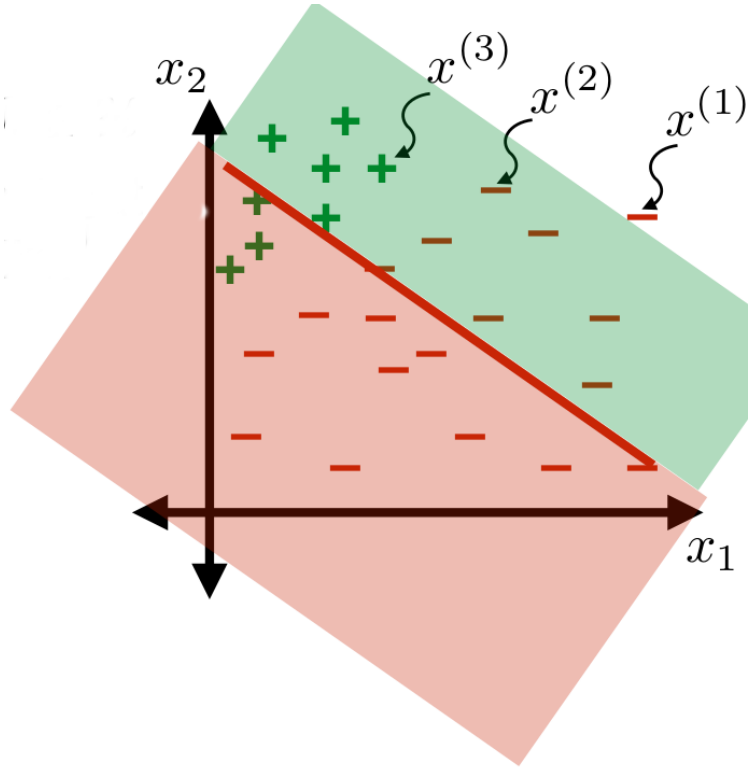

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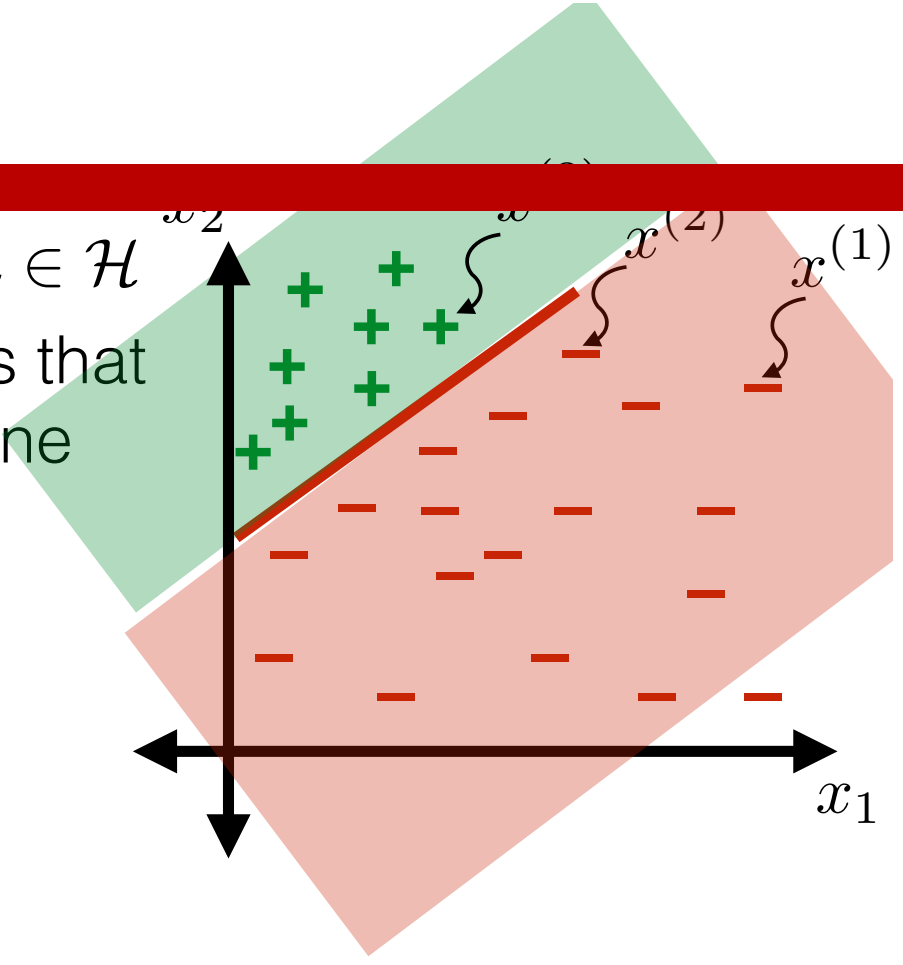
Math facts!



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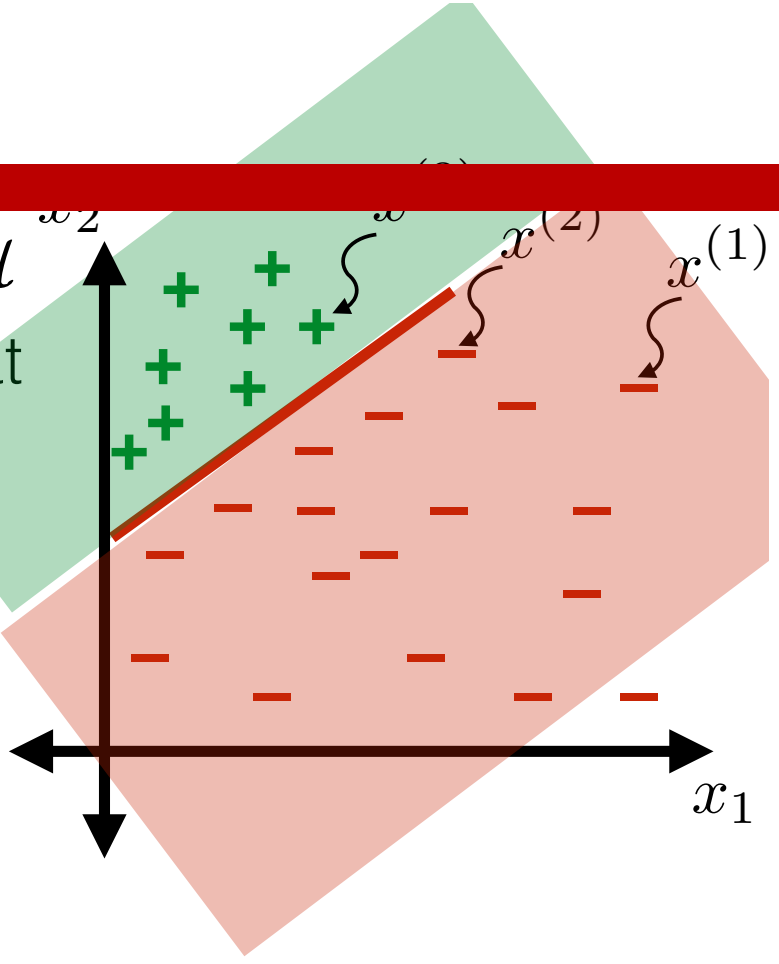

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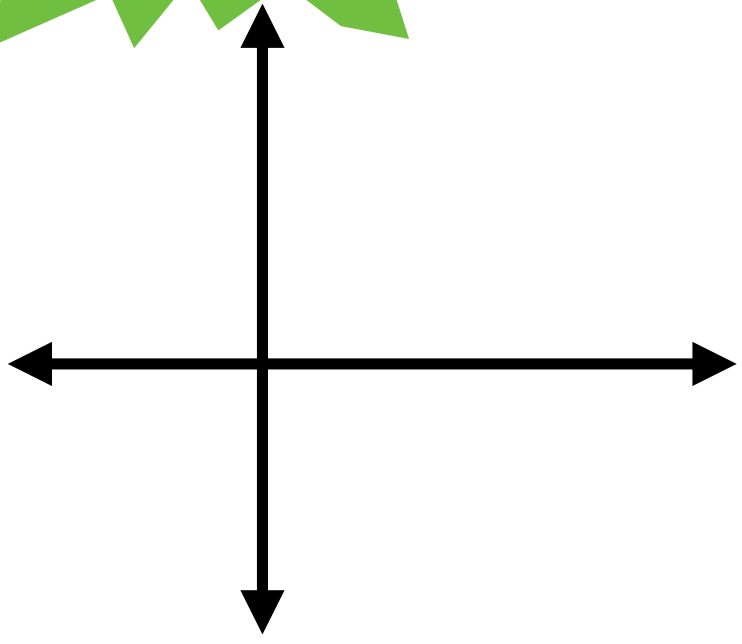

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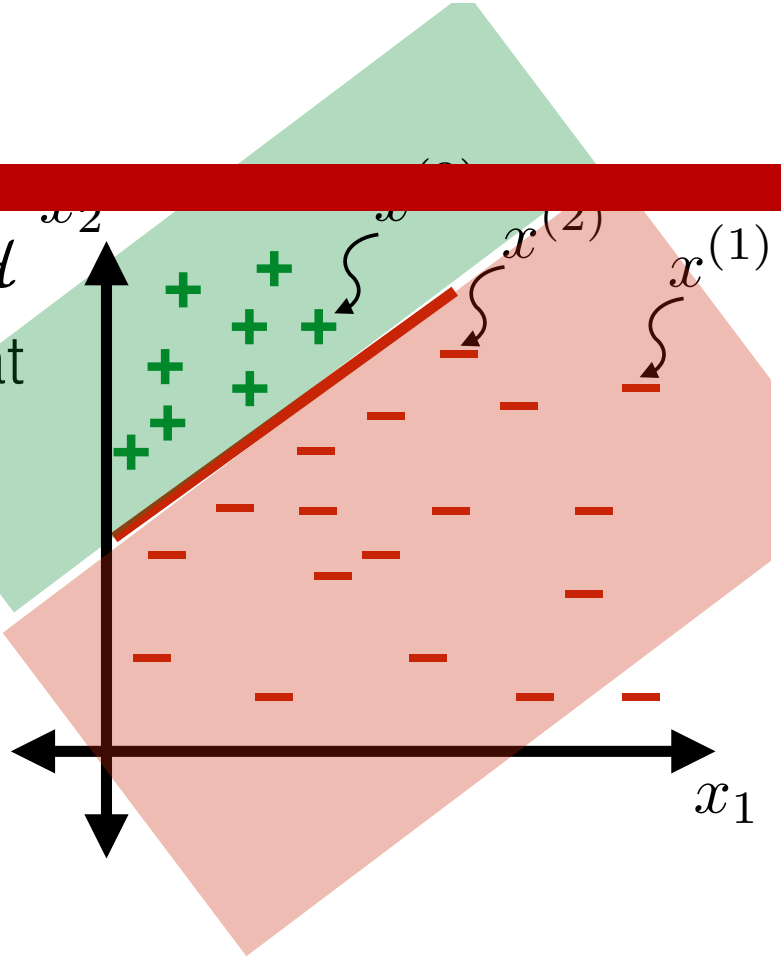

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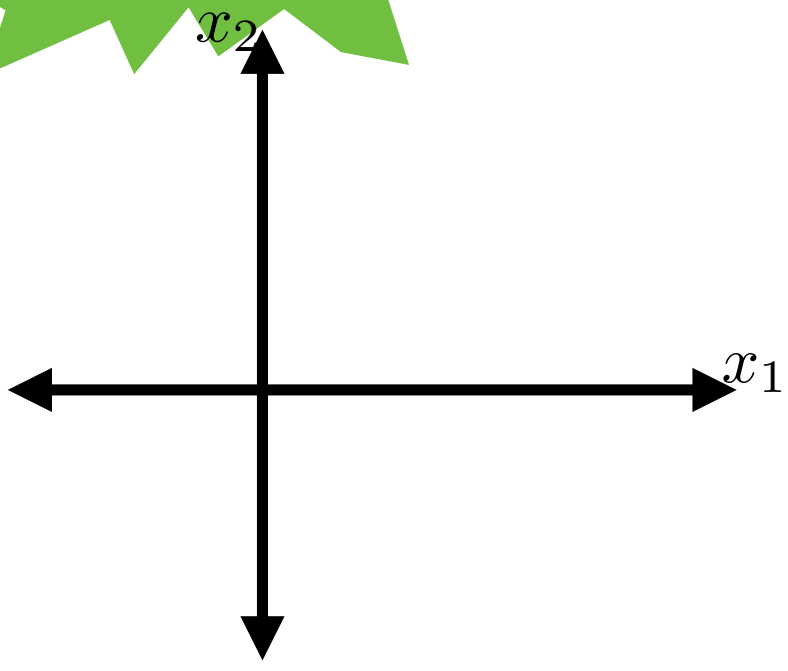

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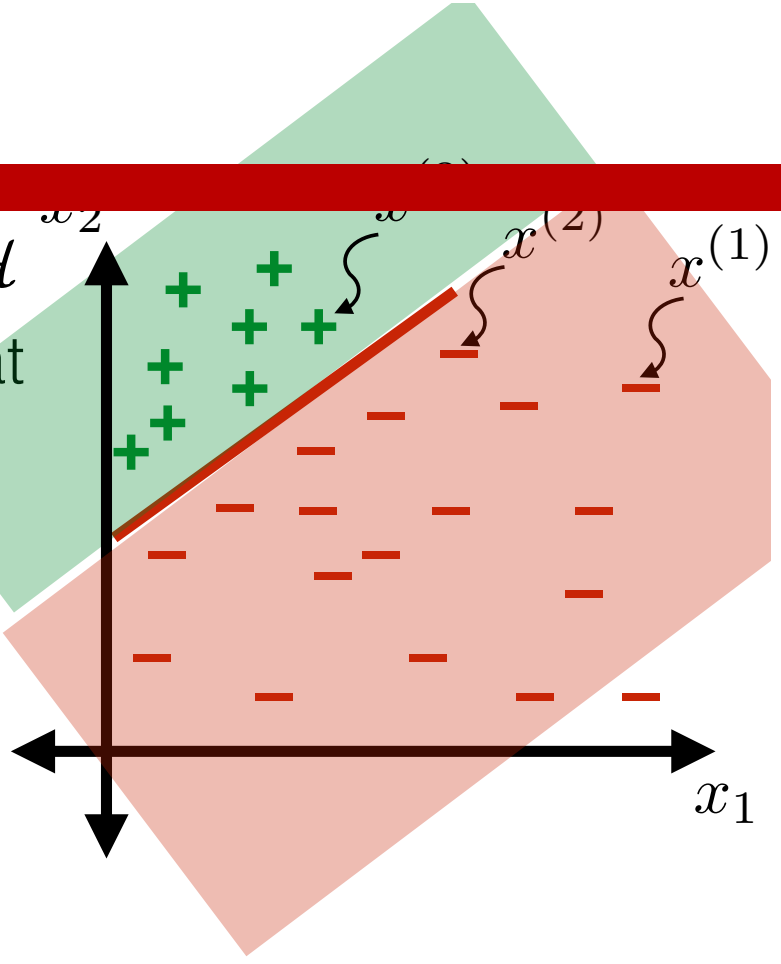

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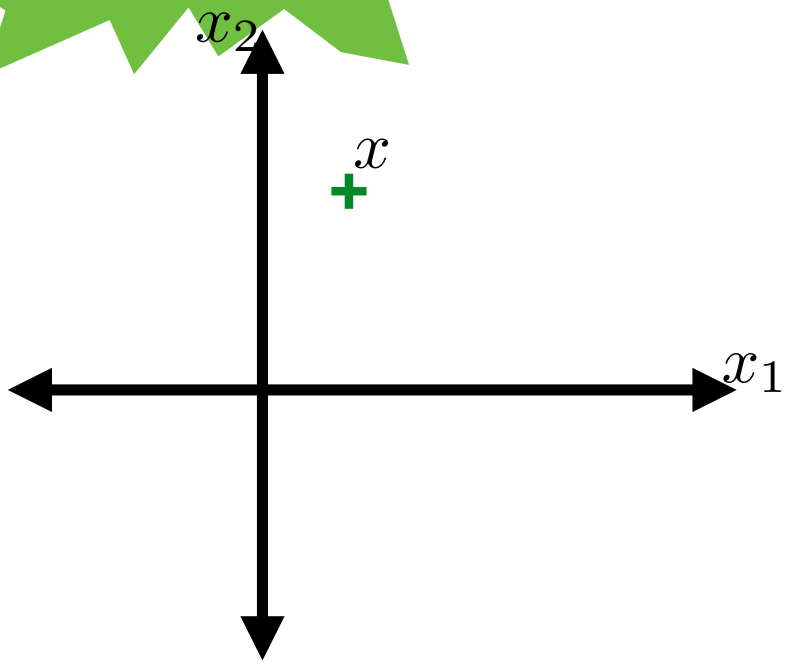

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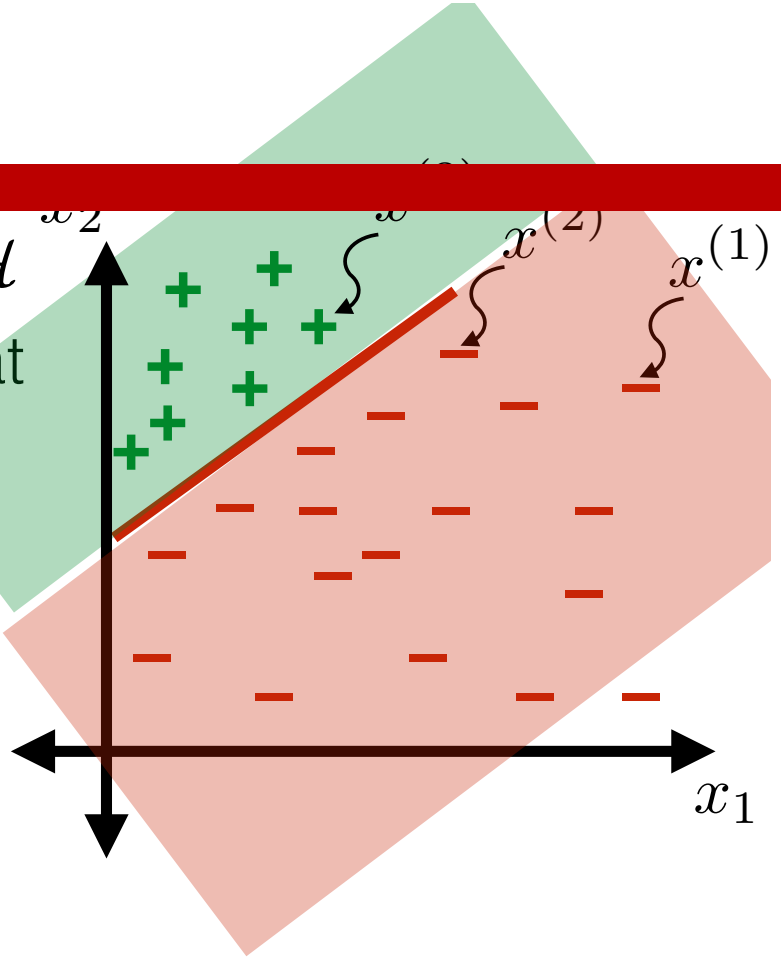

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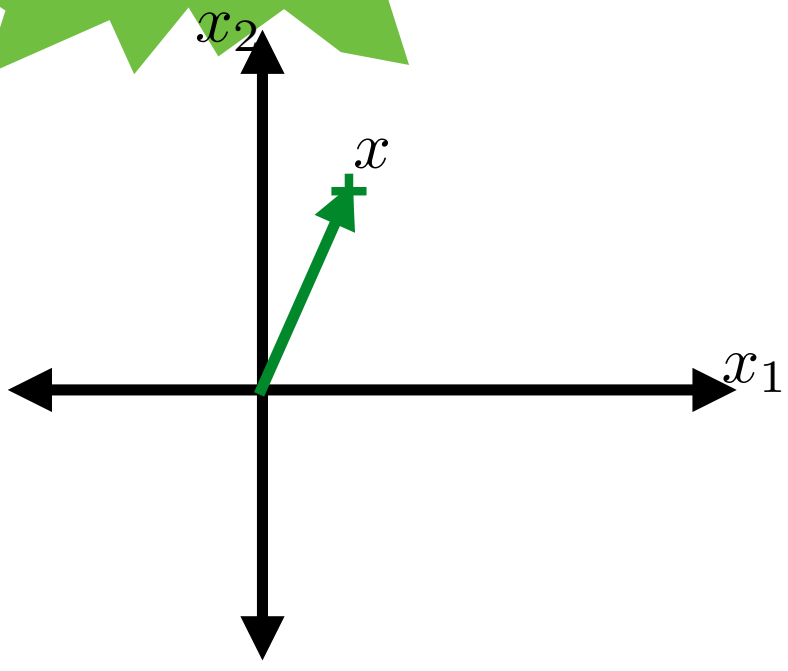

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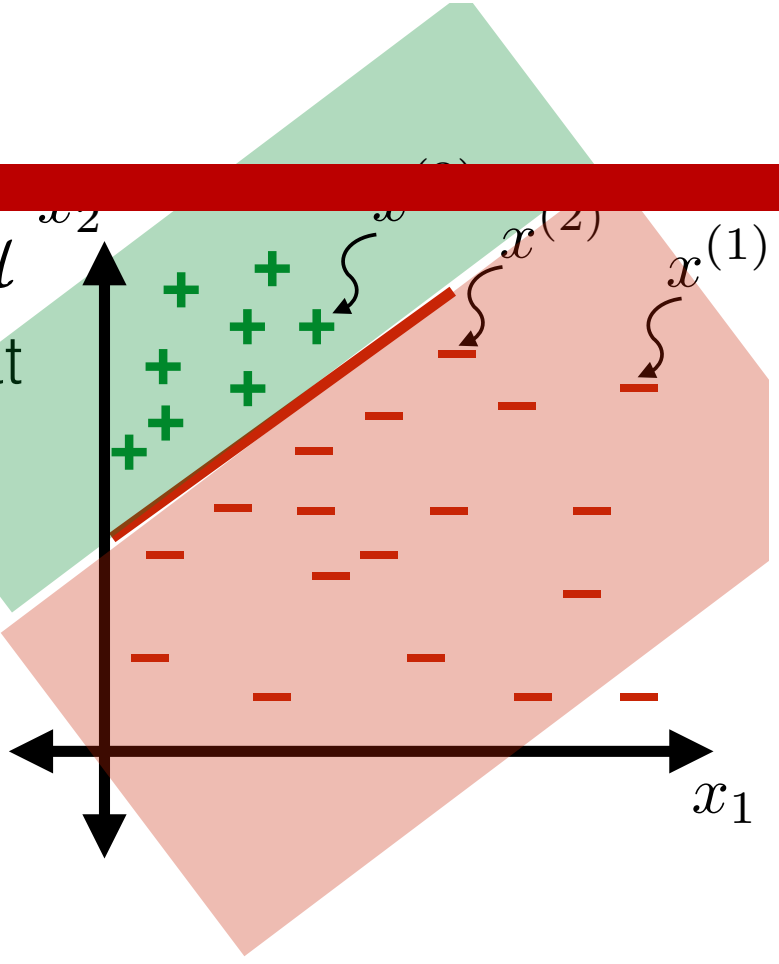

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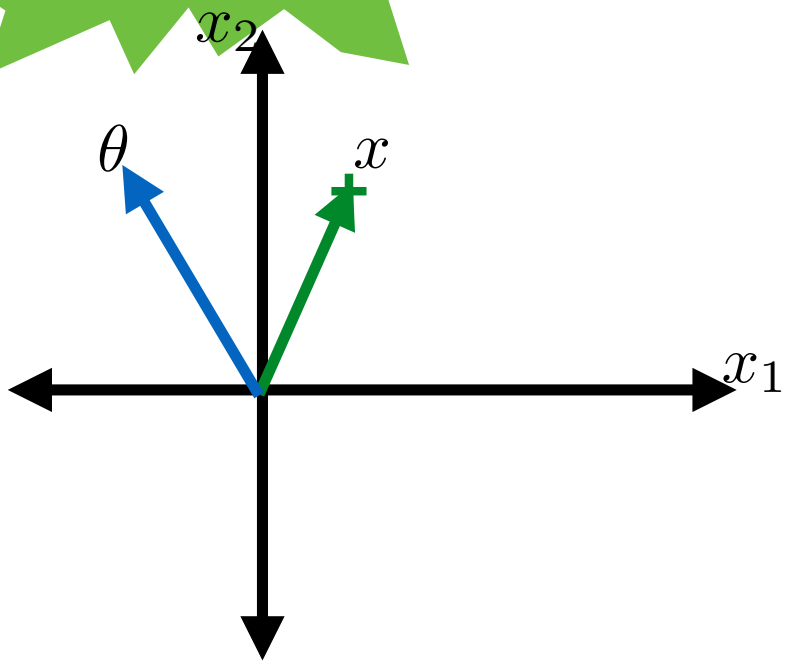

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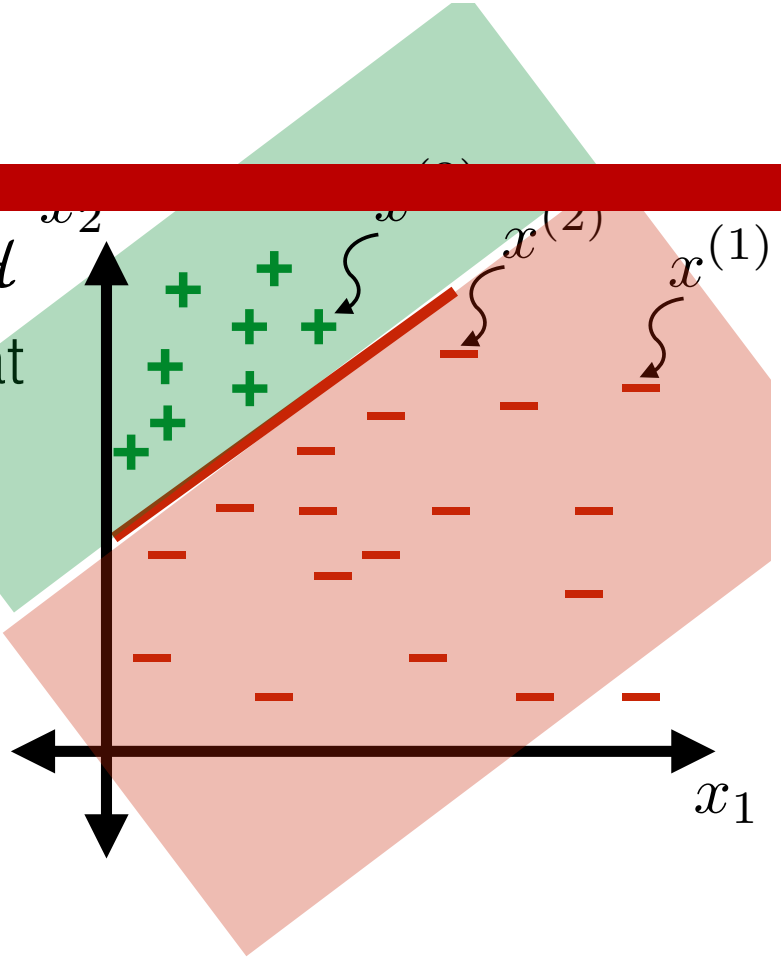

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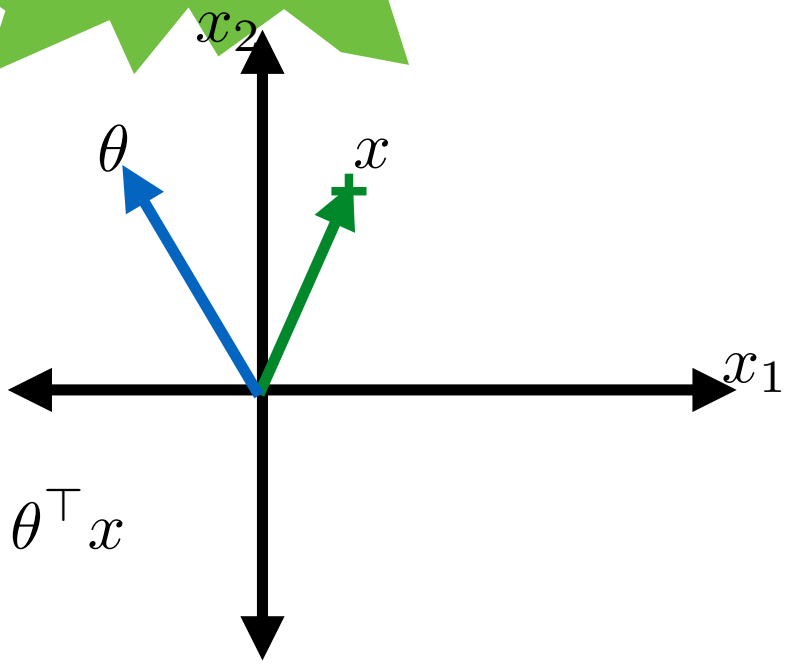

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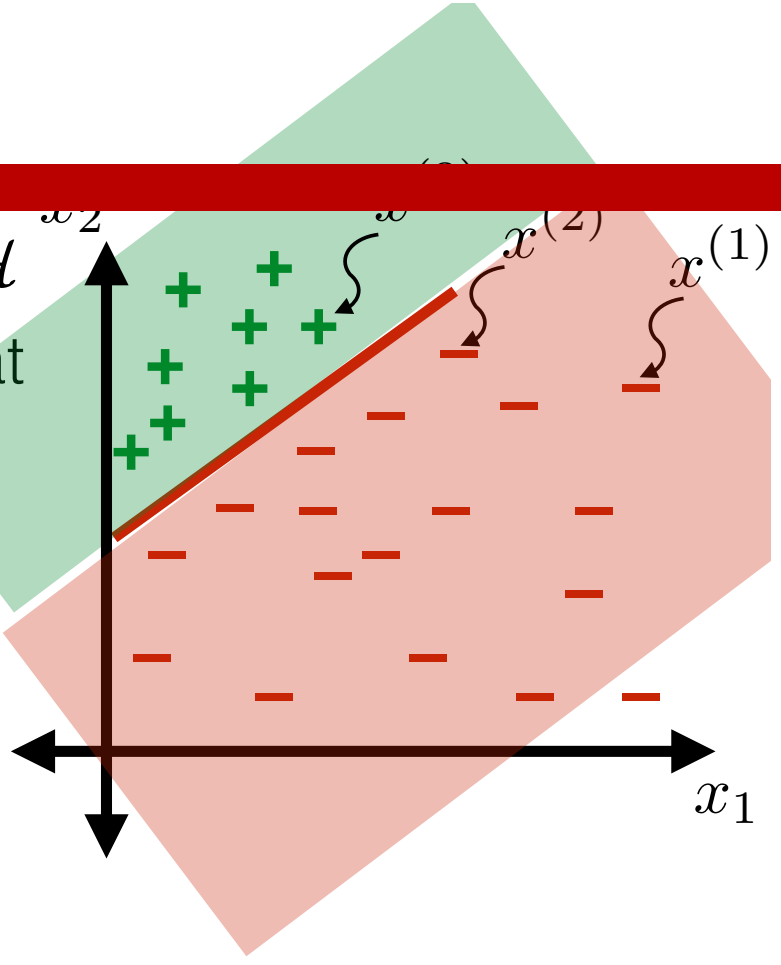

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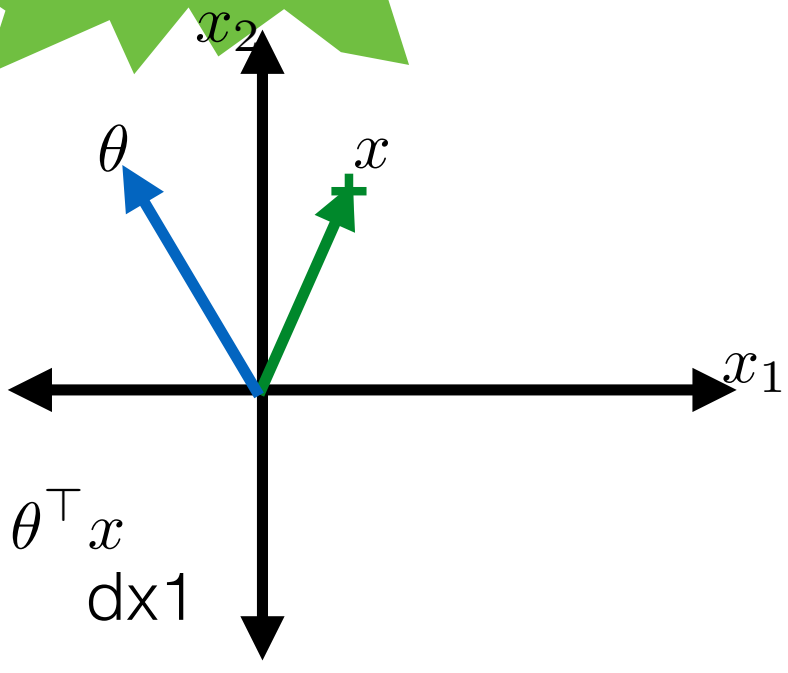

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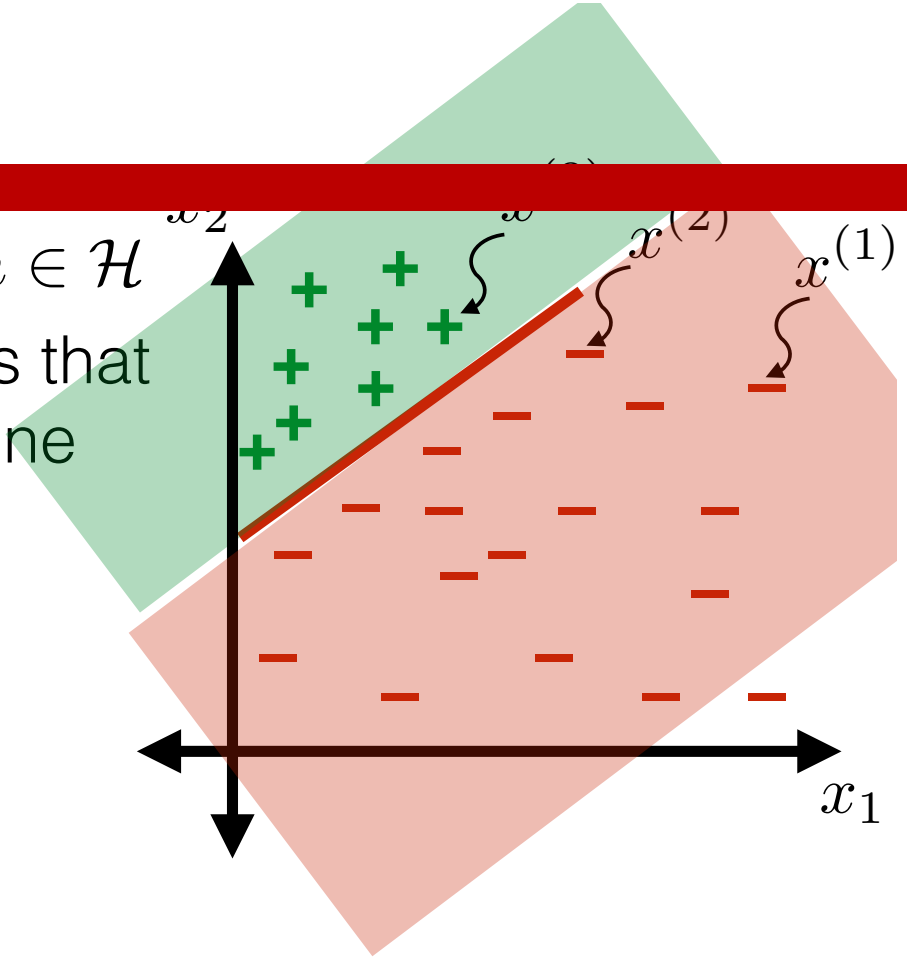

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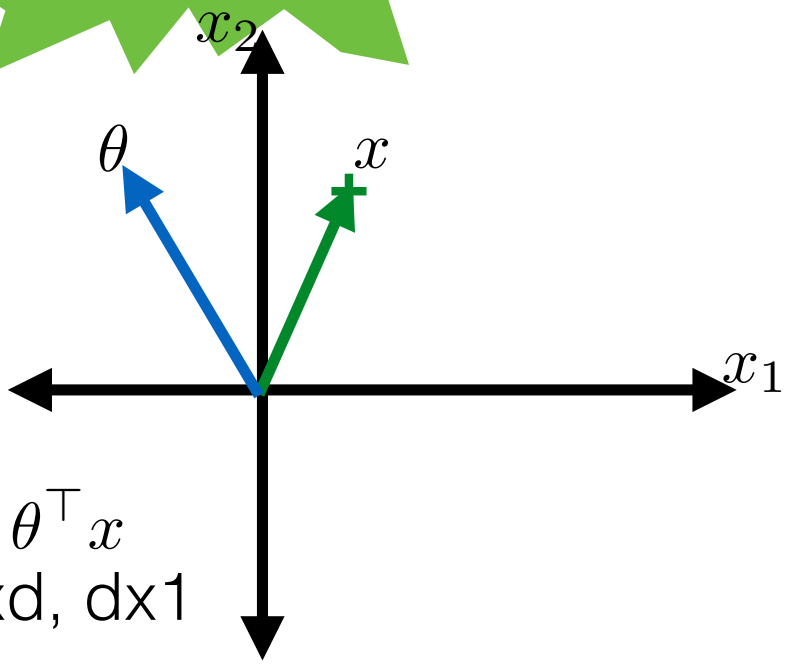

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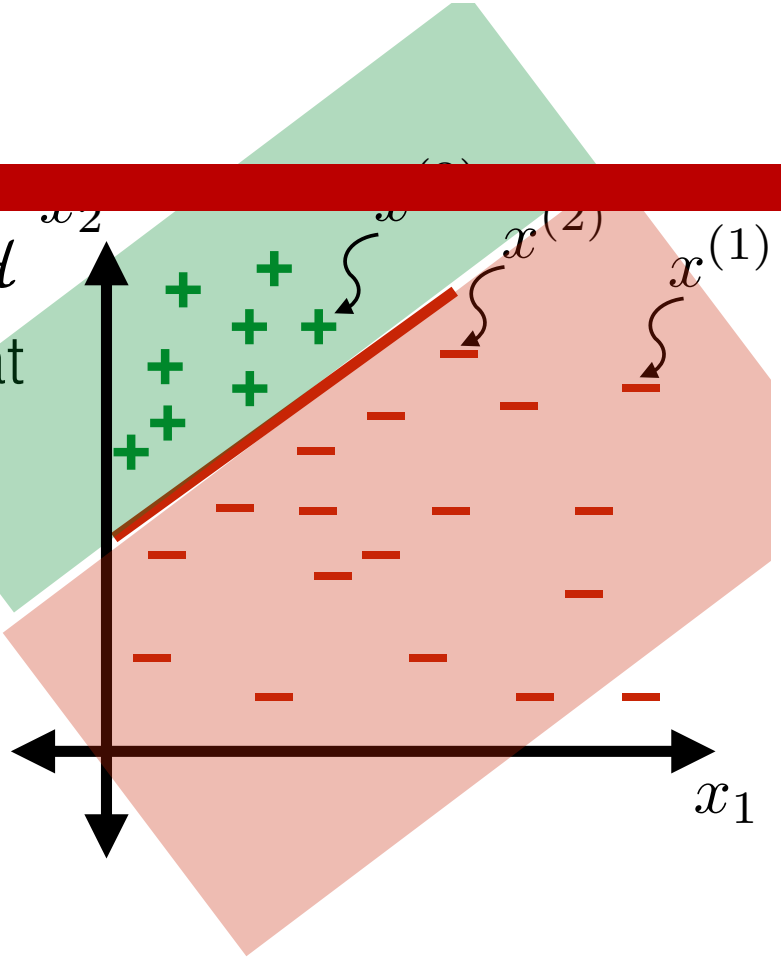

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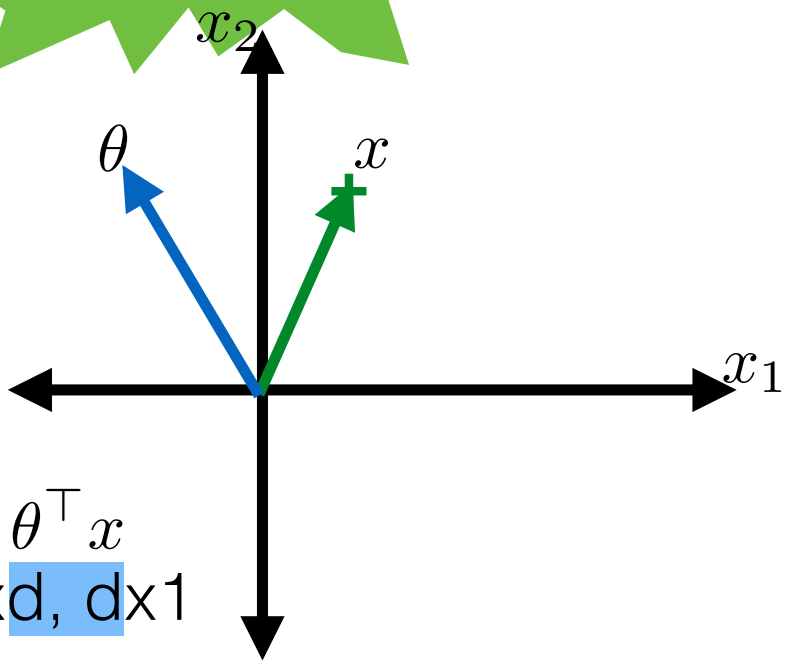

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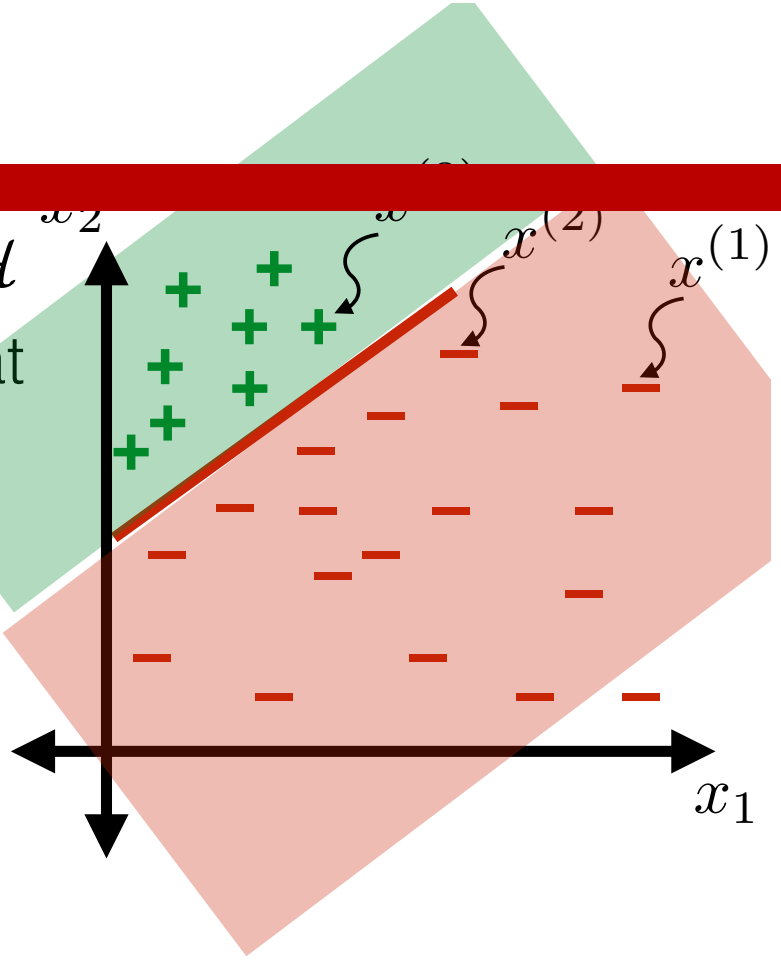

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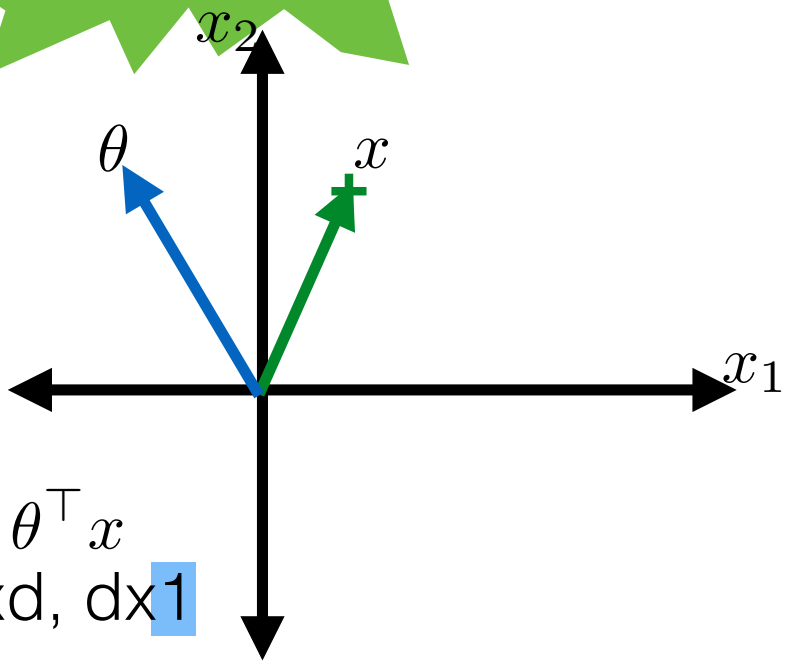

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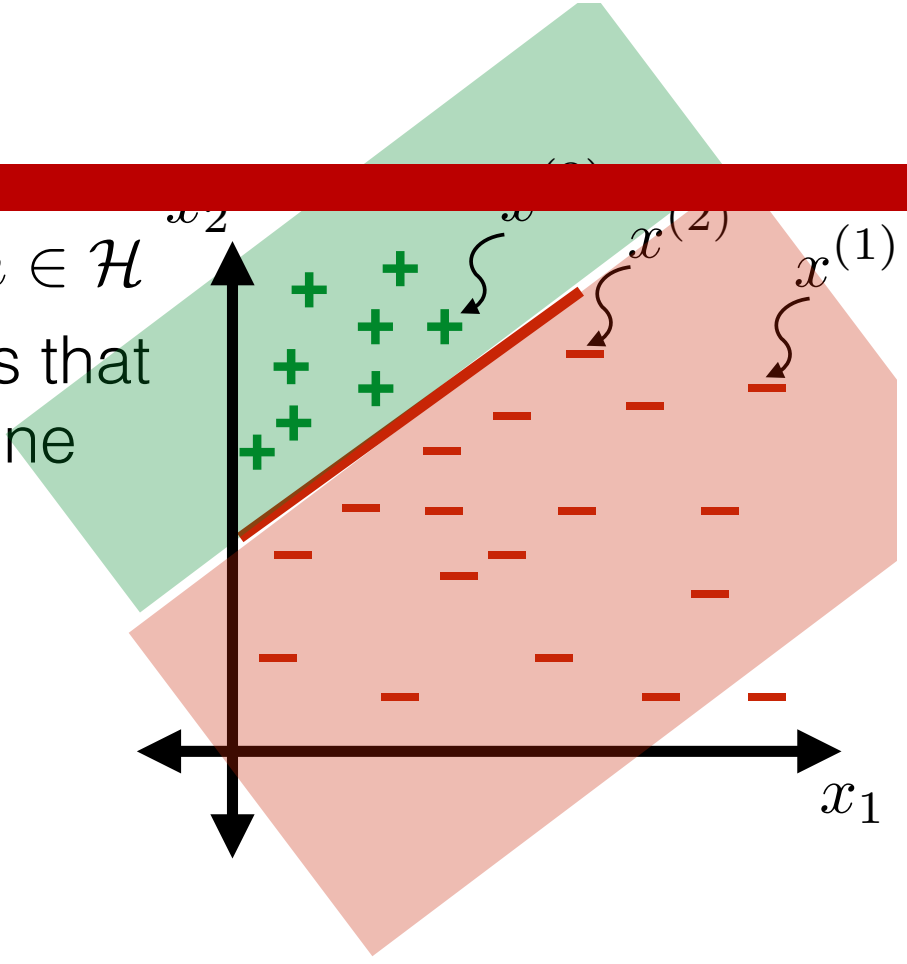

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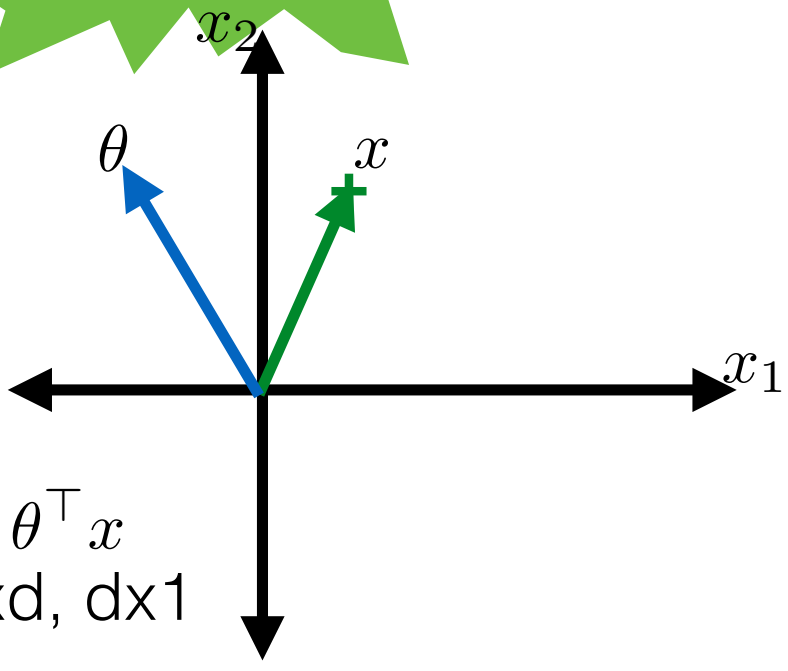

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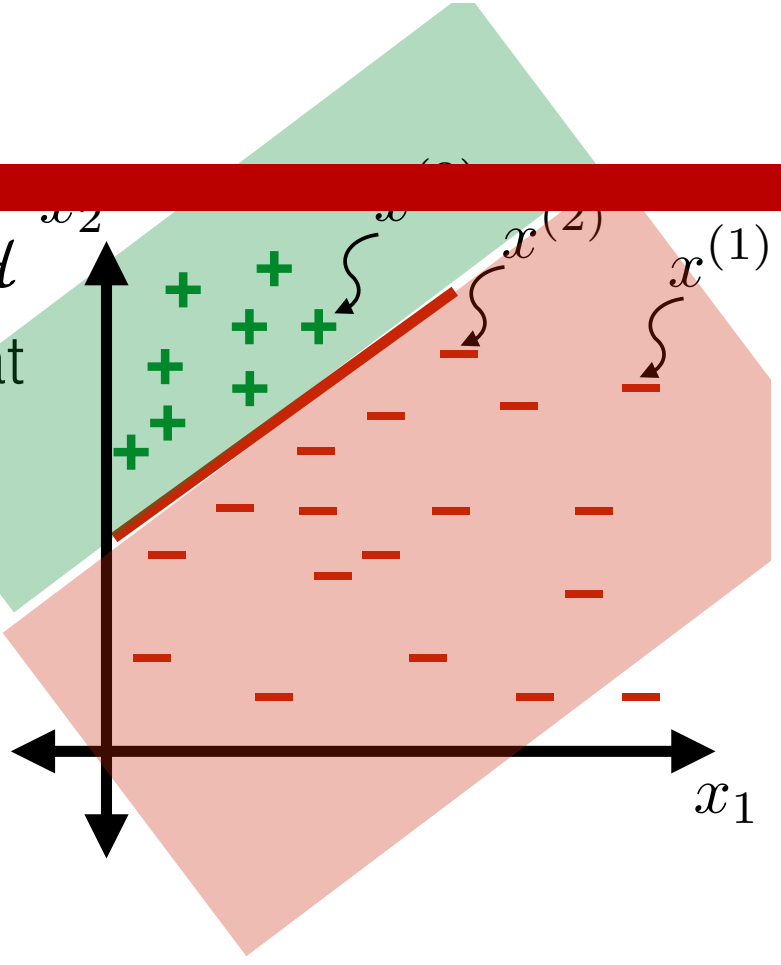

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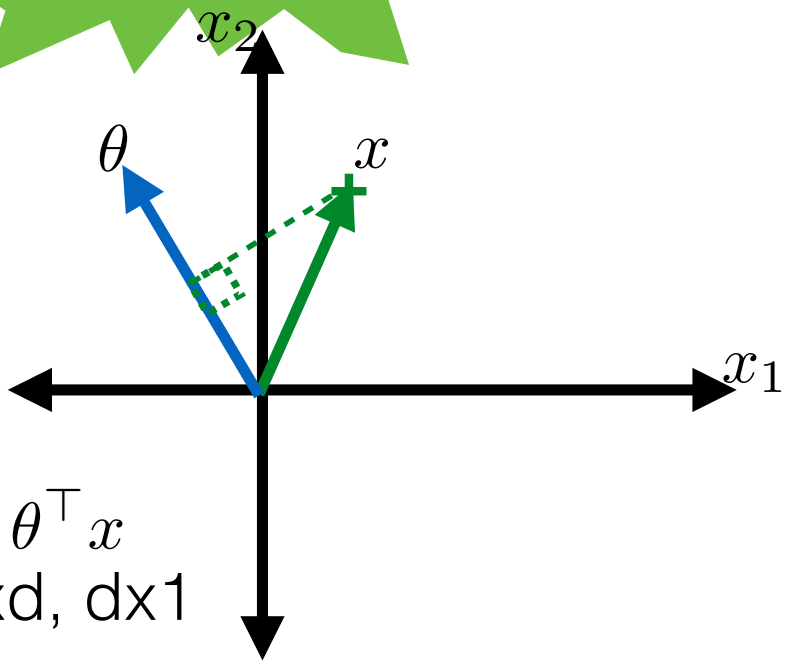

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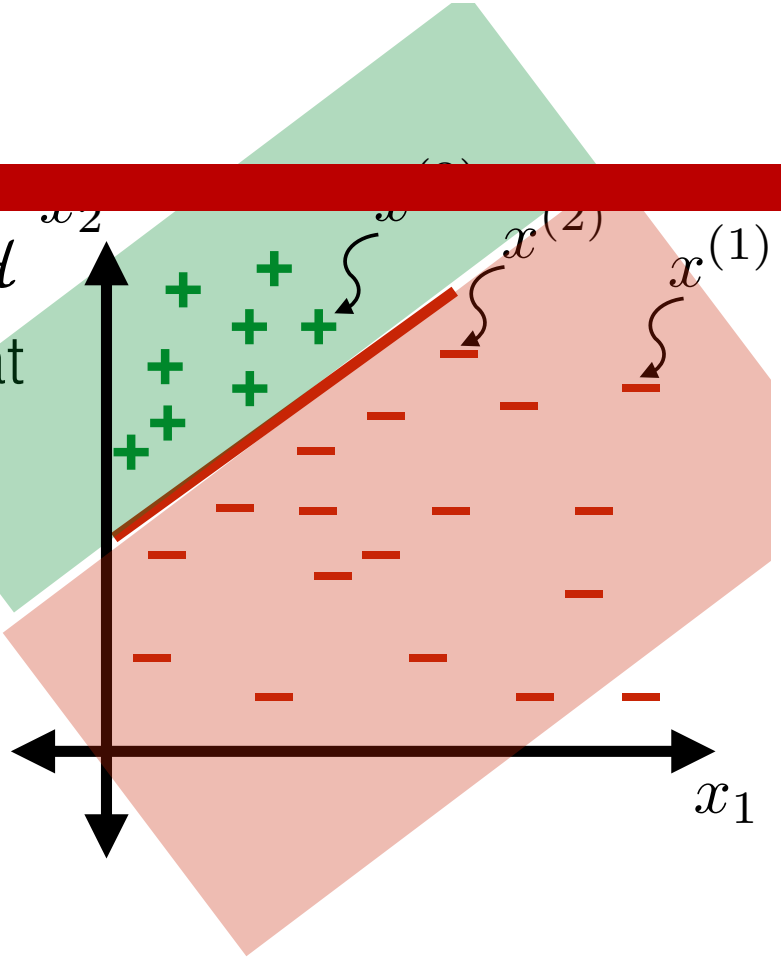

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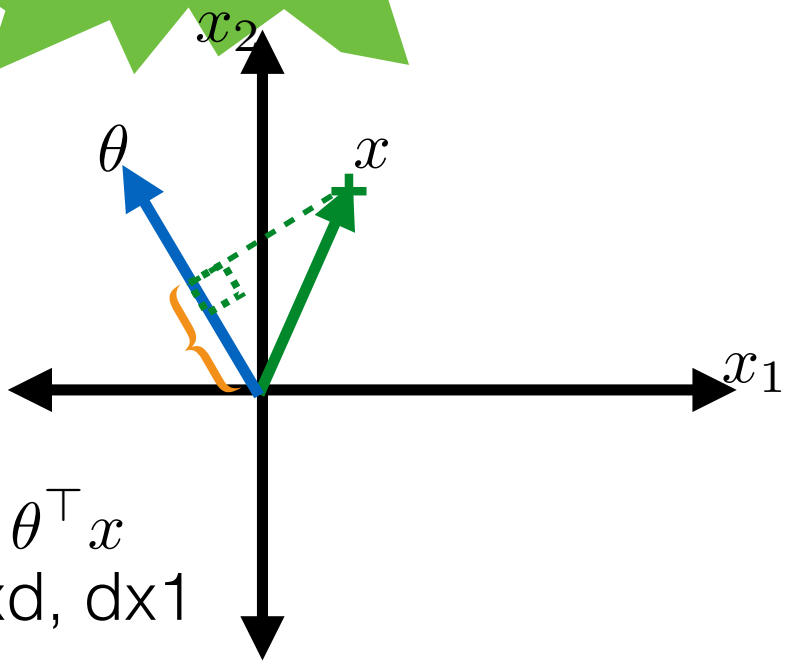

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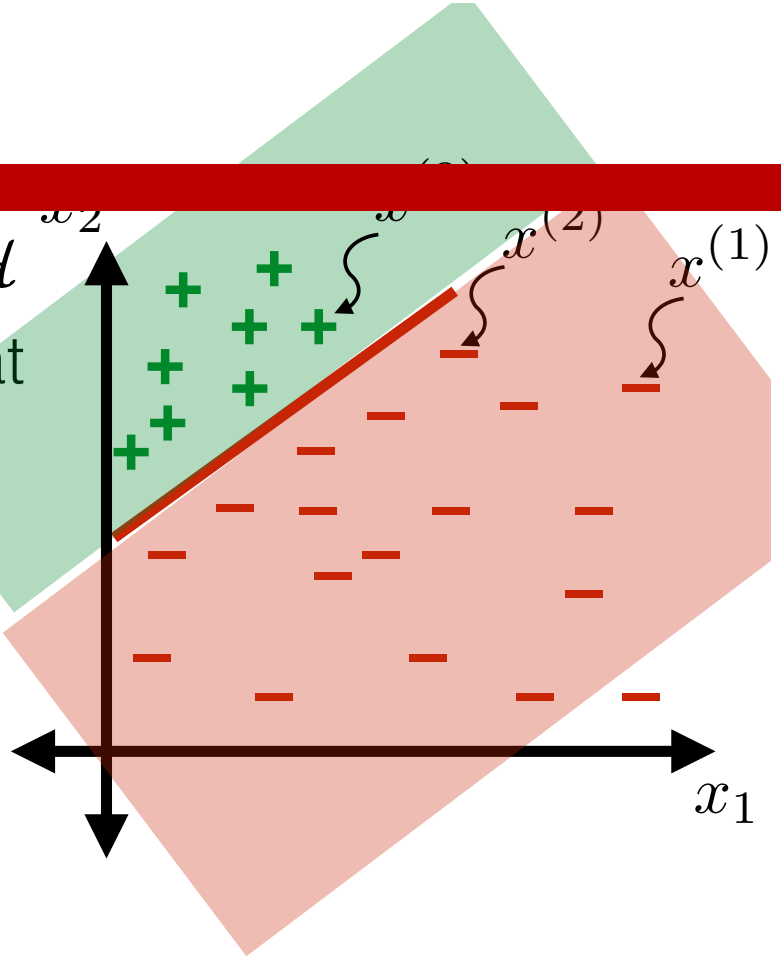

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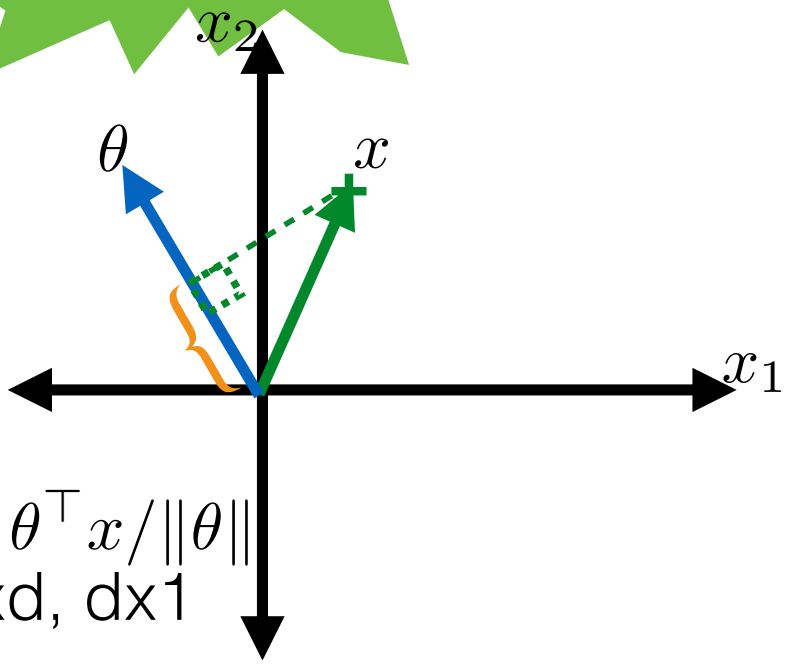

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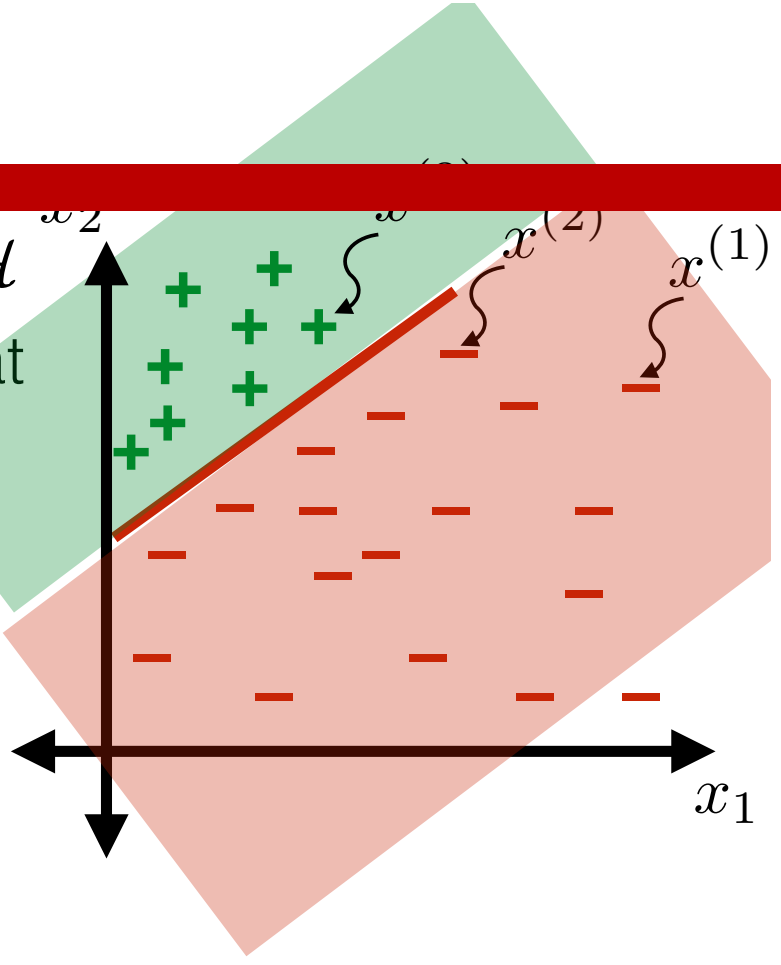

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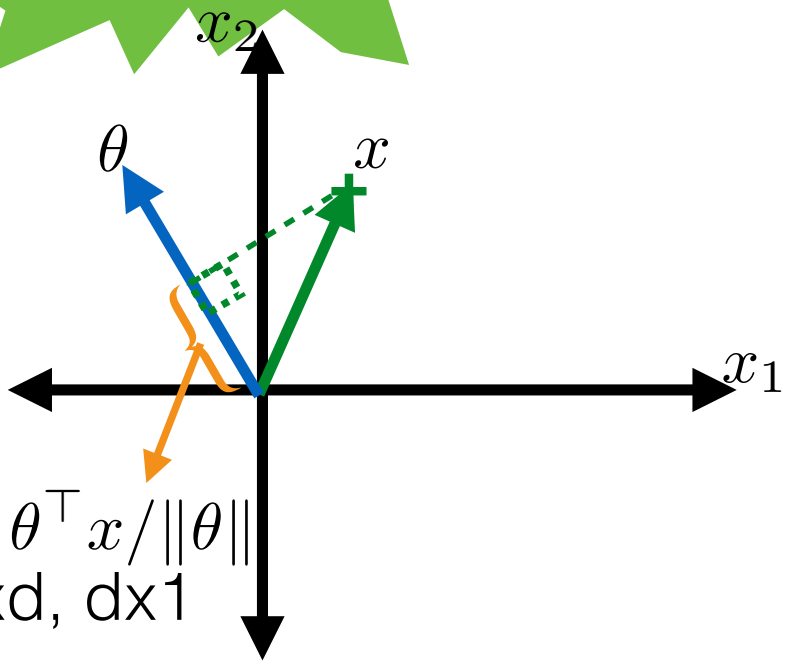

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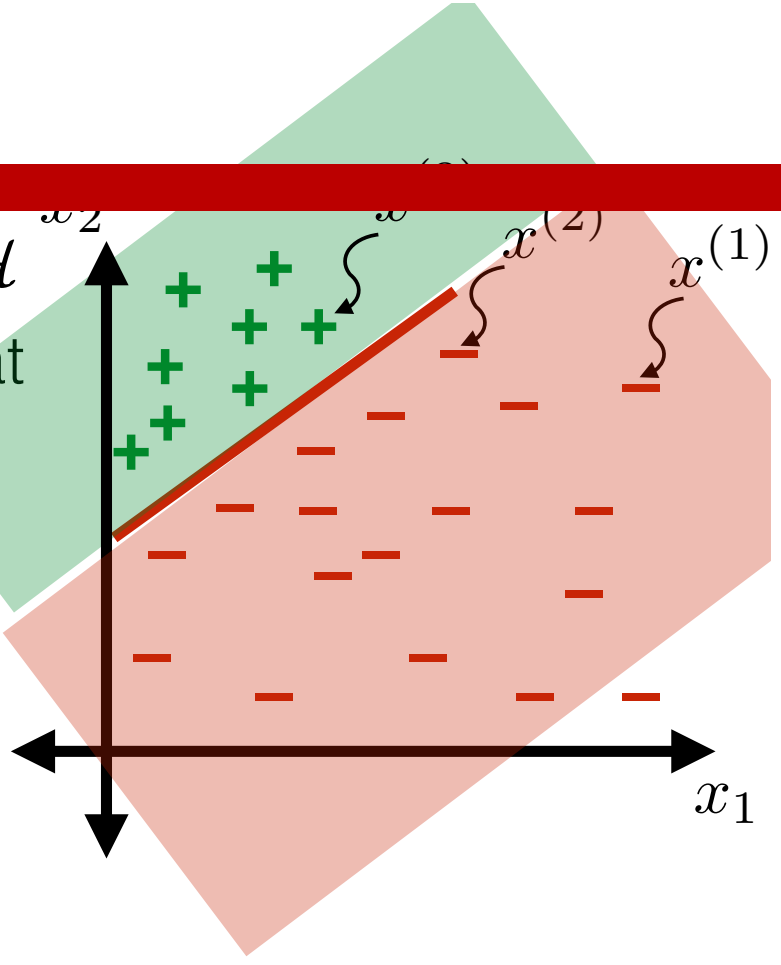

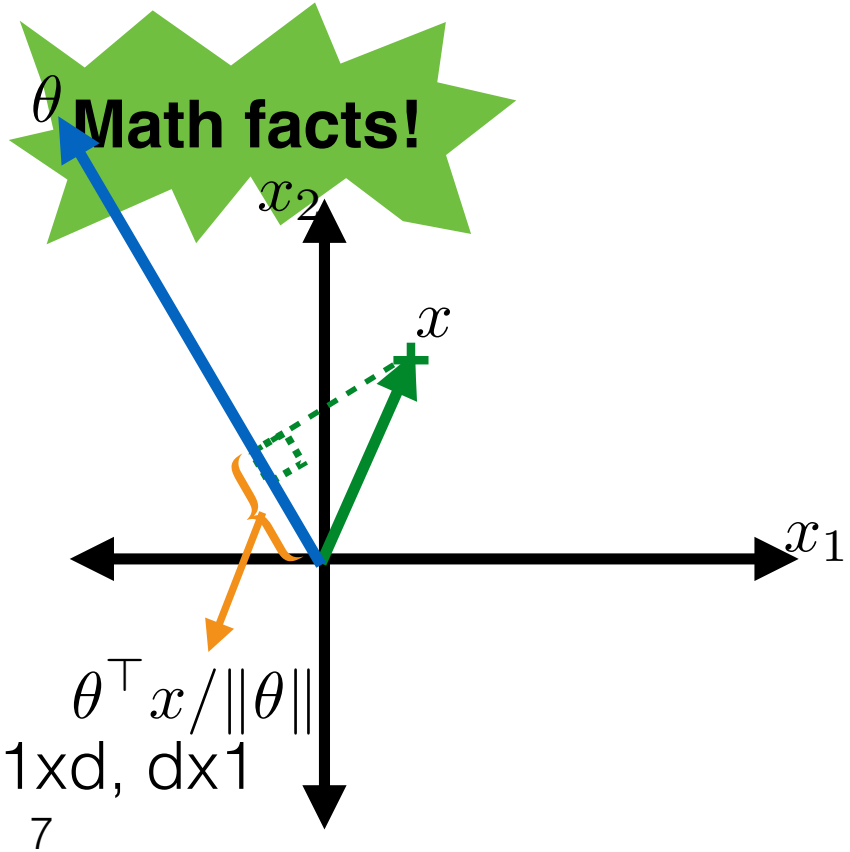

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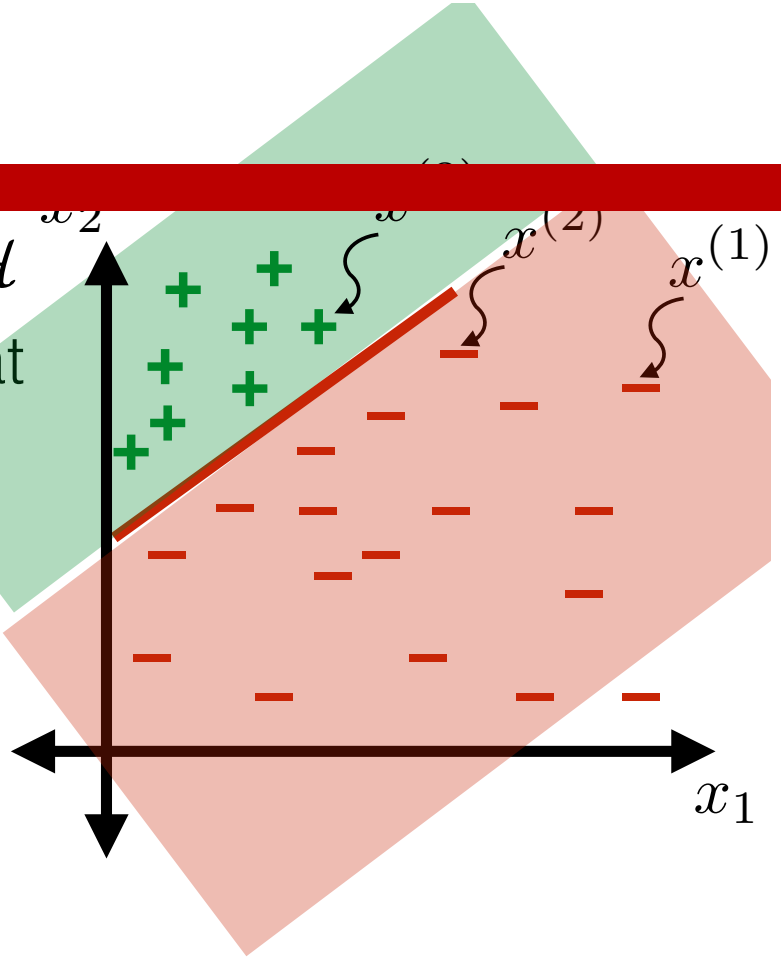

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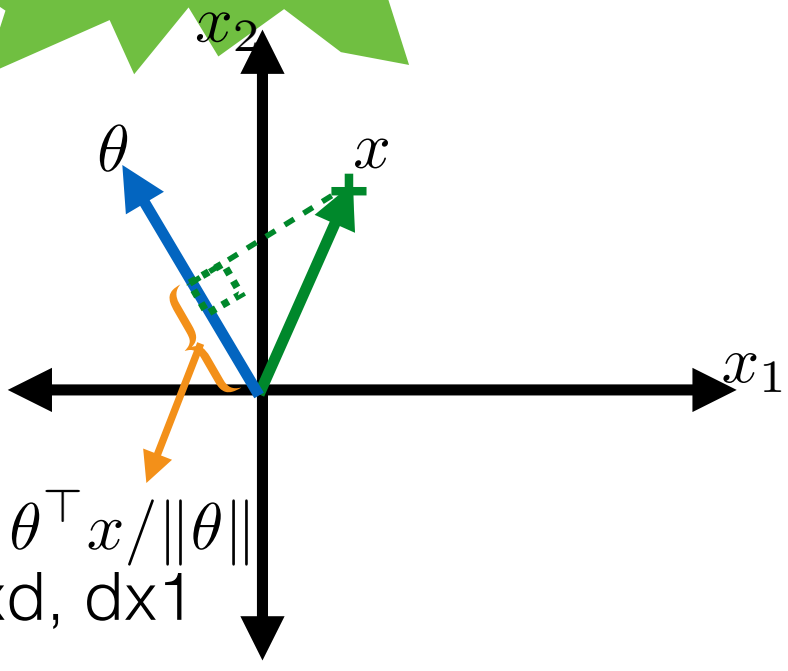

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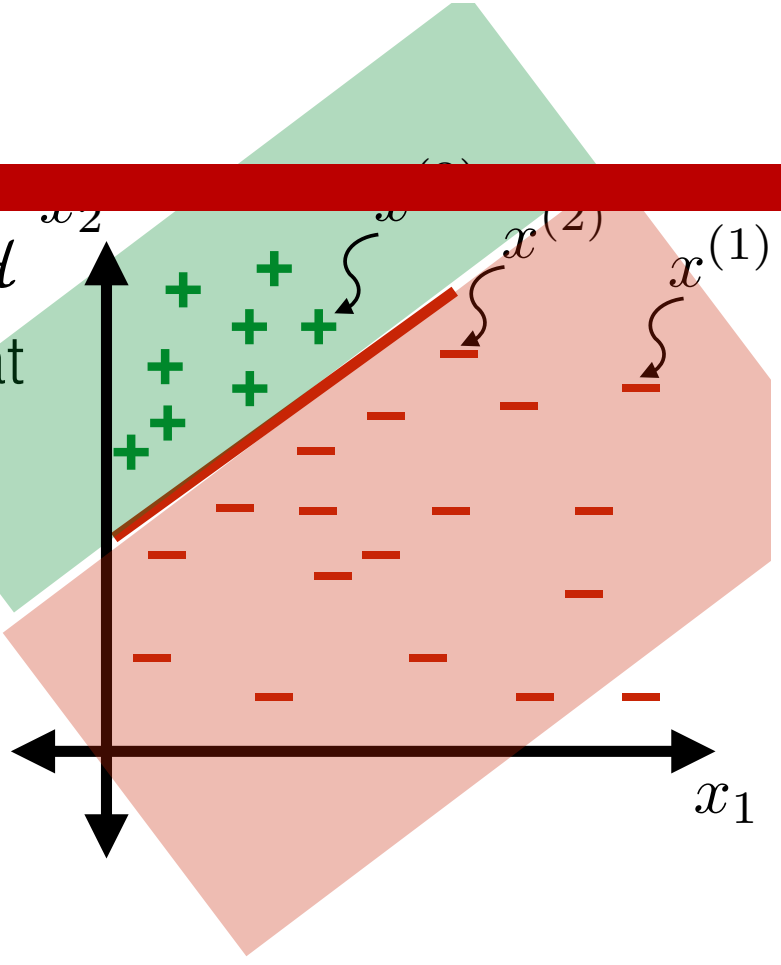

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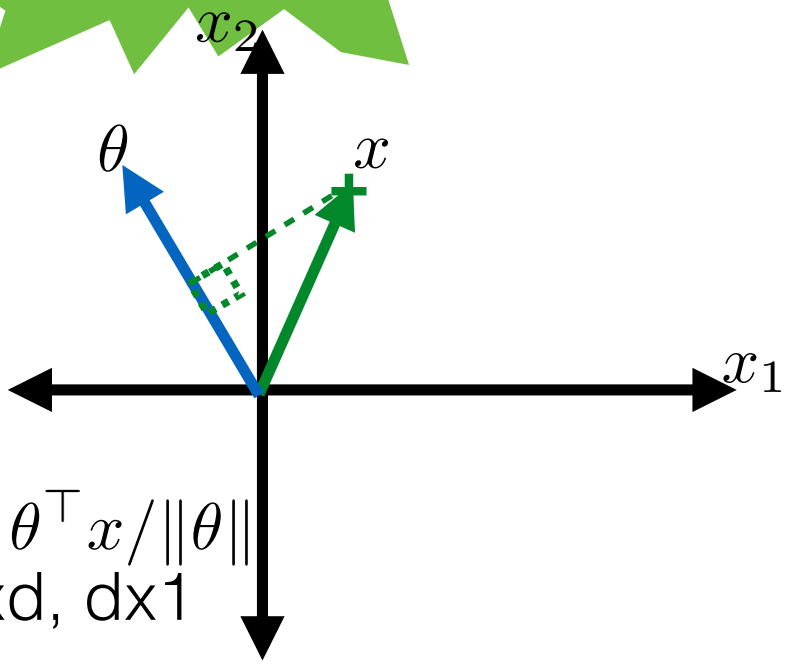

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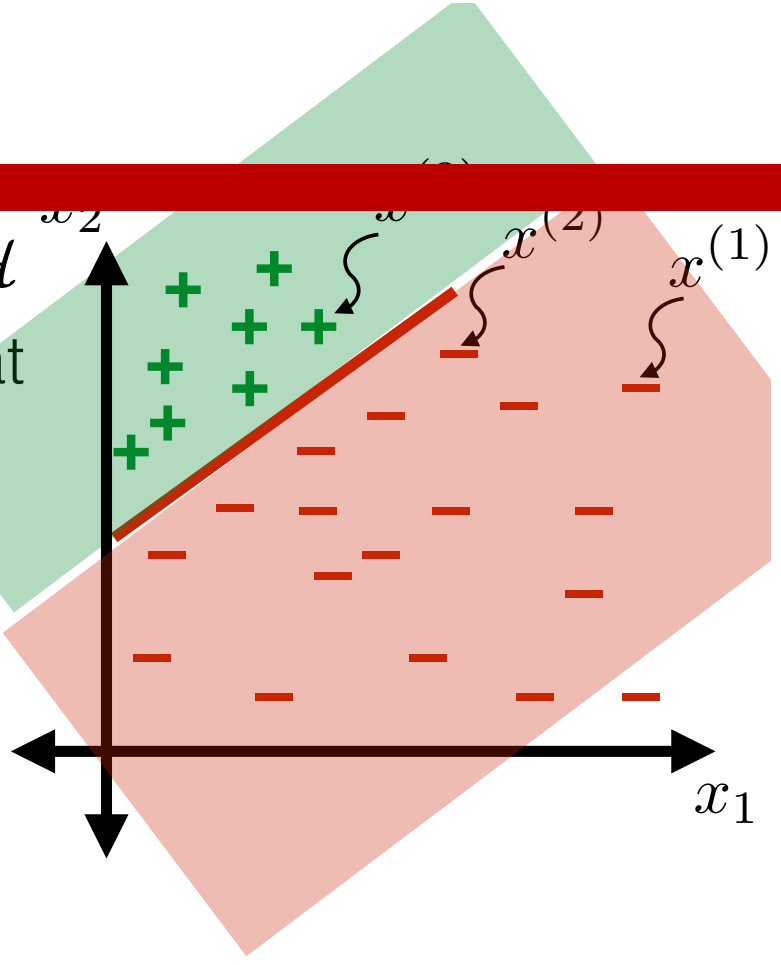

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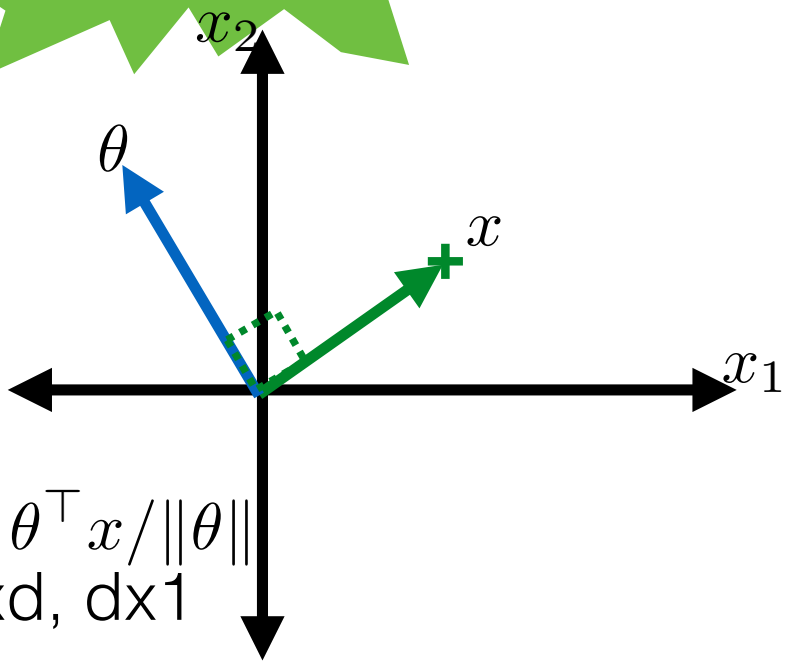

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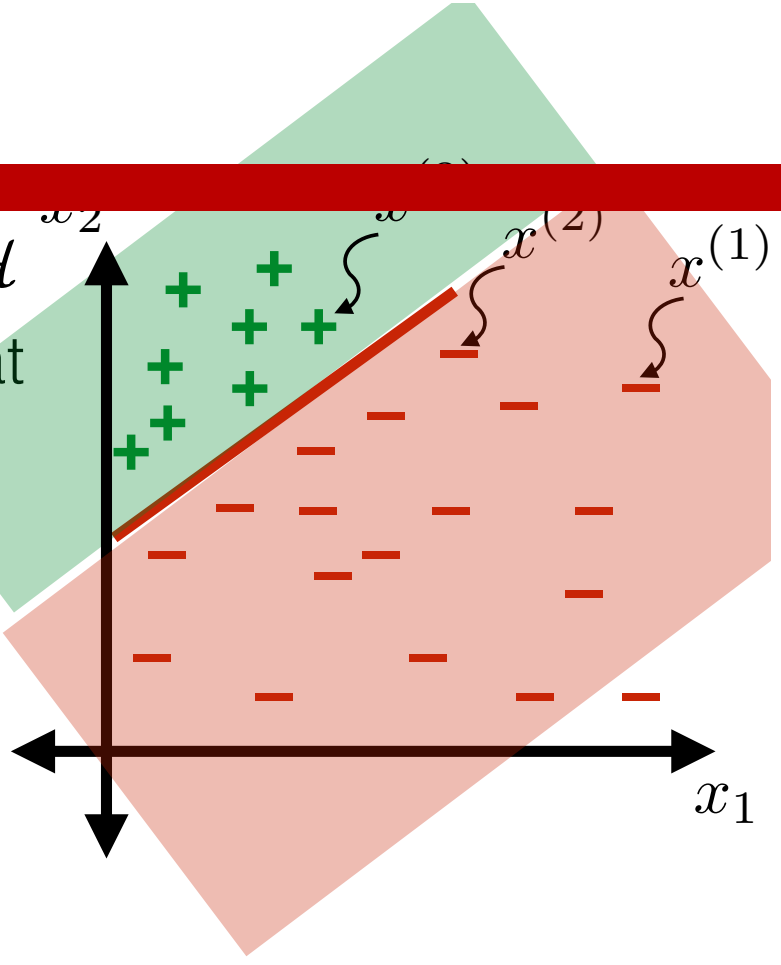

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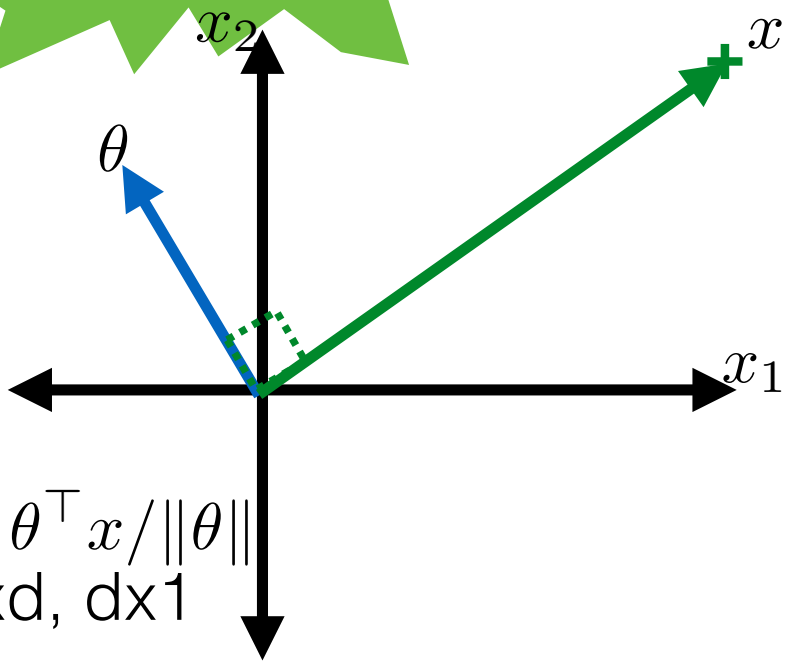

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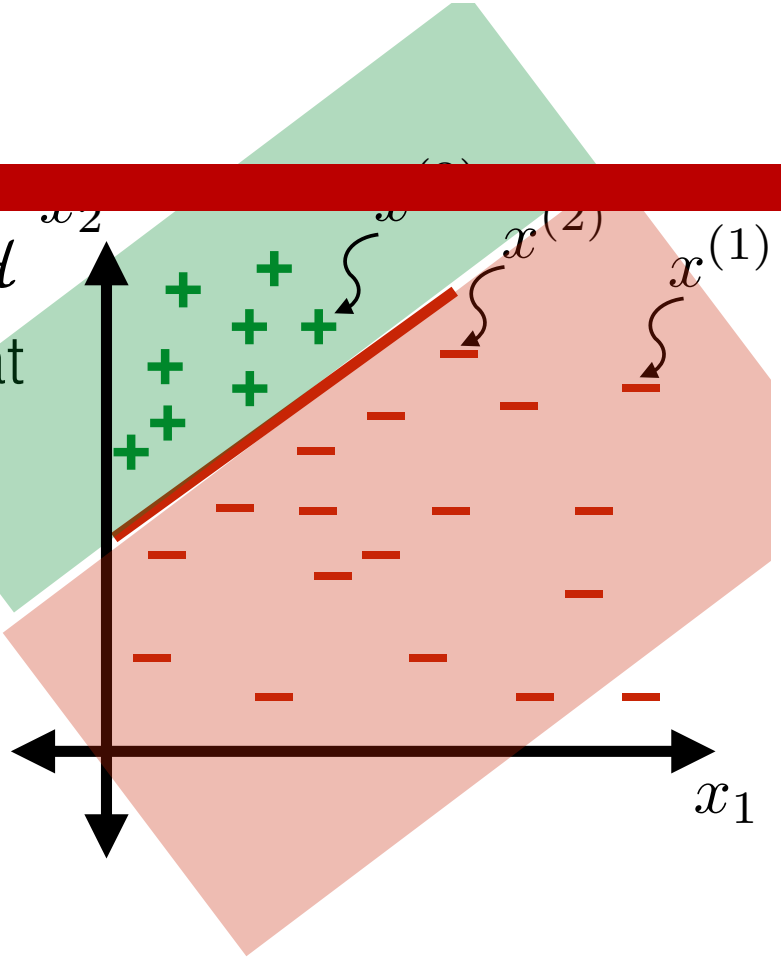

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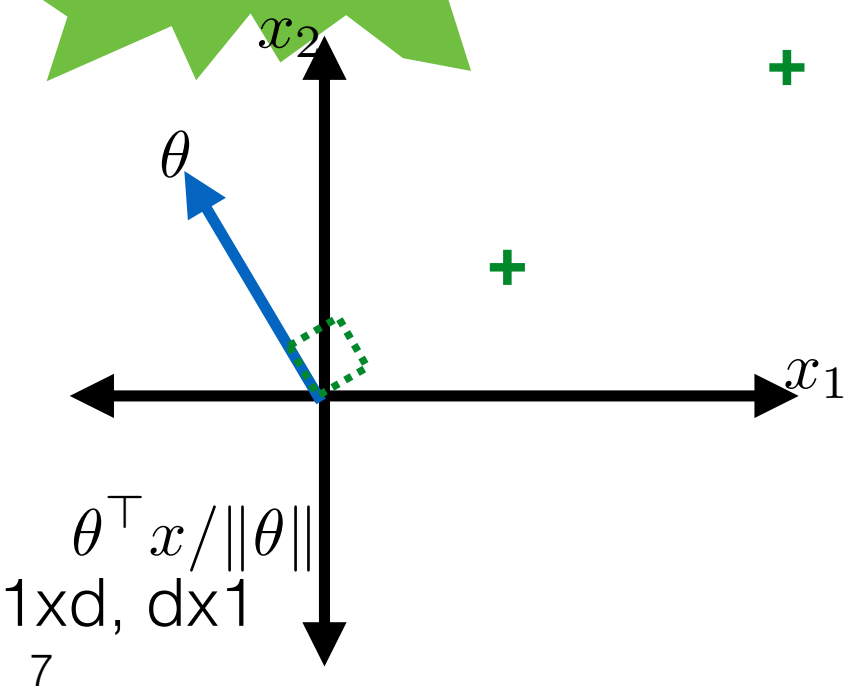

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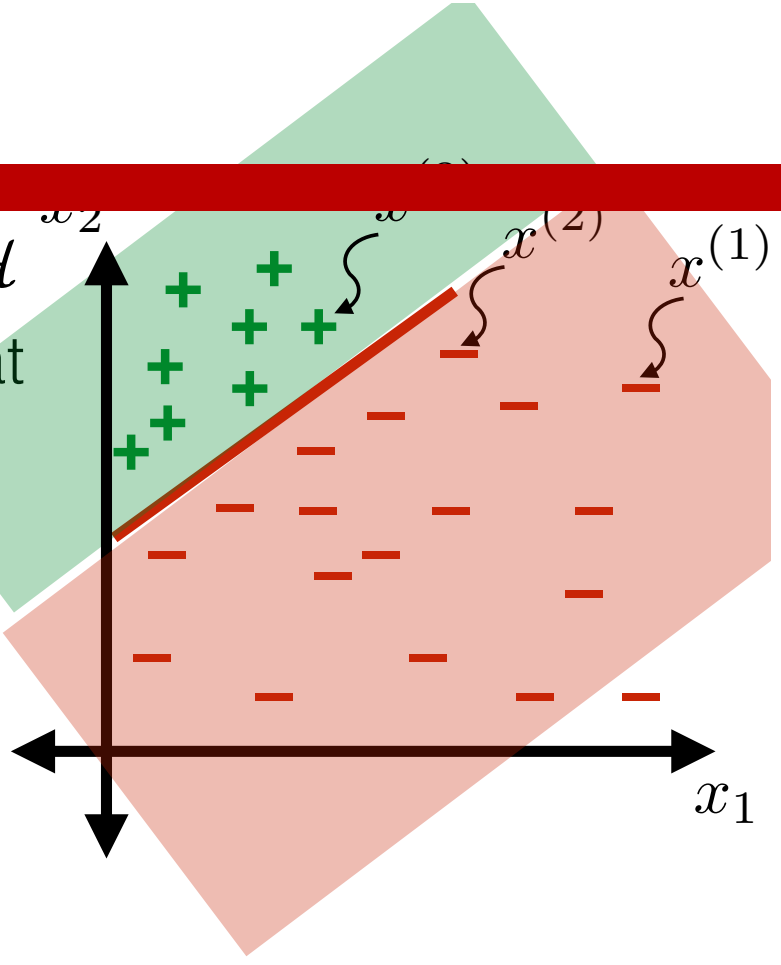

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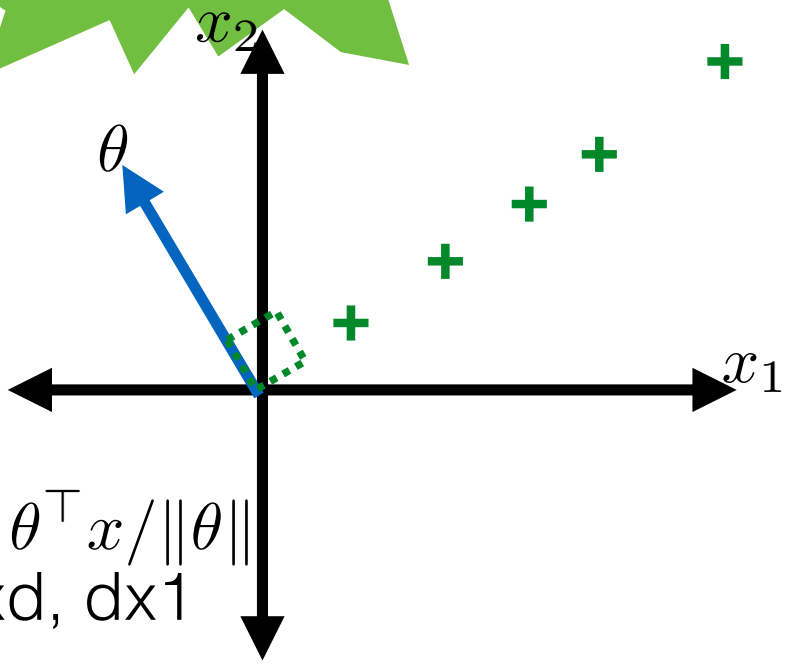

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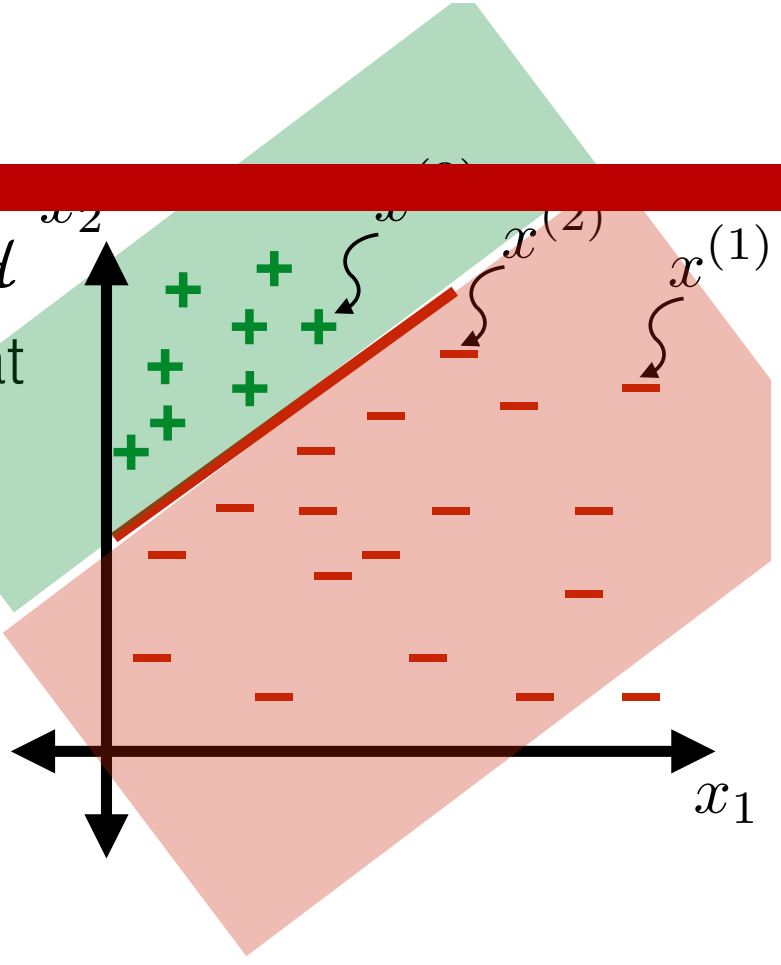

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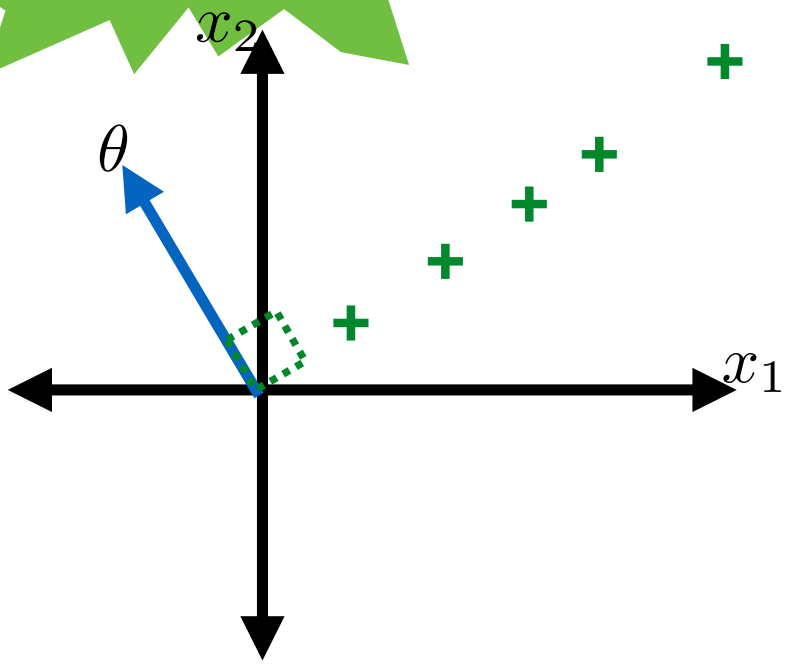

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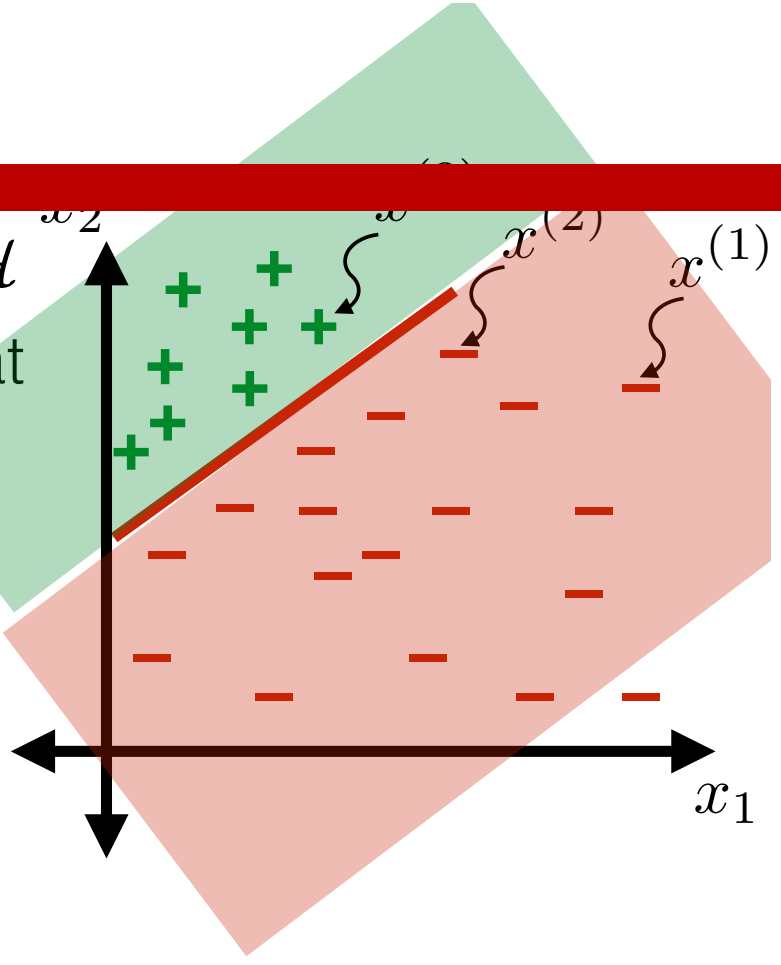

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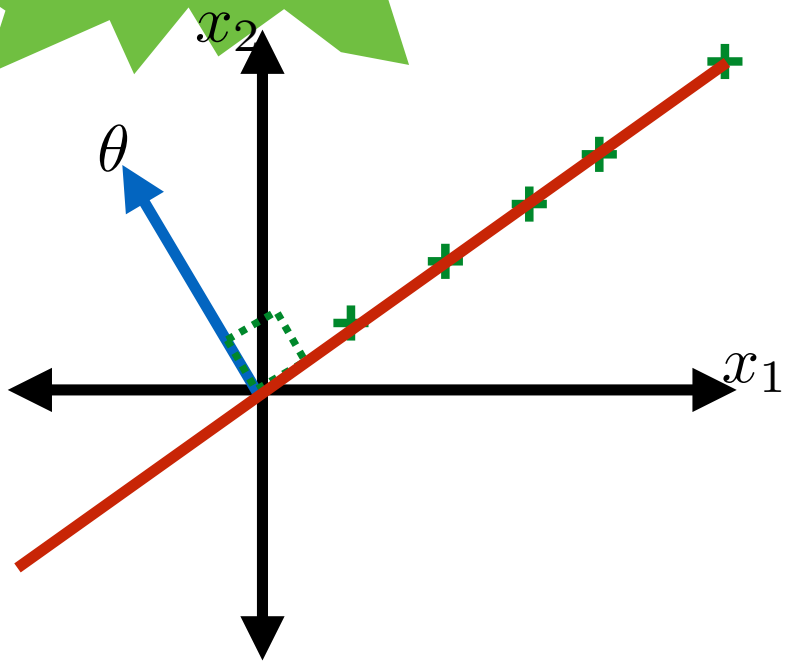

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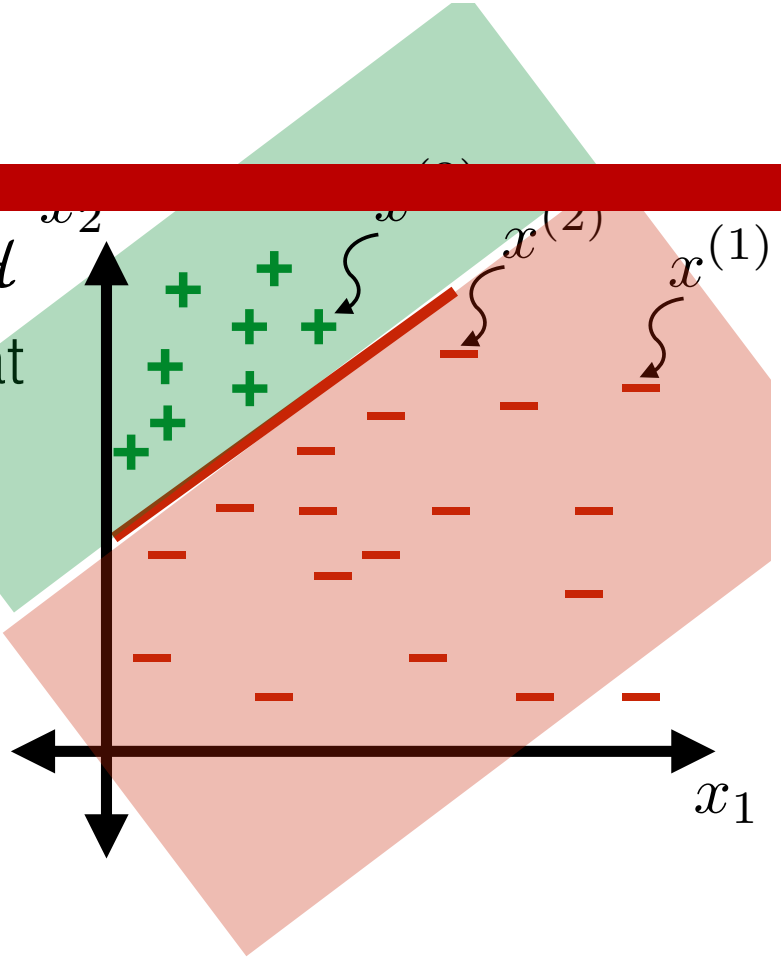

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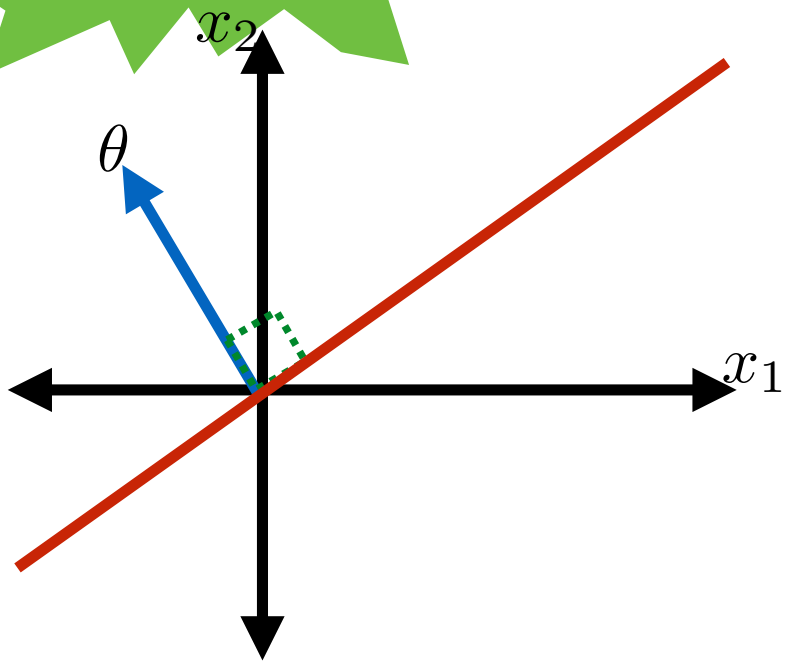

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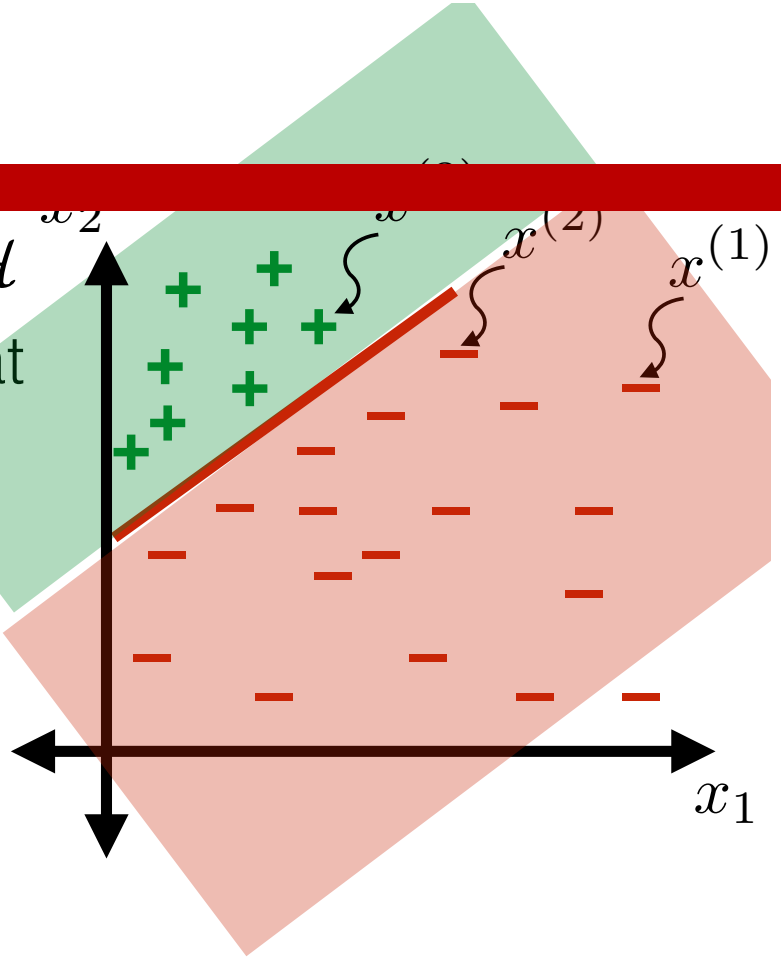

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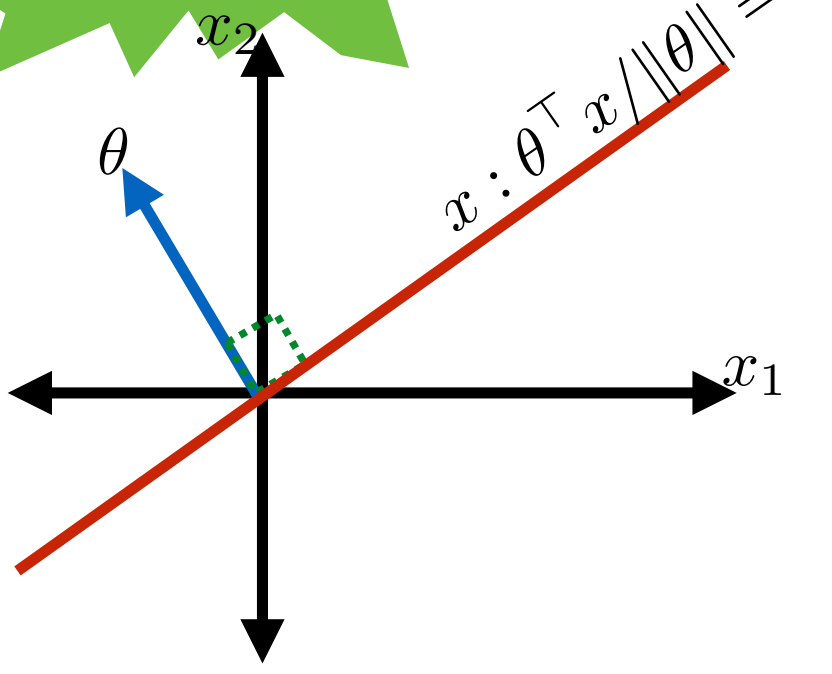

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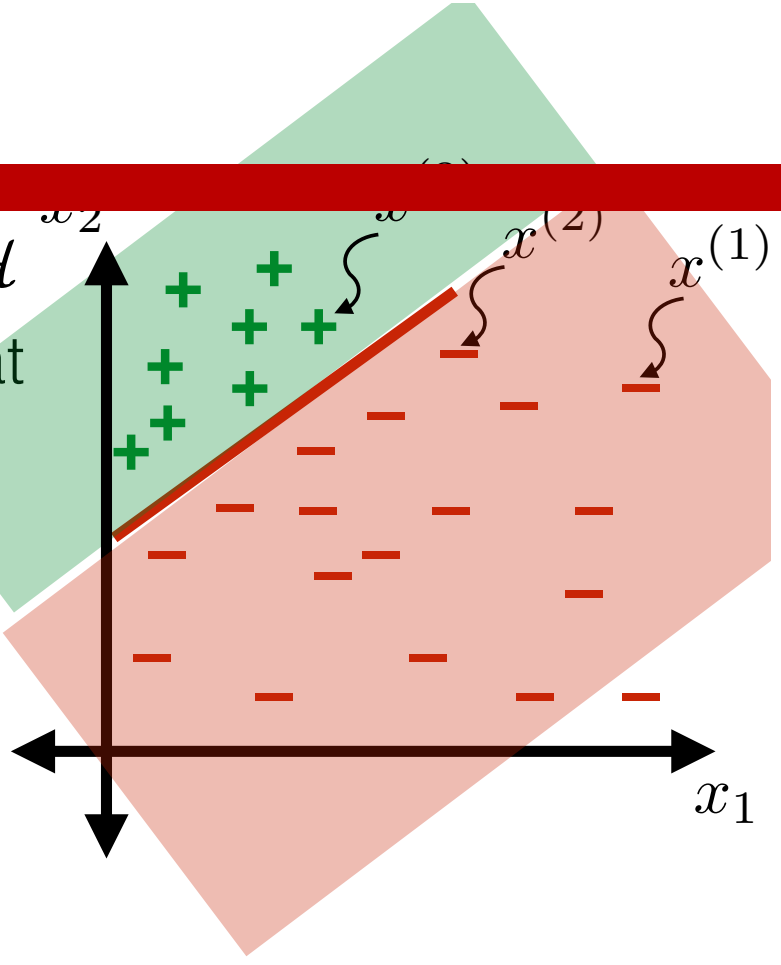

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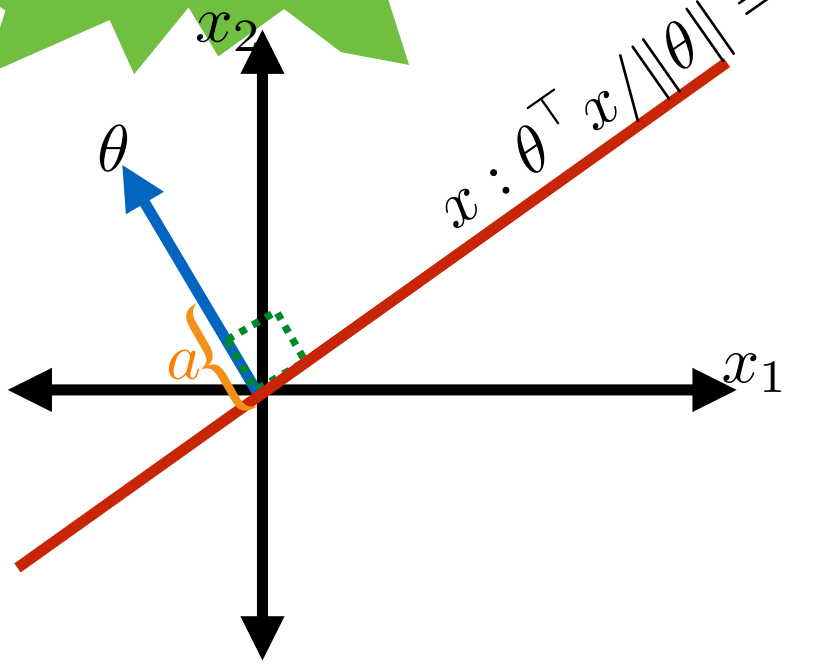

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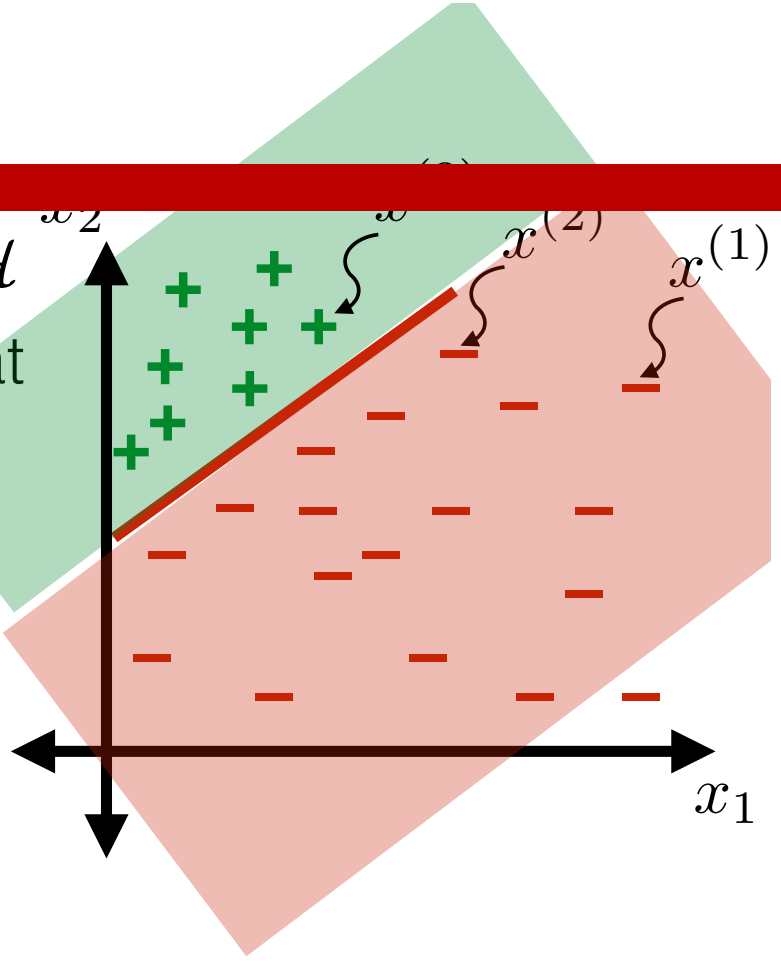

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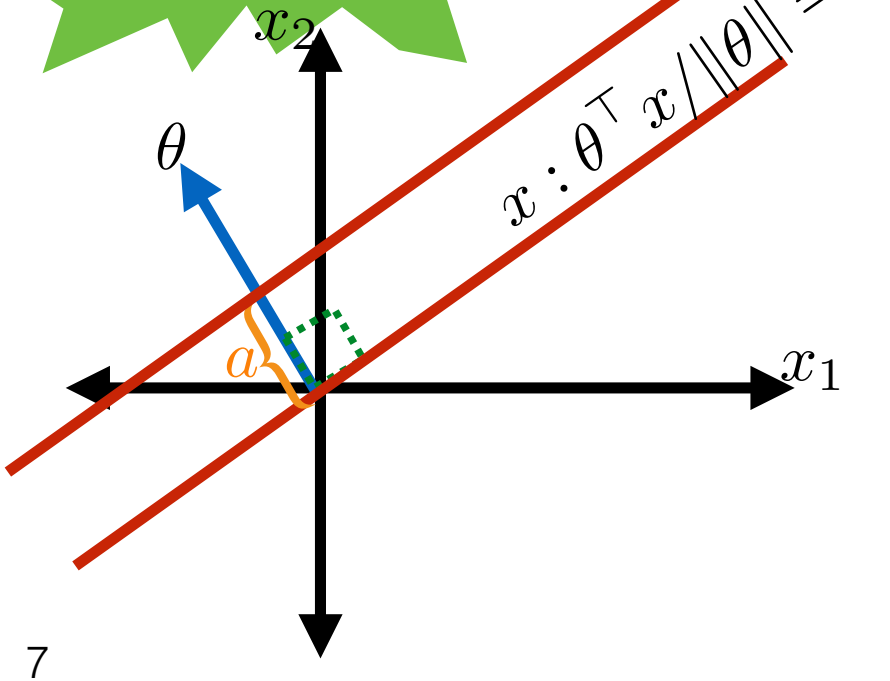

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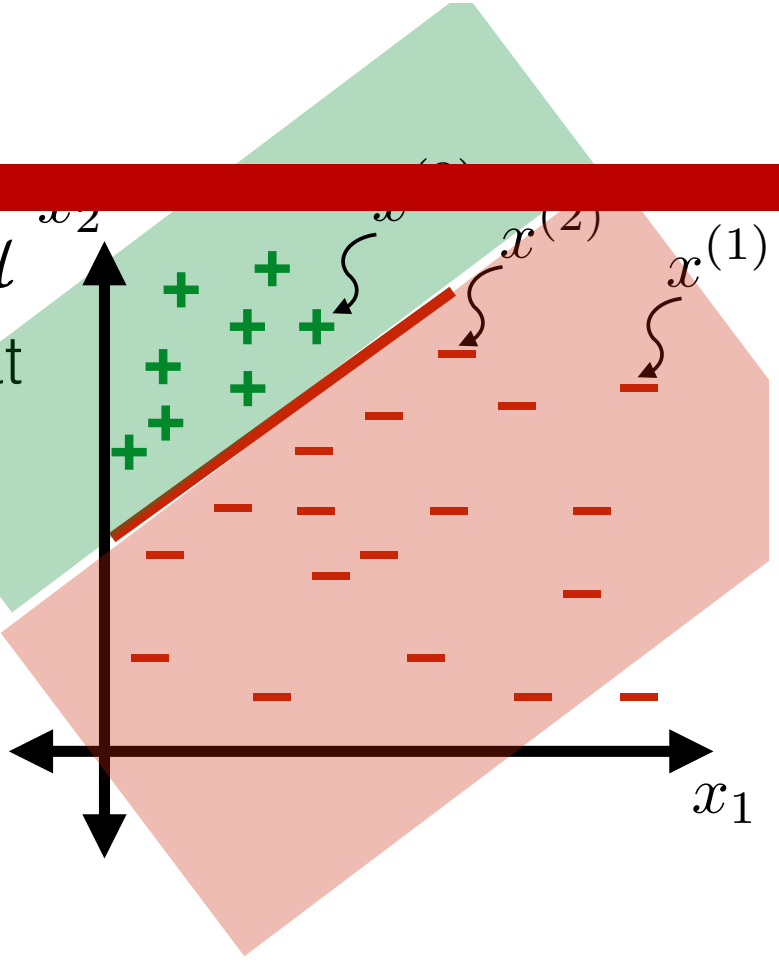

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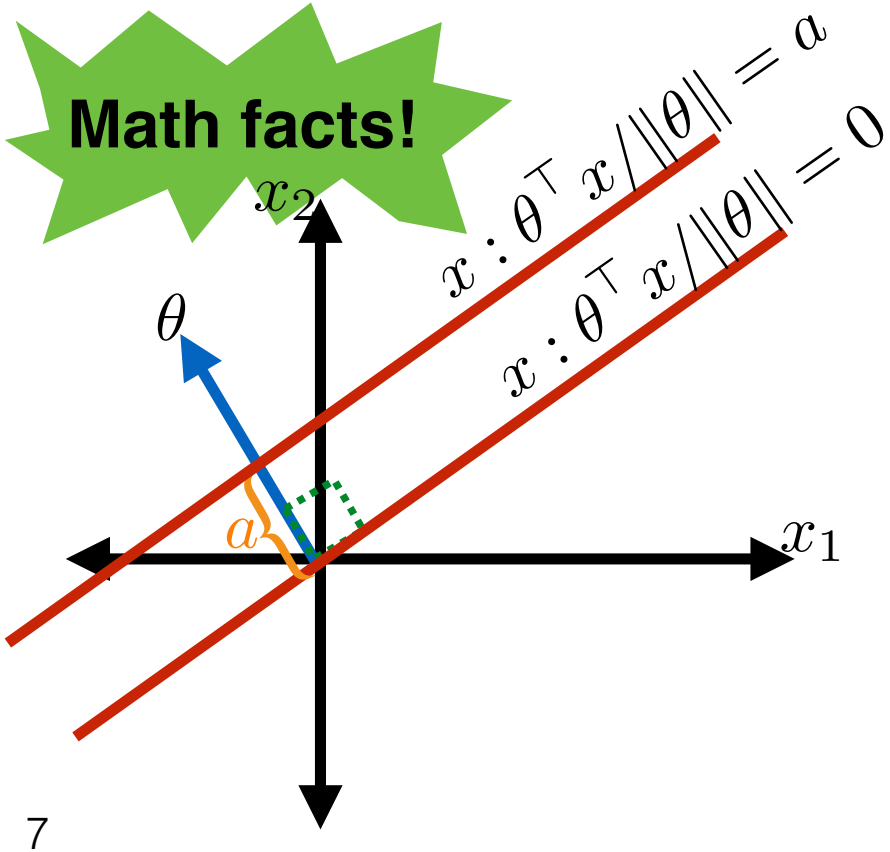

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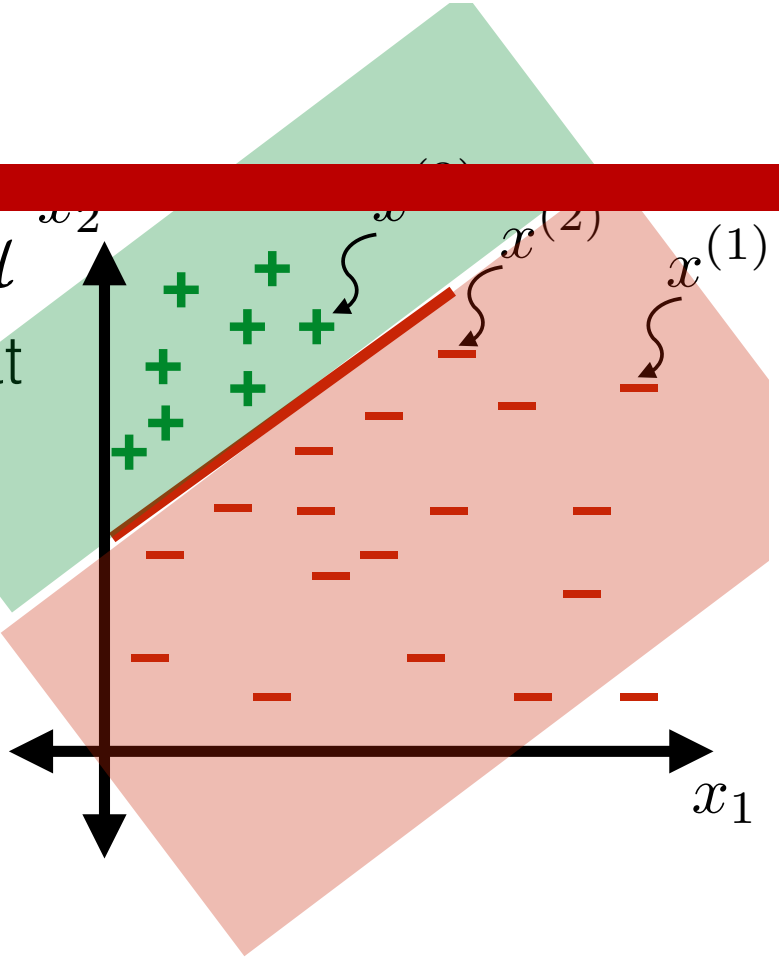

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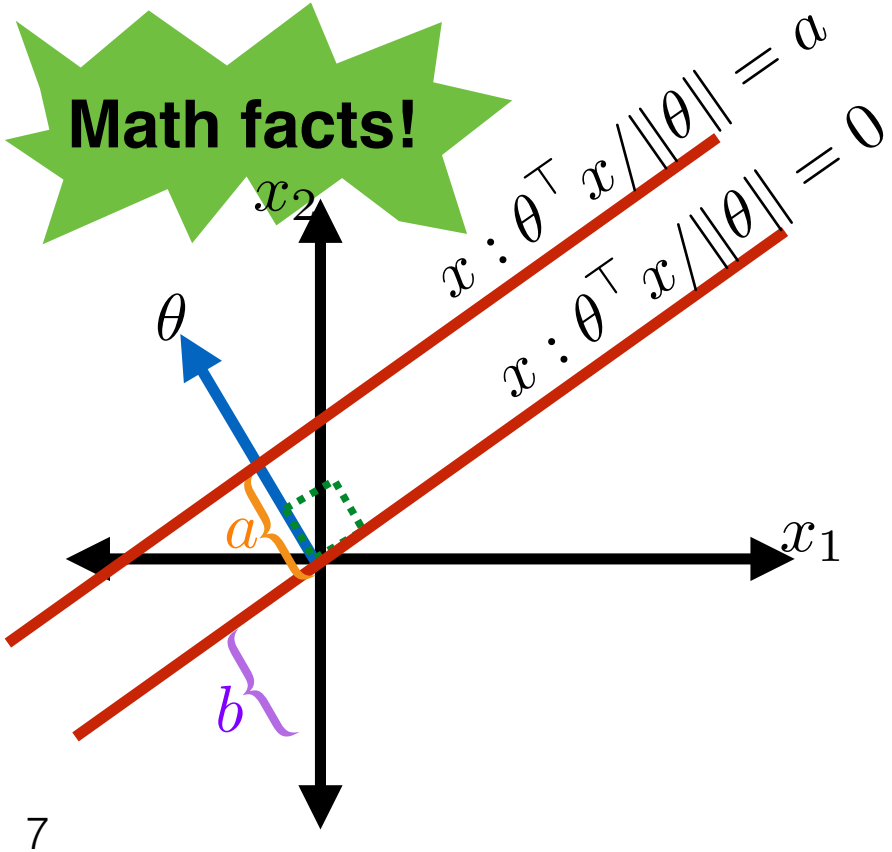

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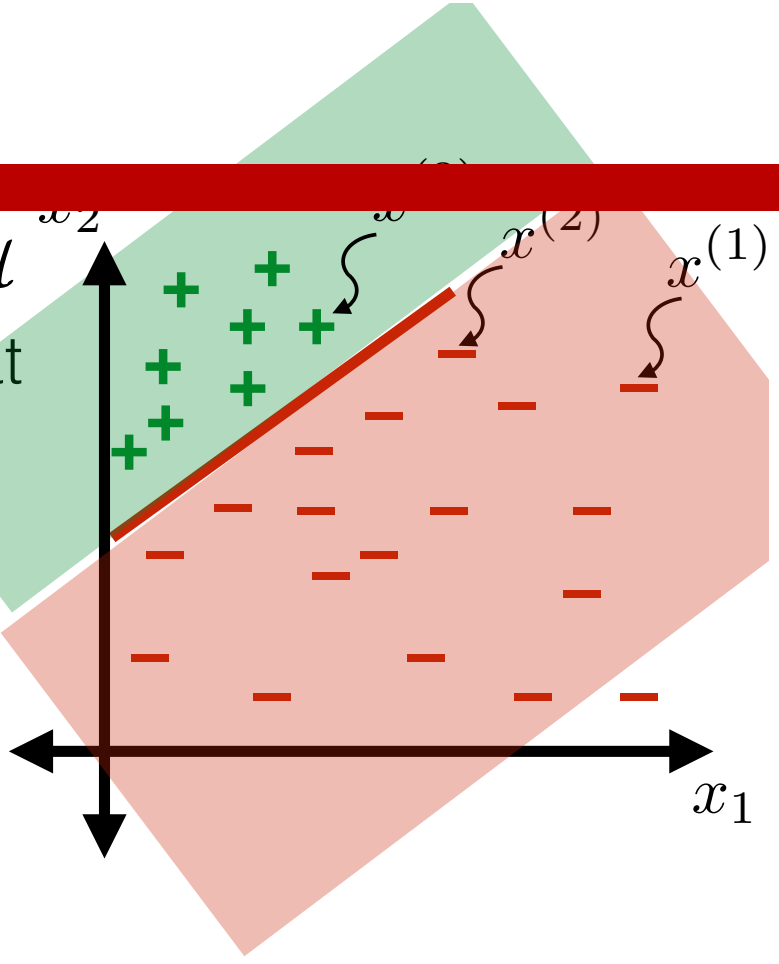

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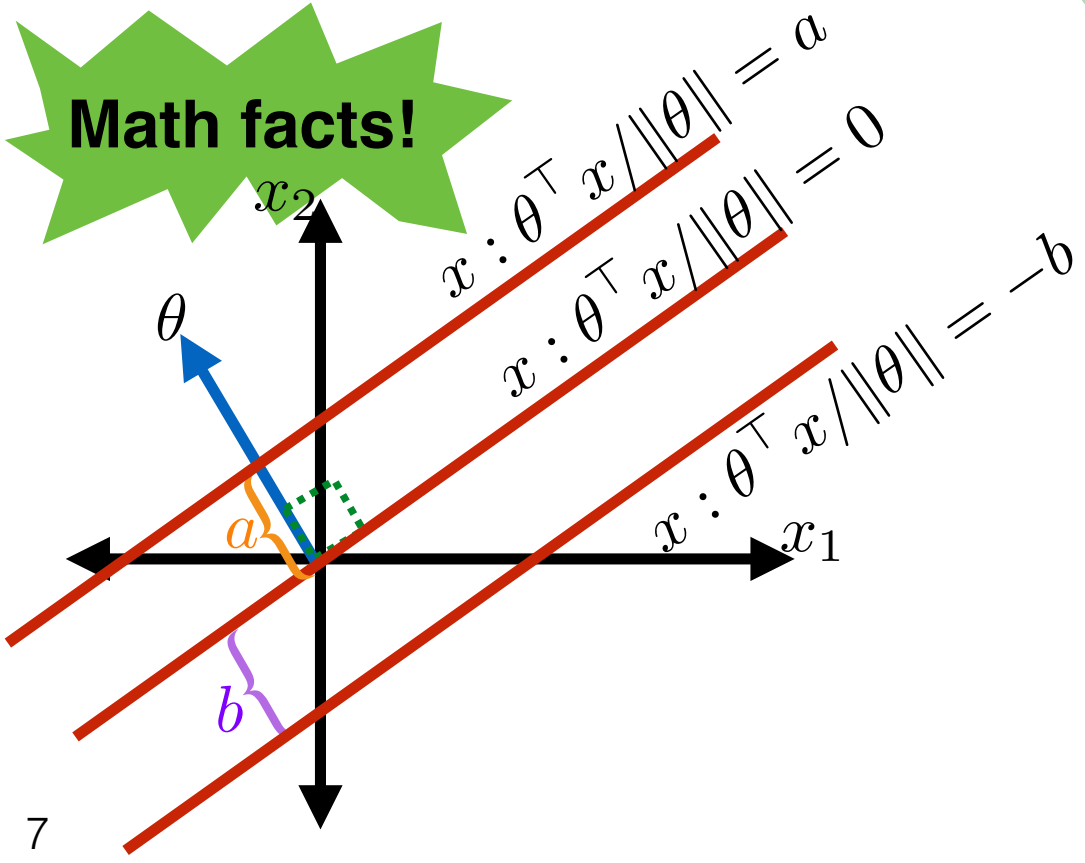

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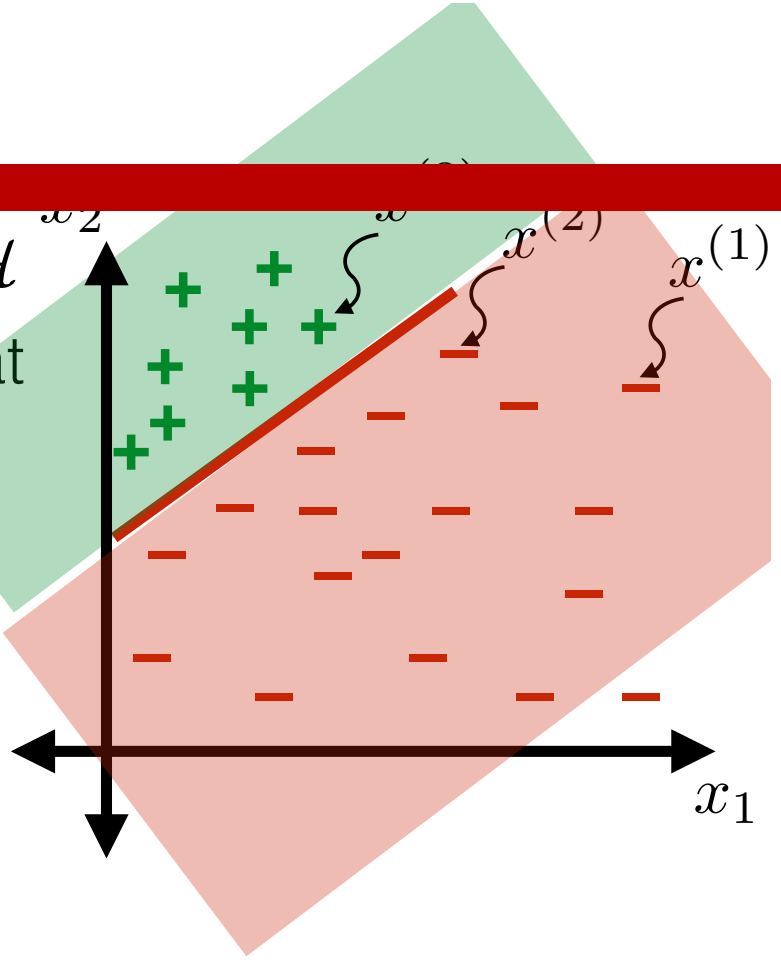

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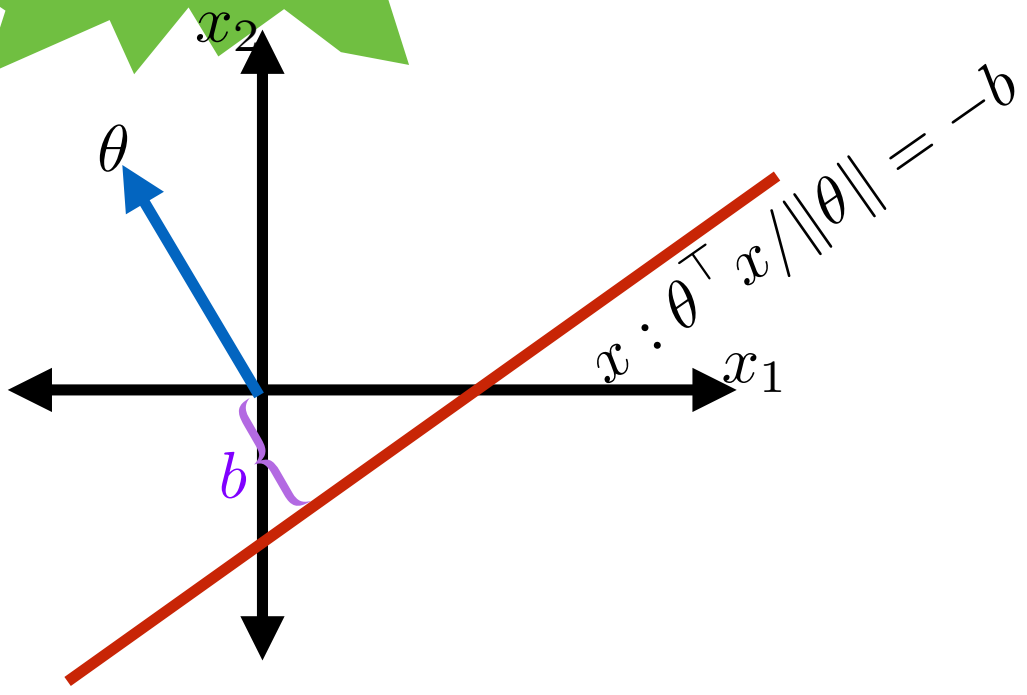

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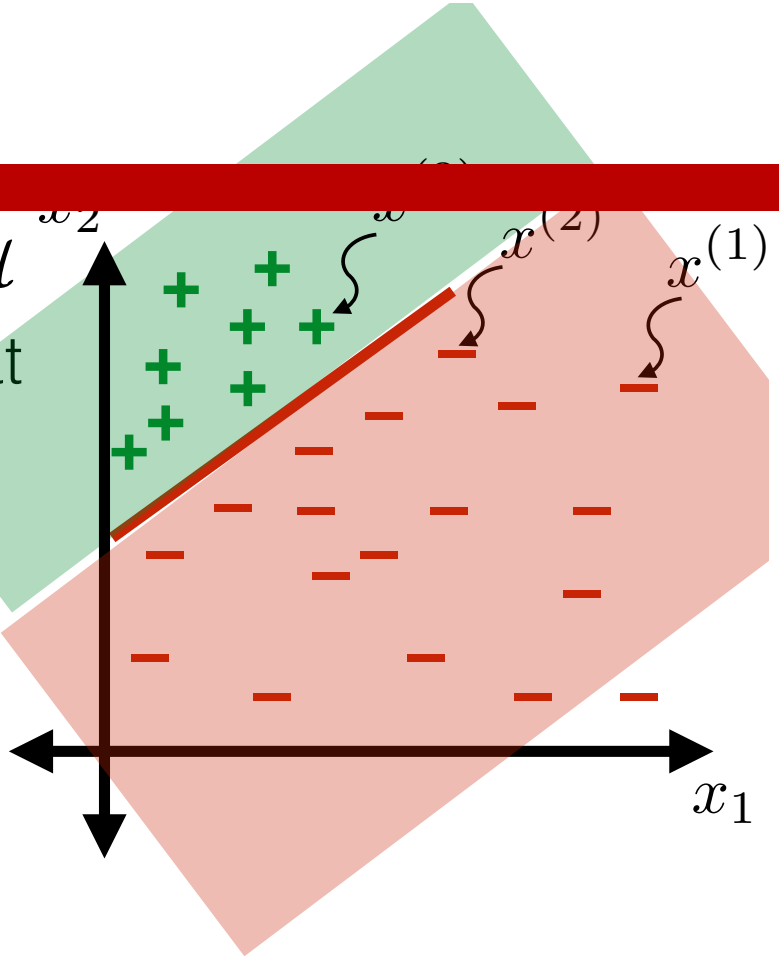

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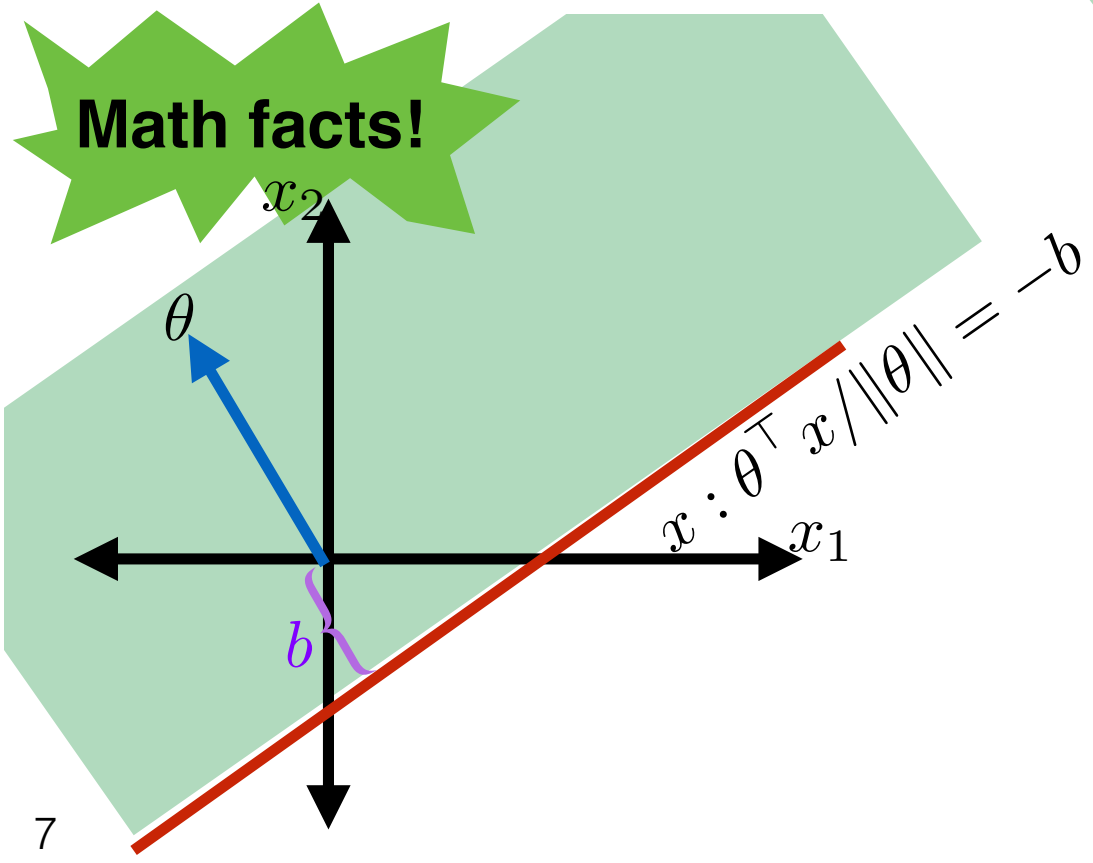

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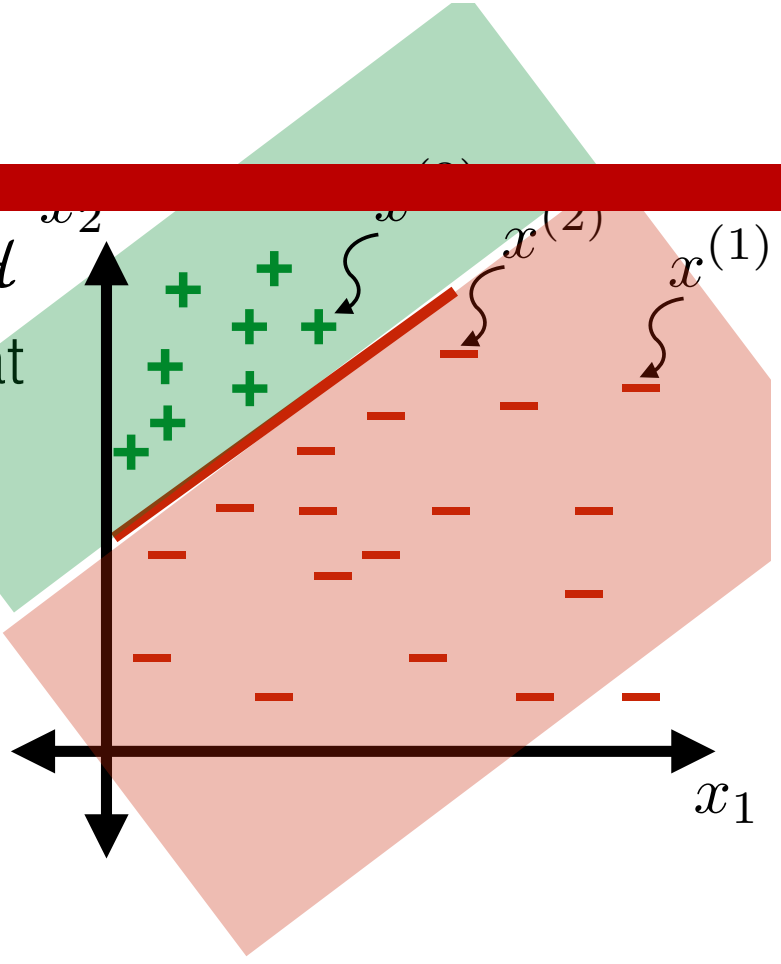

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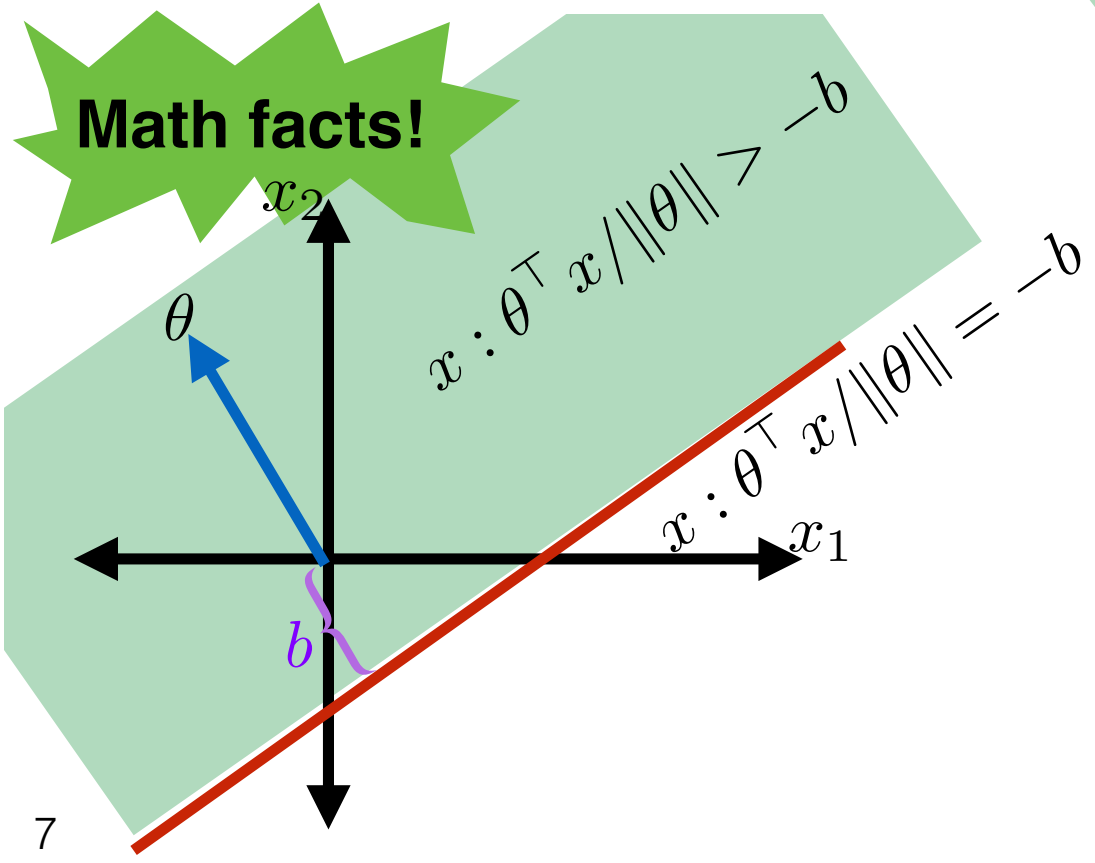

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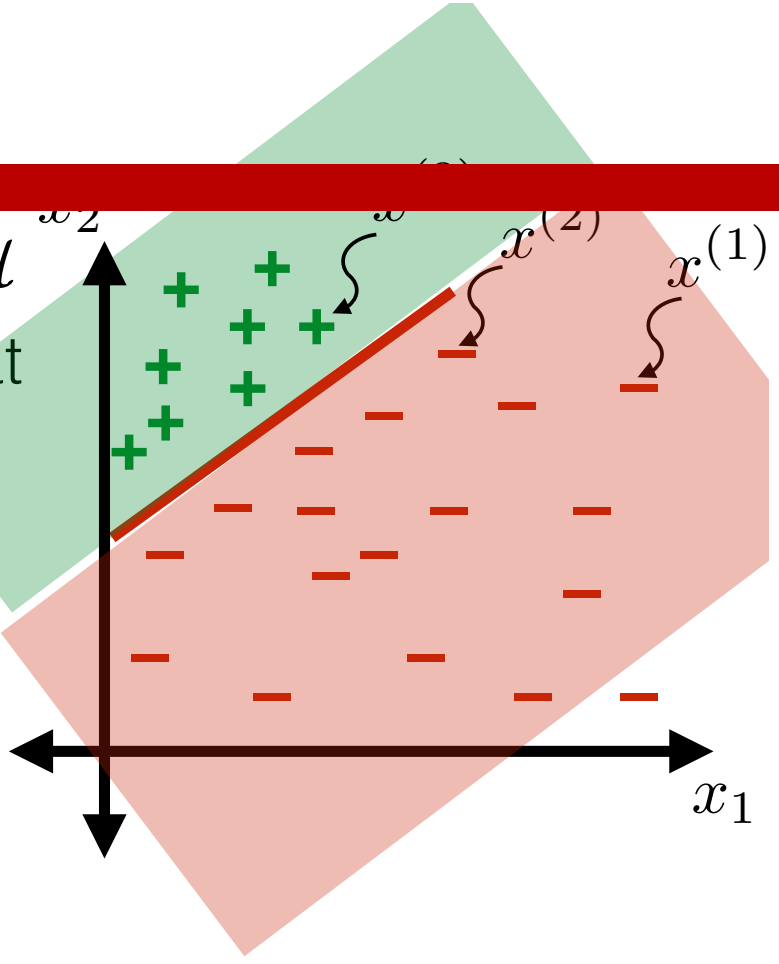

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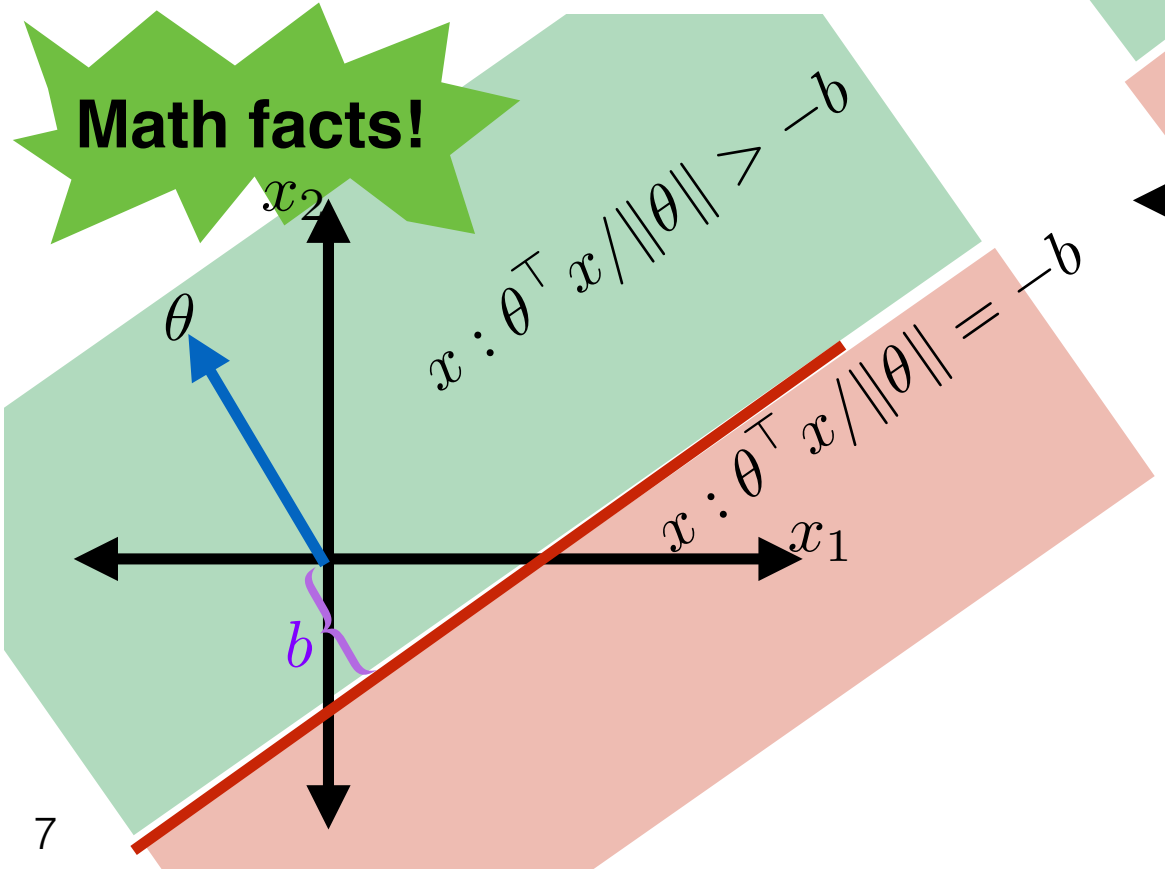

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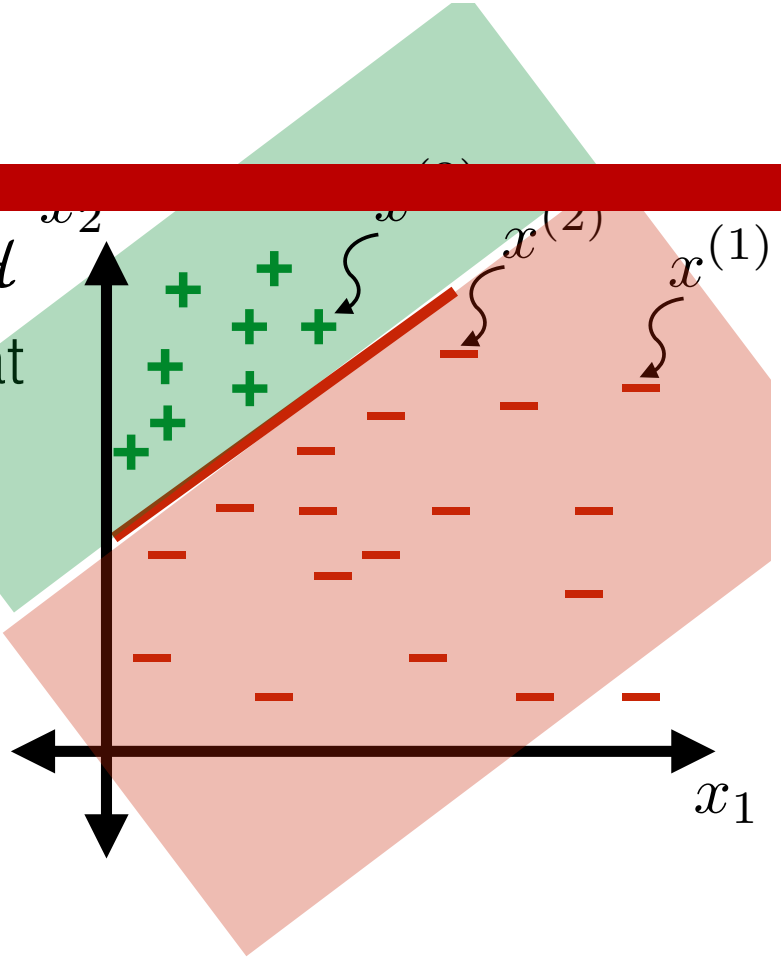

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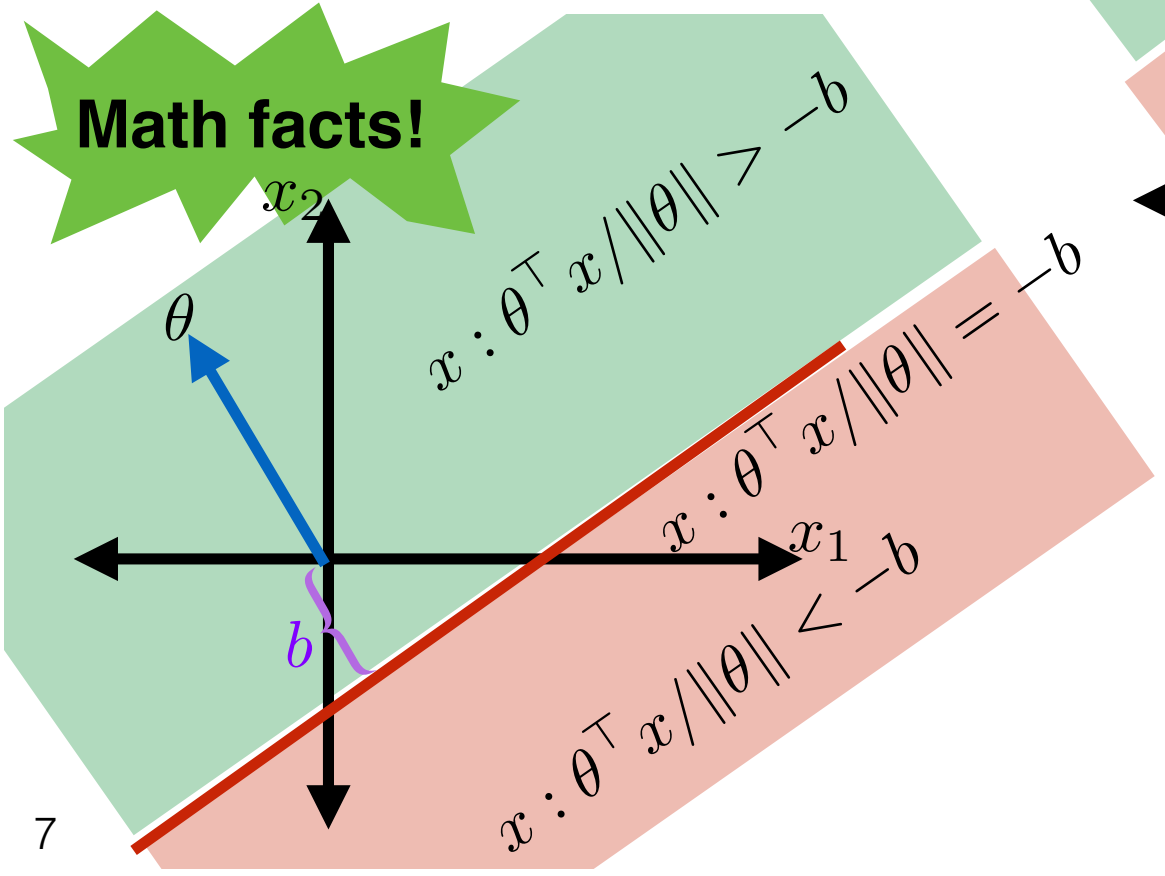

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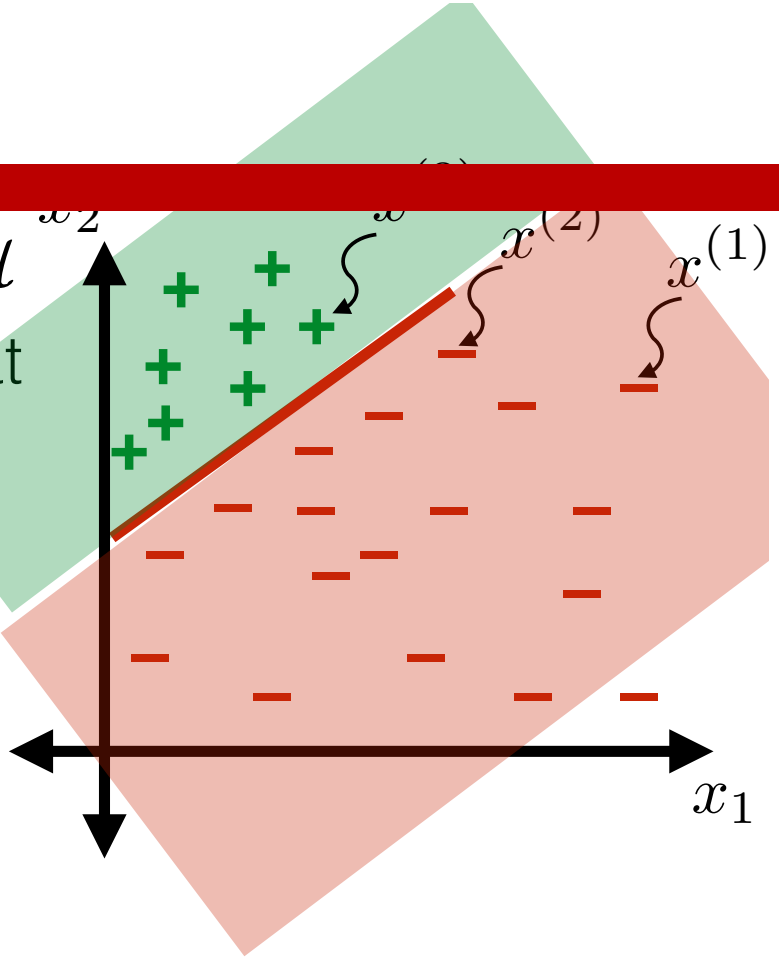

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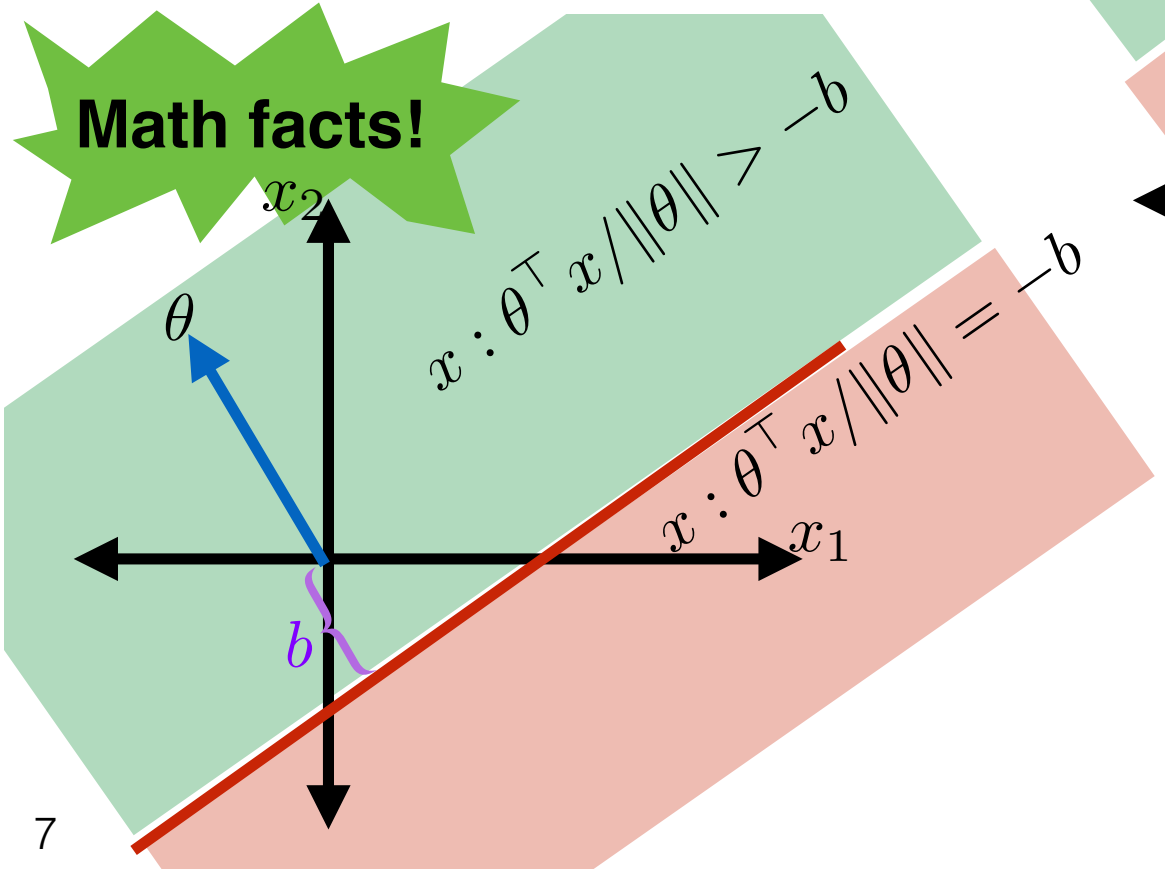

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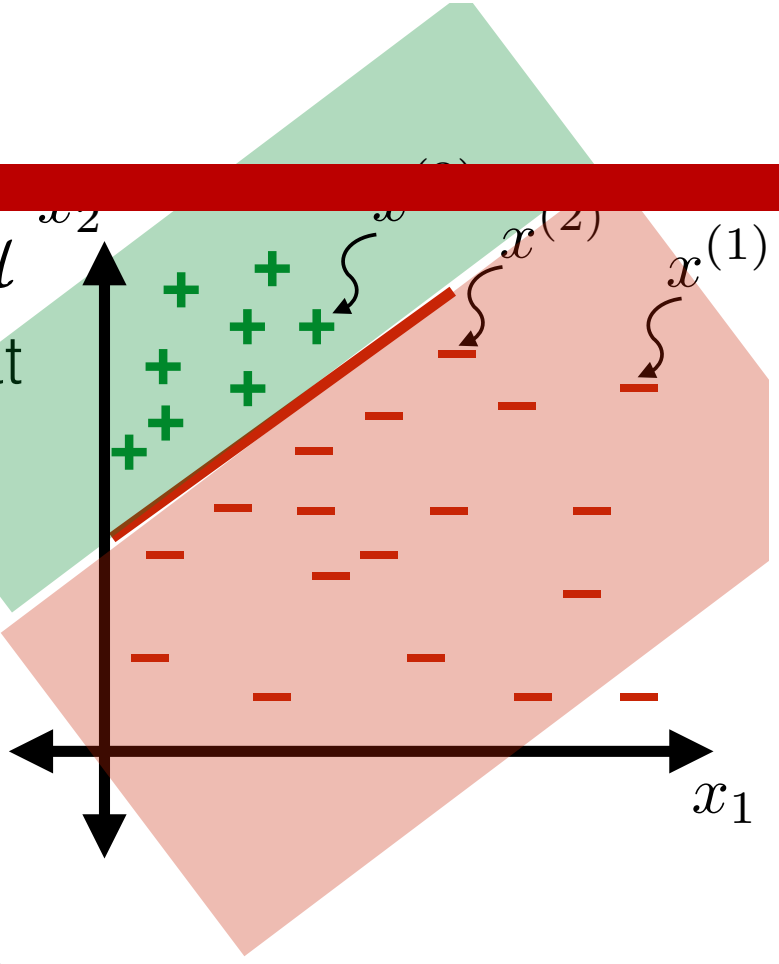

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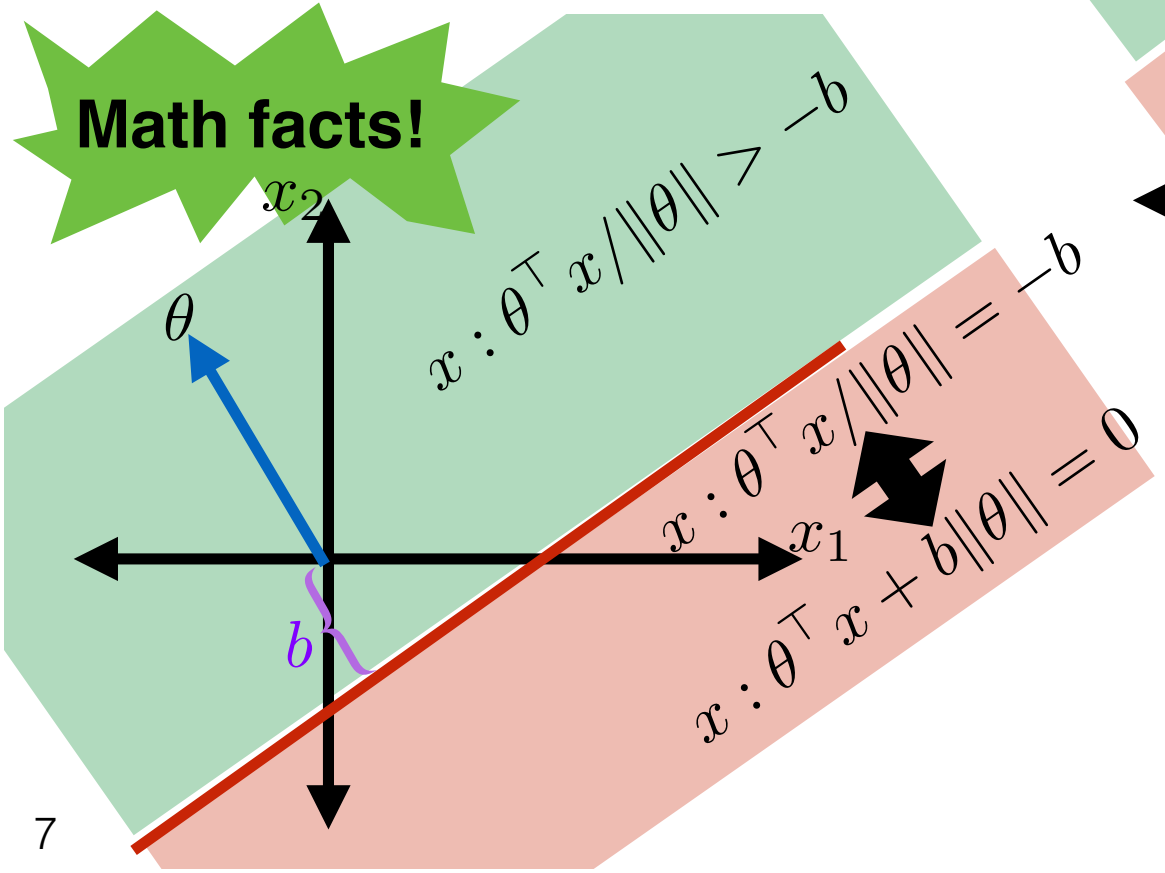

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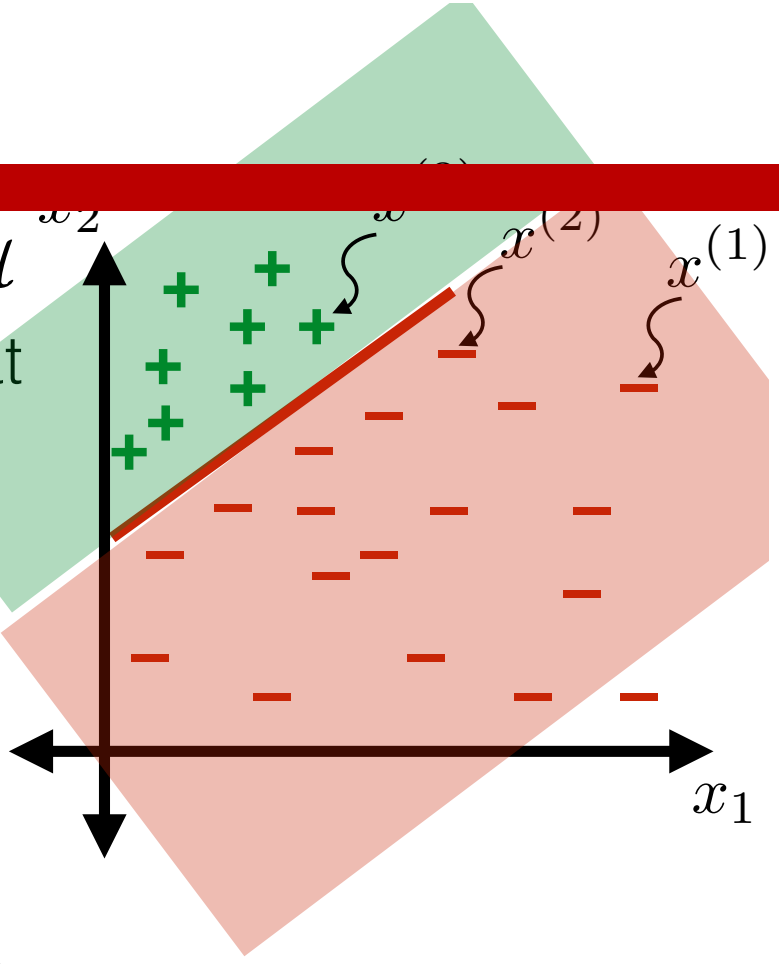

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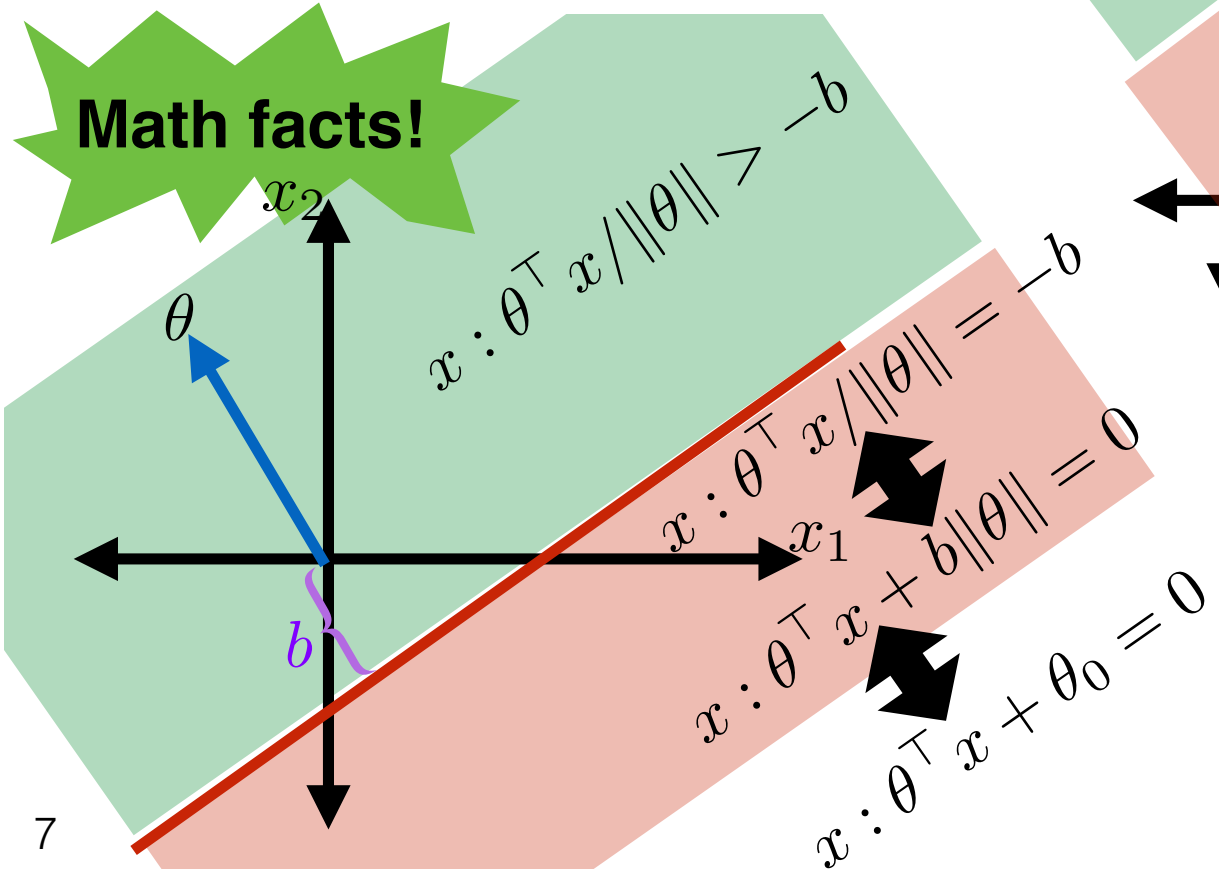

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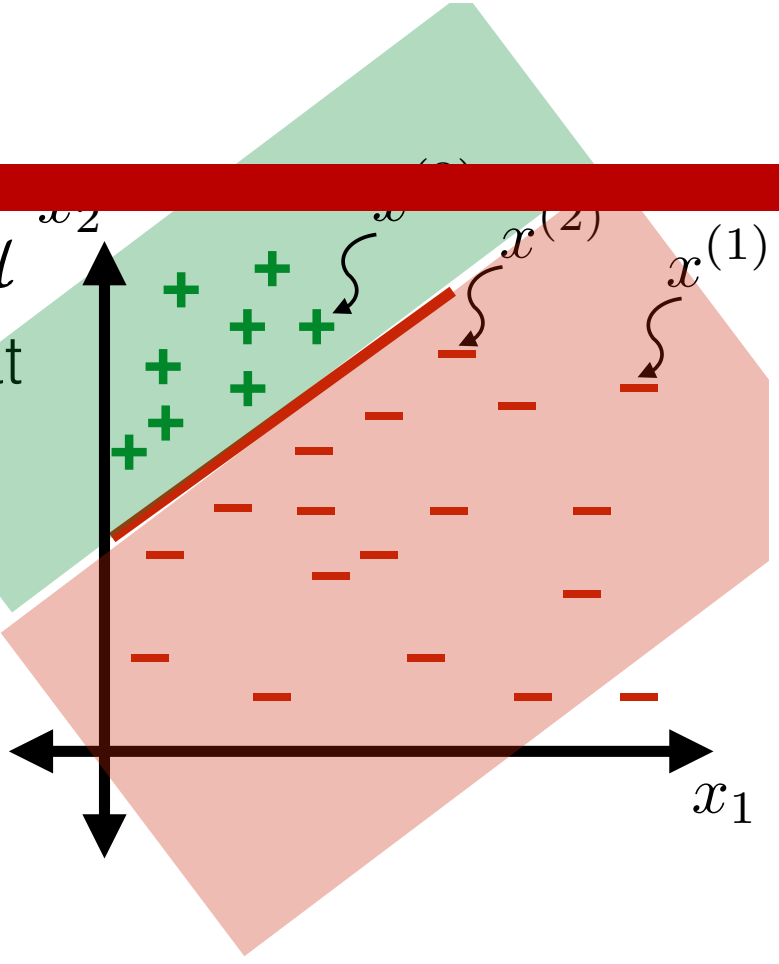

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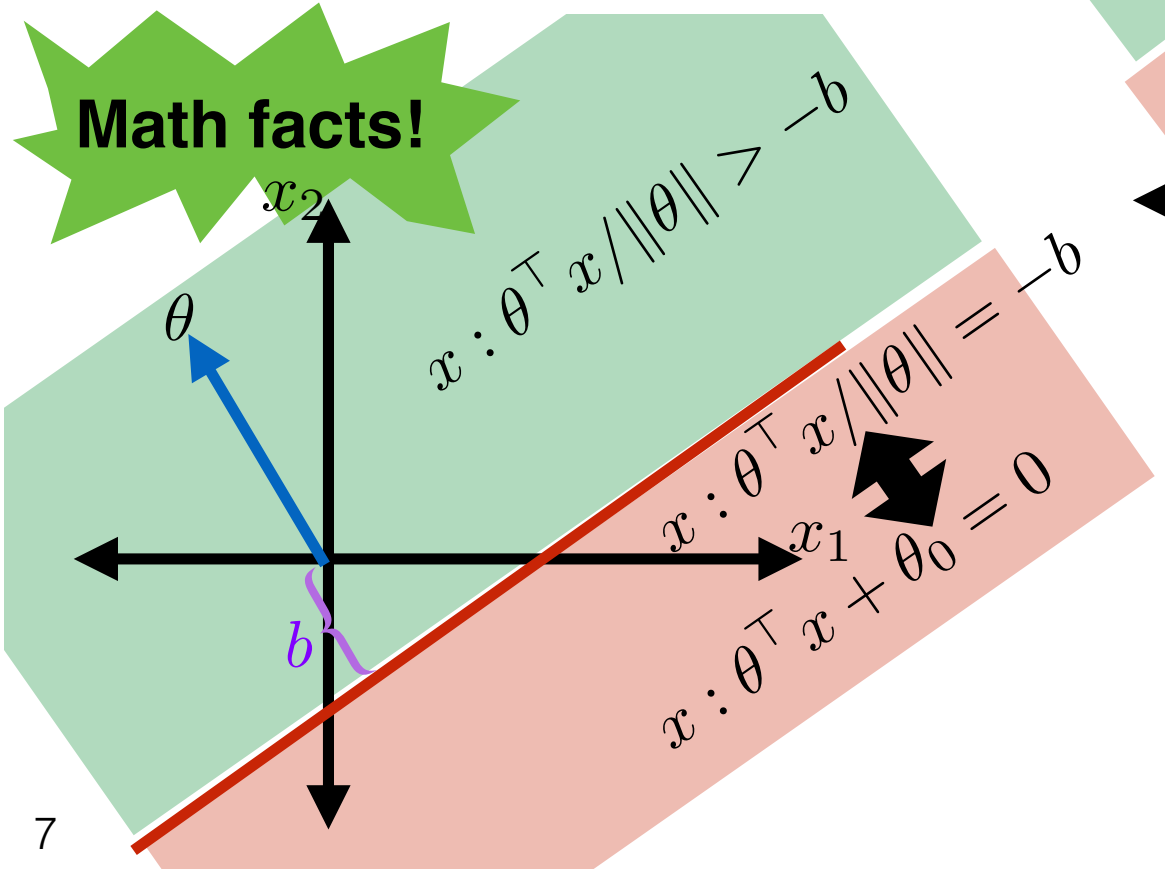

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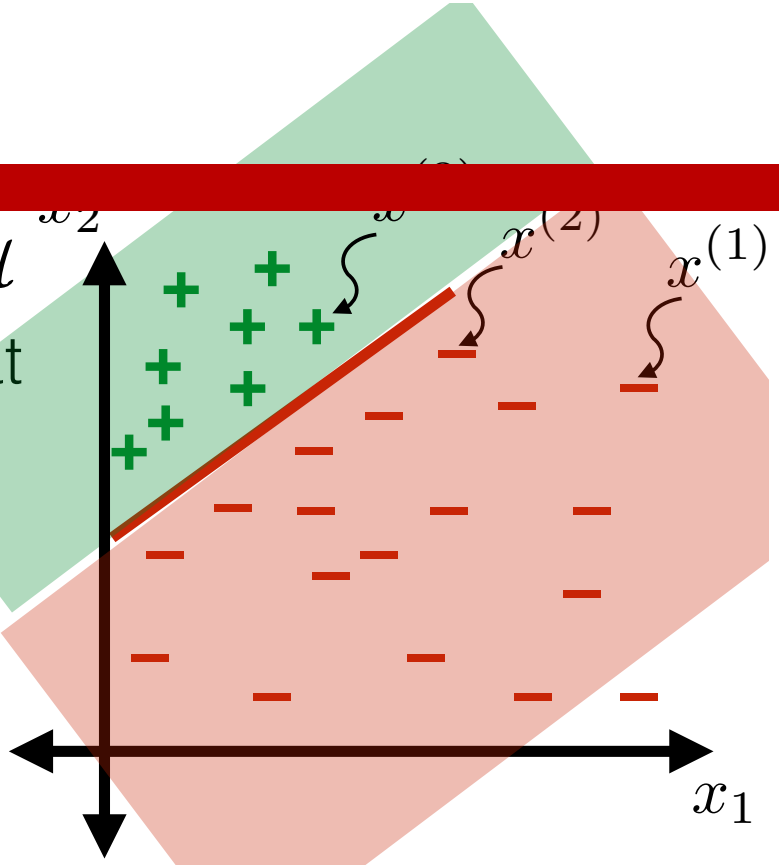

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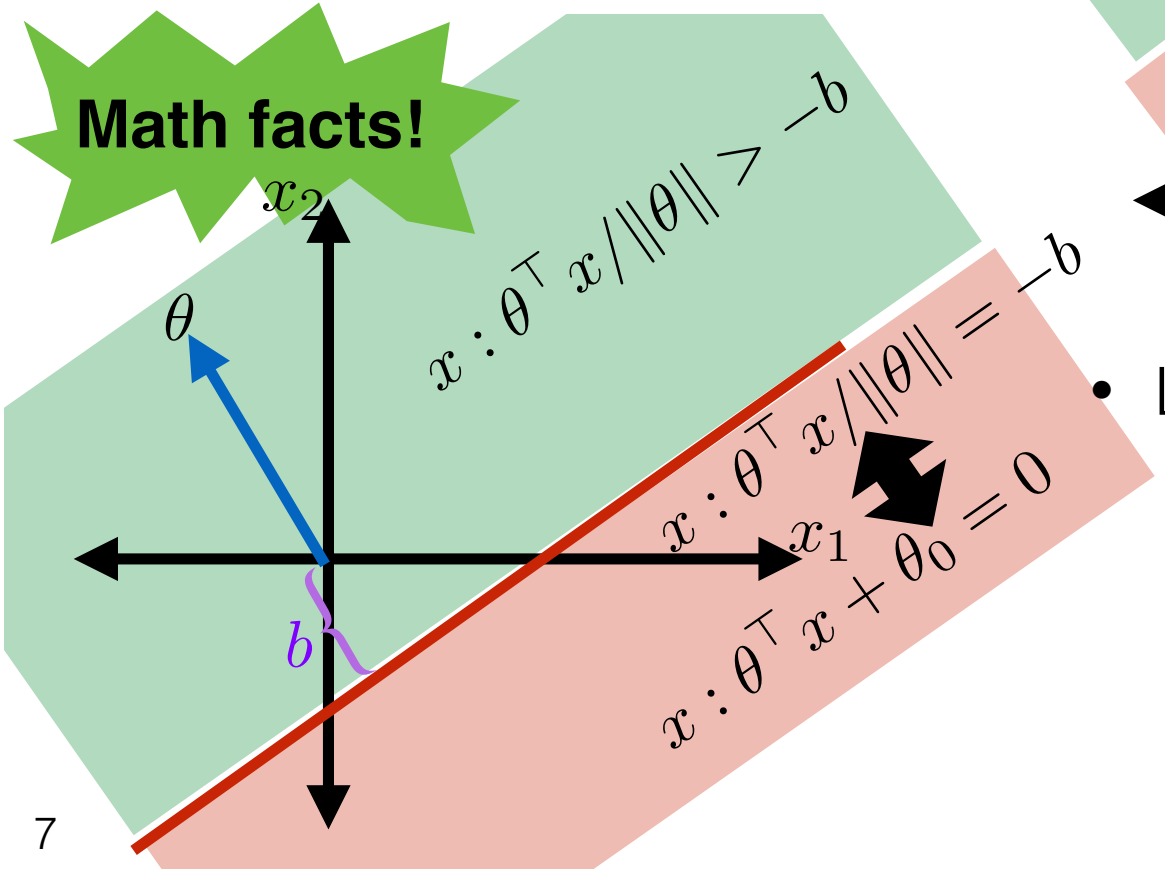

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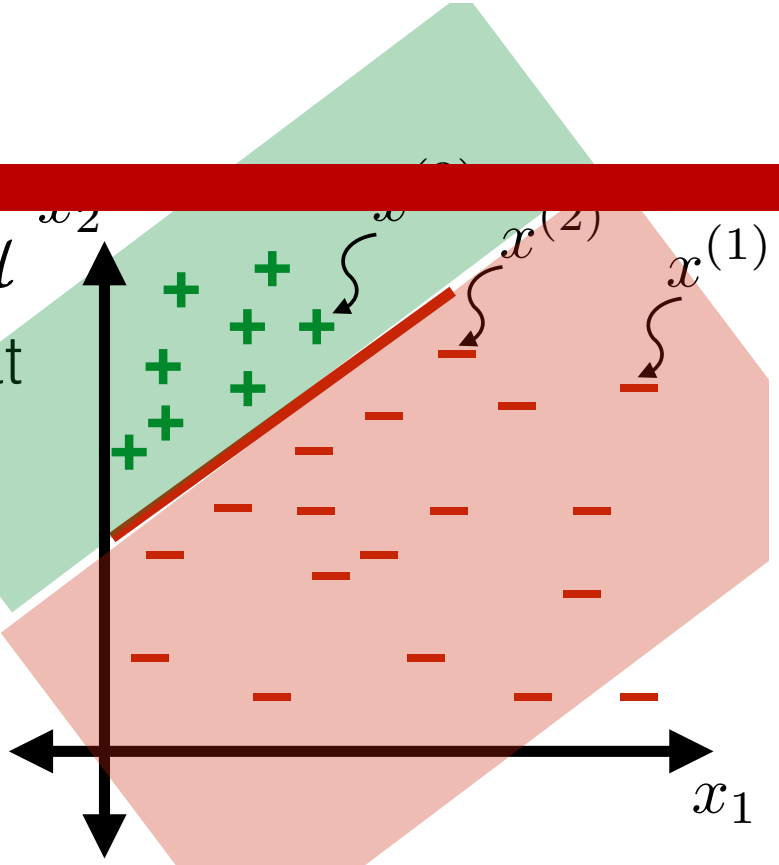

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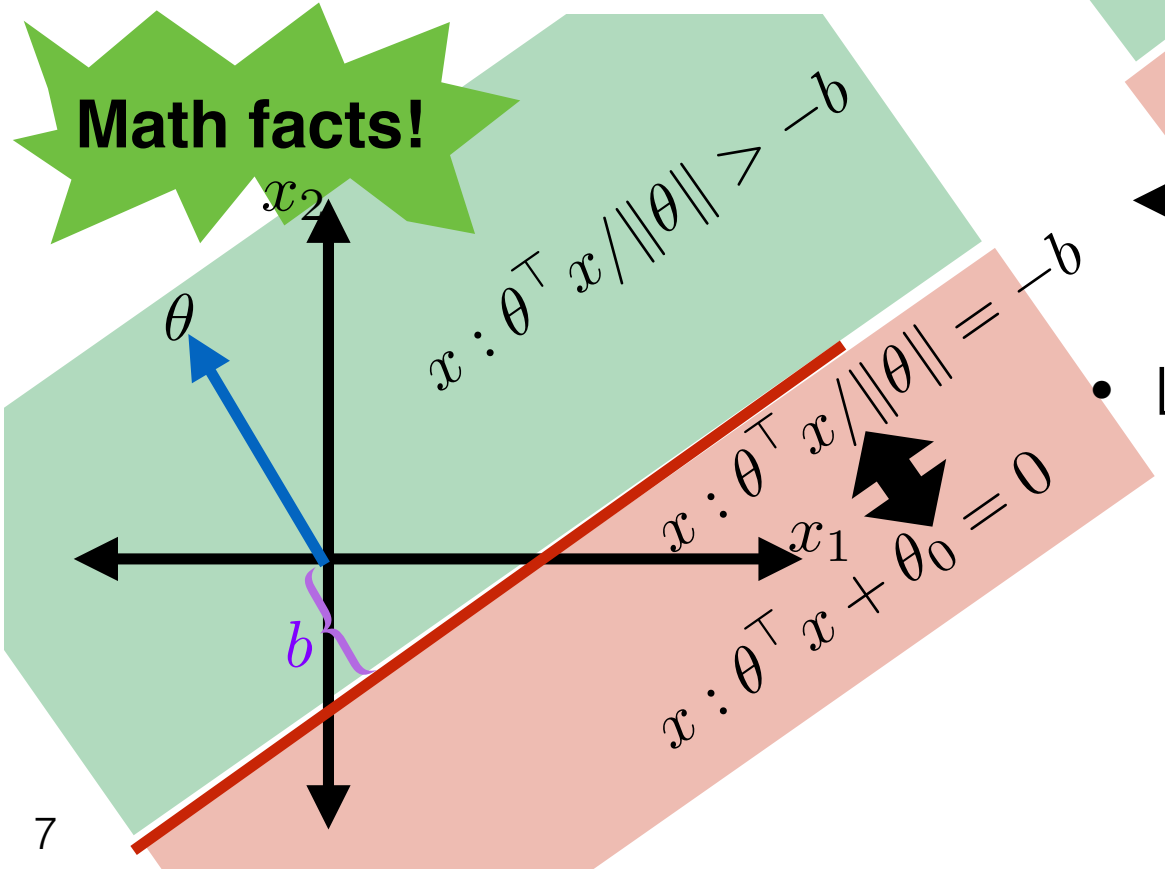

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$$h(x) = \text{sign}(\theta^\top x + \theta_0)$$

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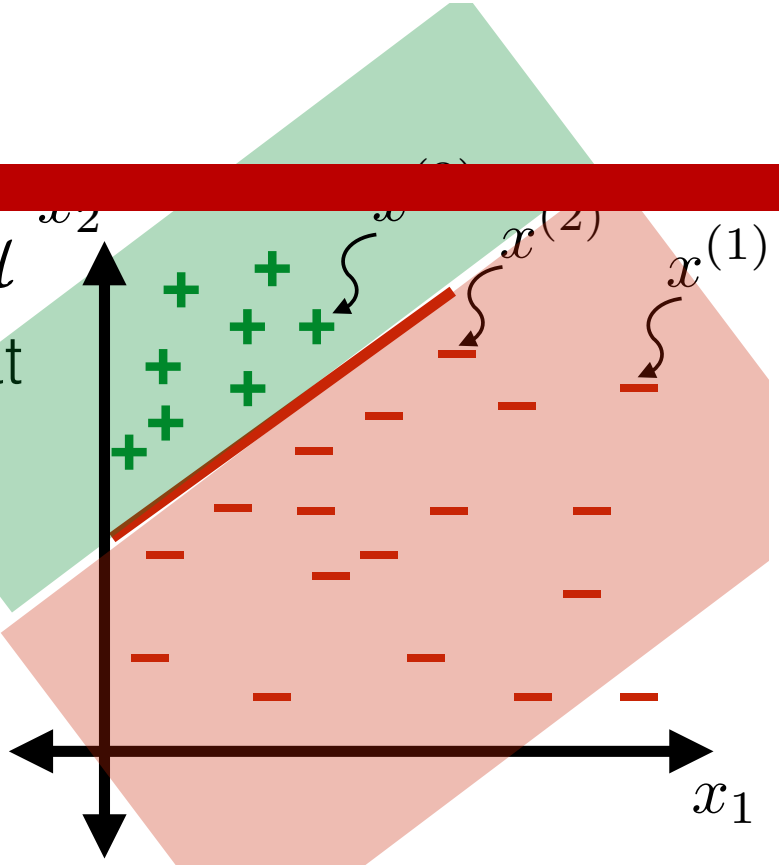

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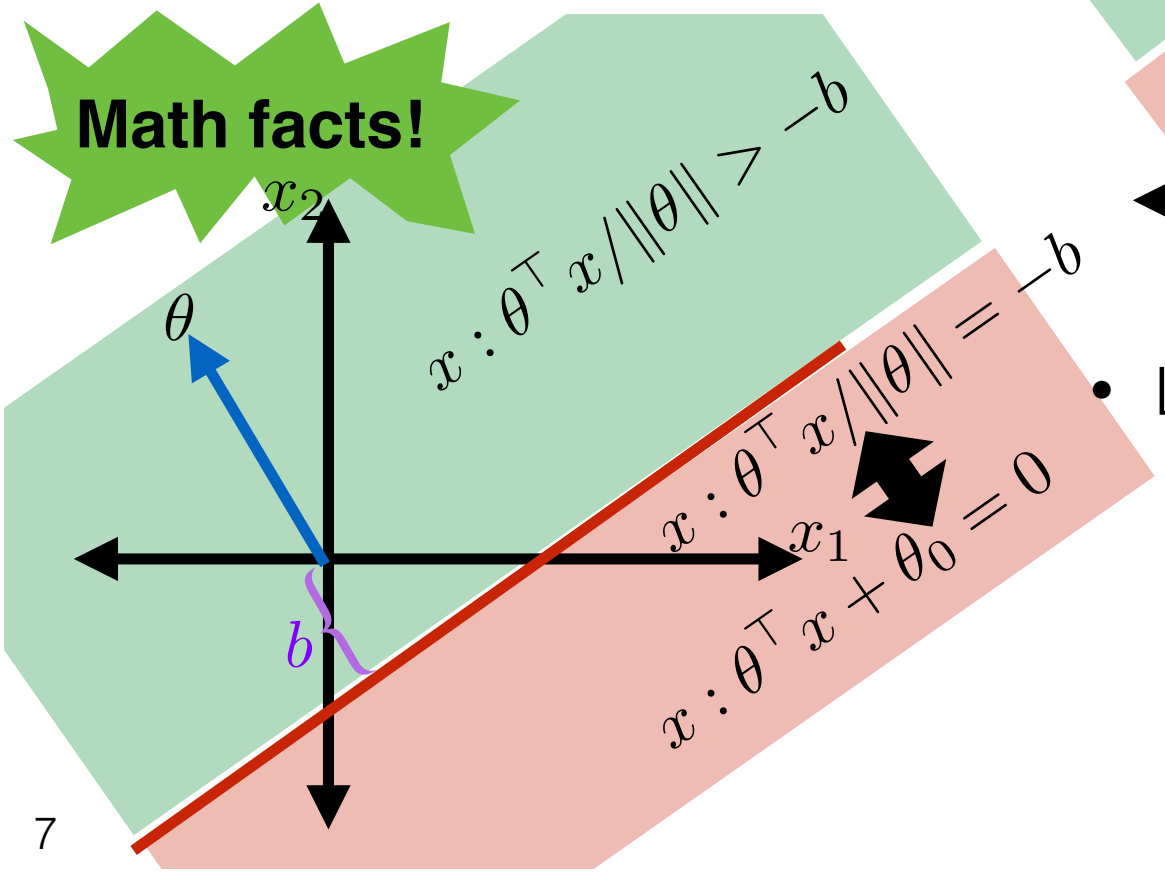

- Linear classifier:

$$h(x) = \text{sign}(\theta^\top x + \theta_0)$$

$$= \begin{cases} +1 \text{ if } \theta^\top x + \theta_0 > 0 \\ -1 \text{ if } \theta^\top x + \theta_0 < 0 \end{cases}$$

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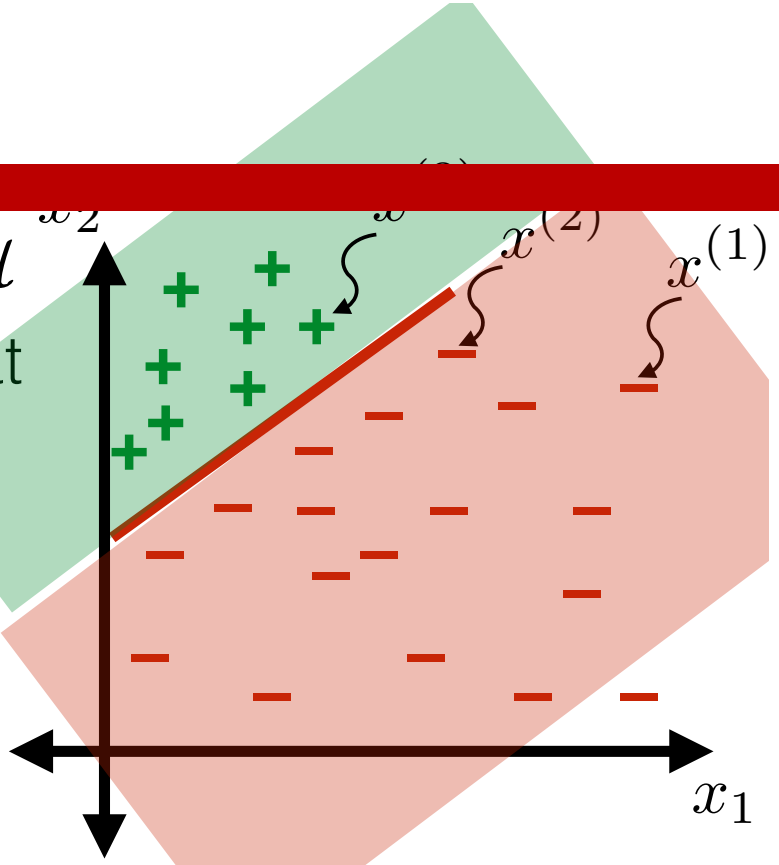
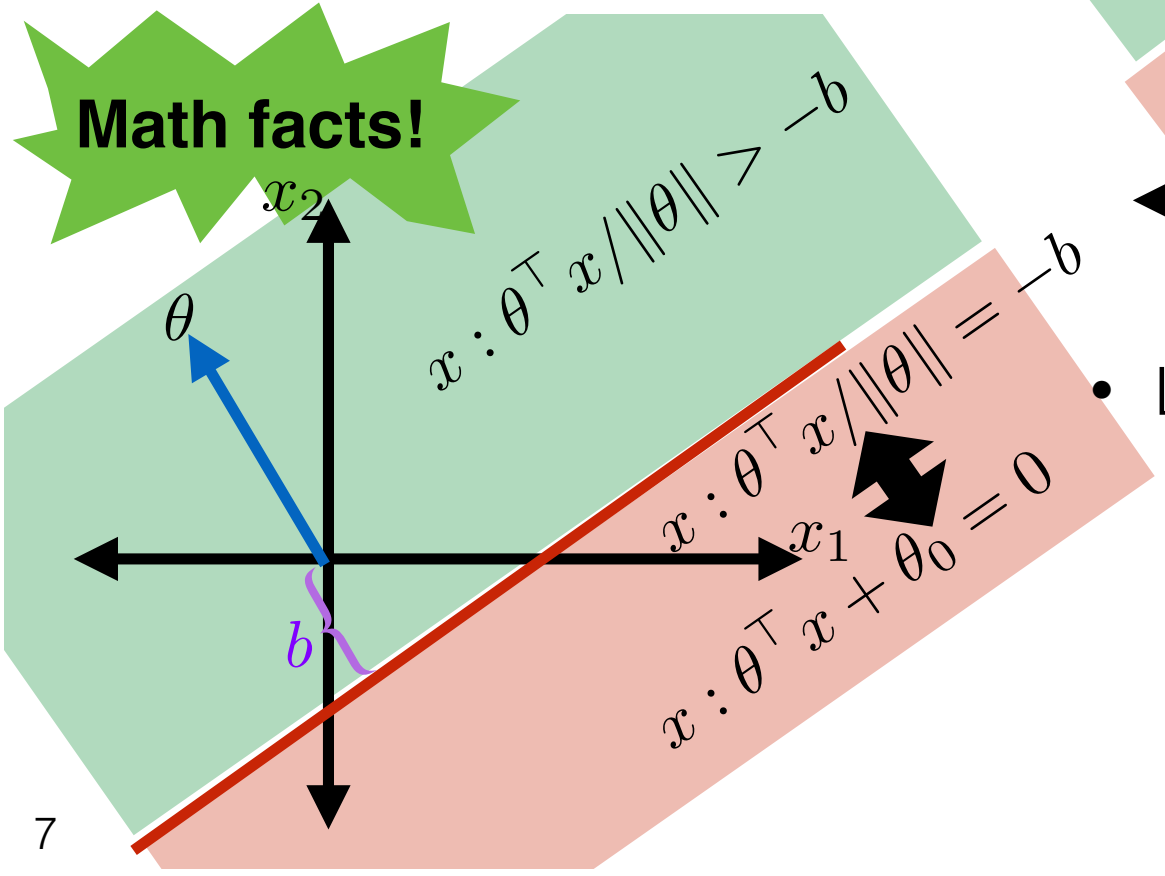


Math facts!



- Linear classifier:

$$h(x) = \text{sign}(\theta^\top x + \theta_0)$$

$$= \begin{cases} +1 \text{ if } \theta^\top x + \theta_0 > 0 \\ -1 \text{ if } \theta^\top x + \theta_0 < 0 \end{cases}$$

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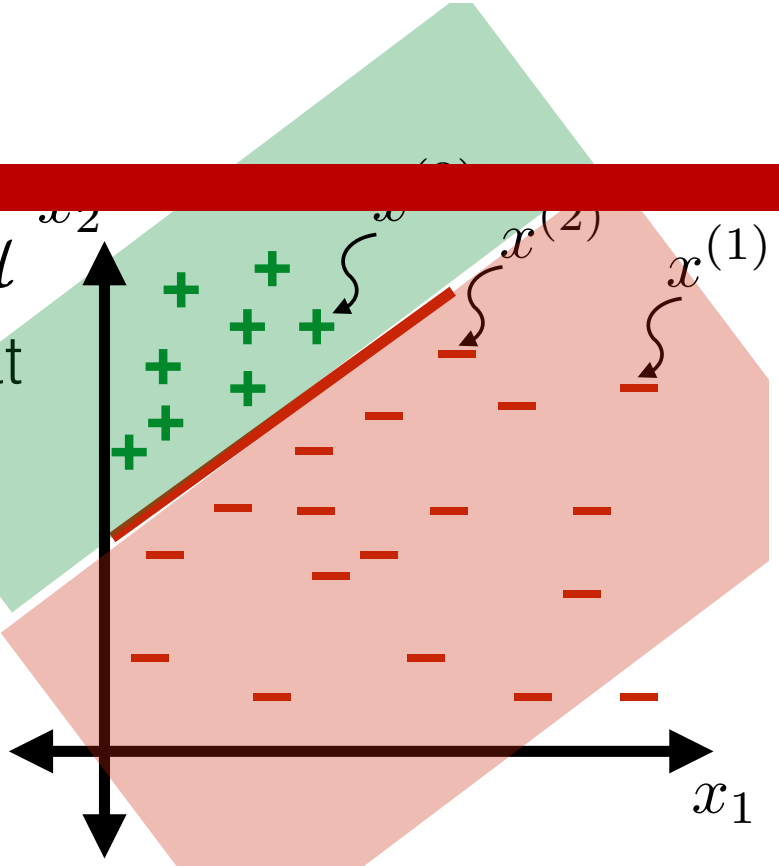

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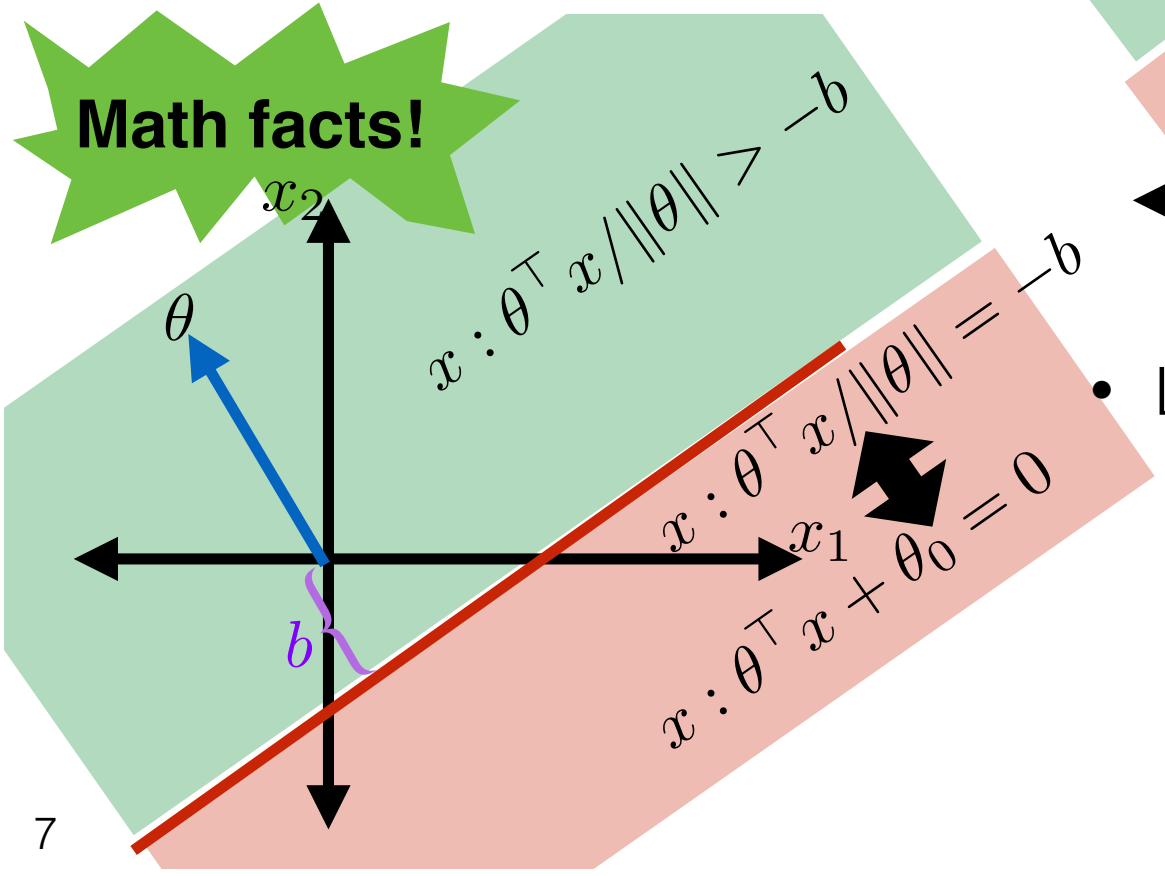

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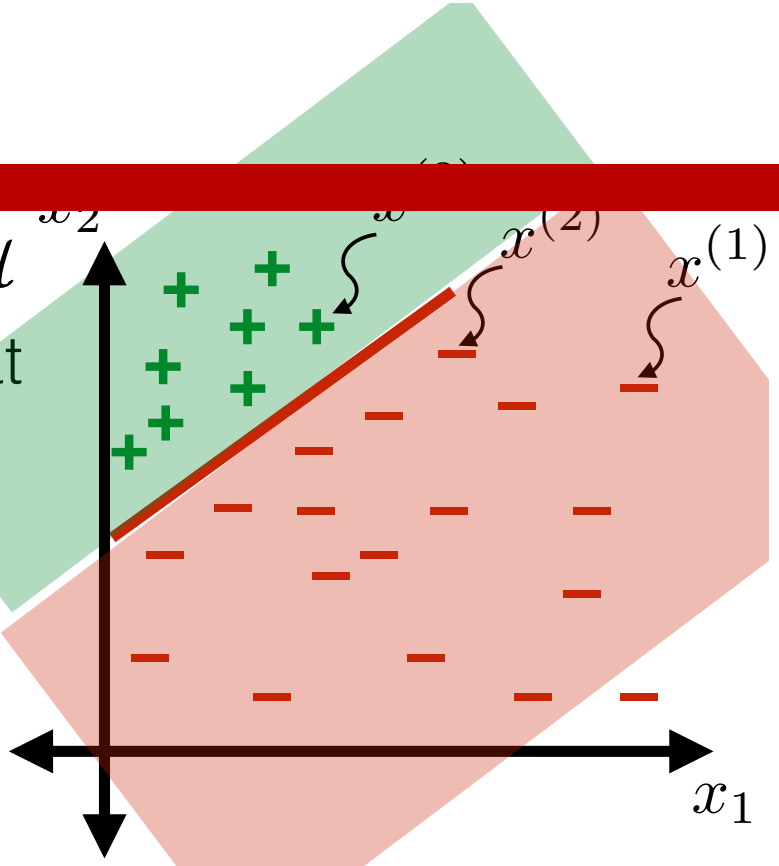

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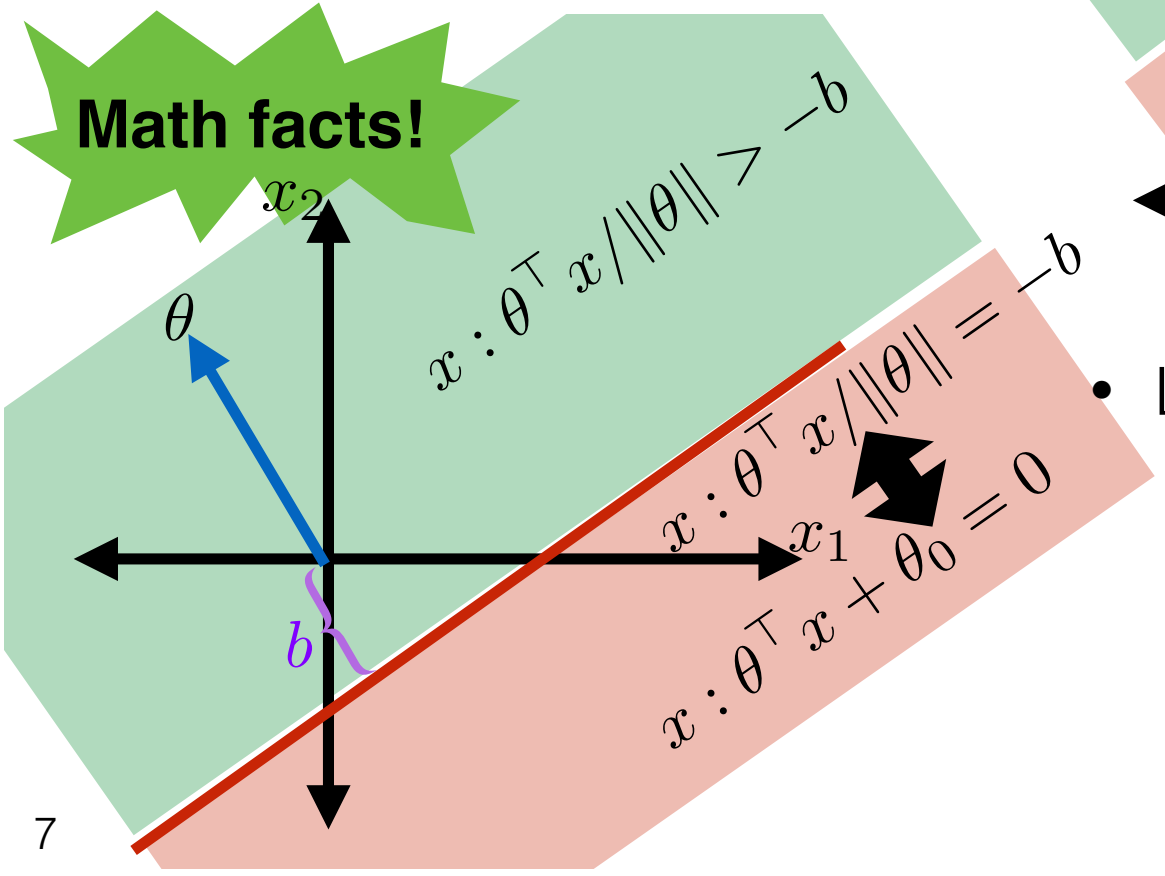

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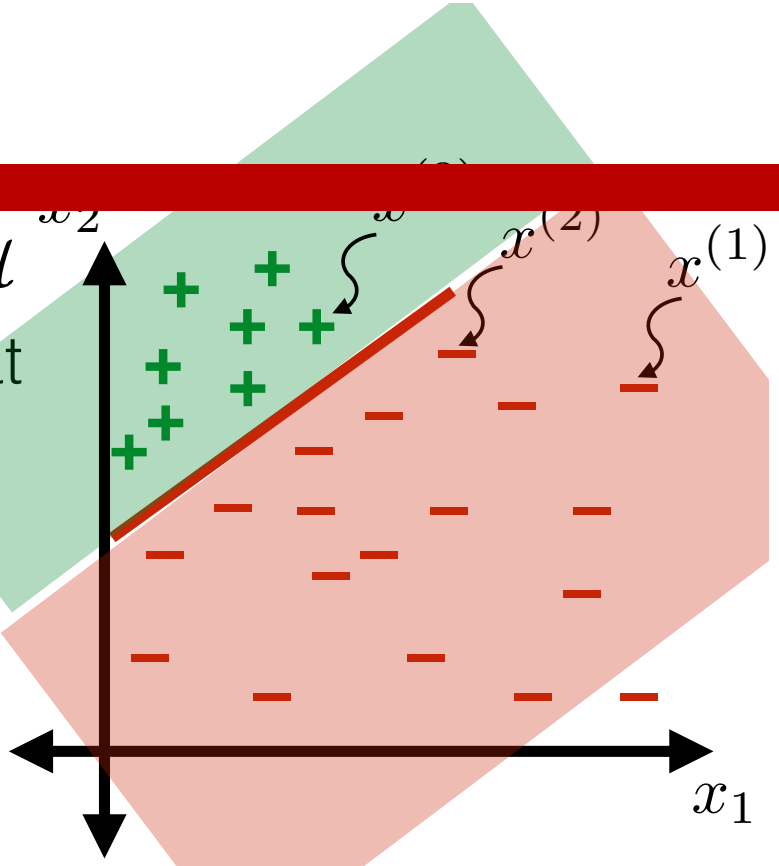

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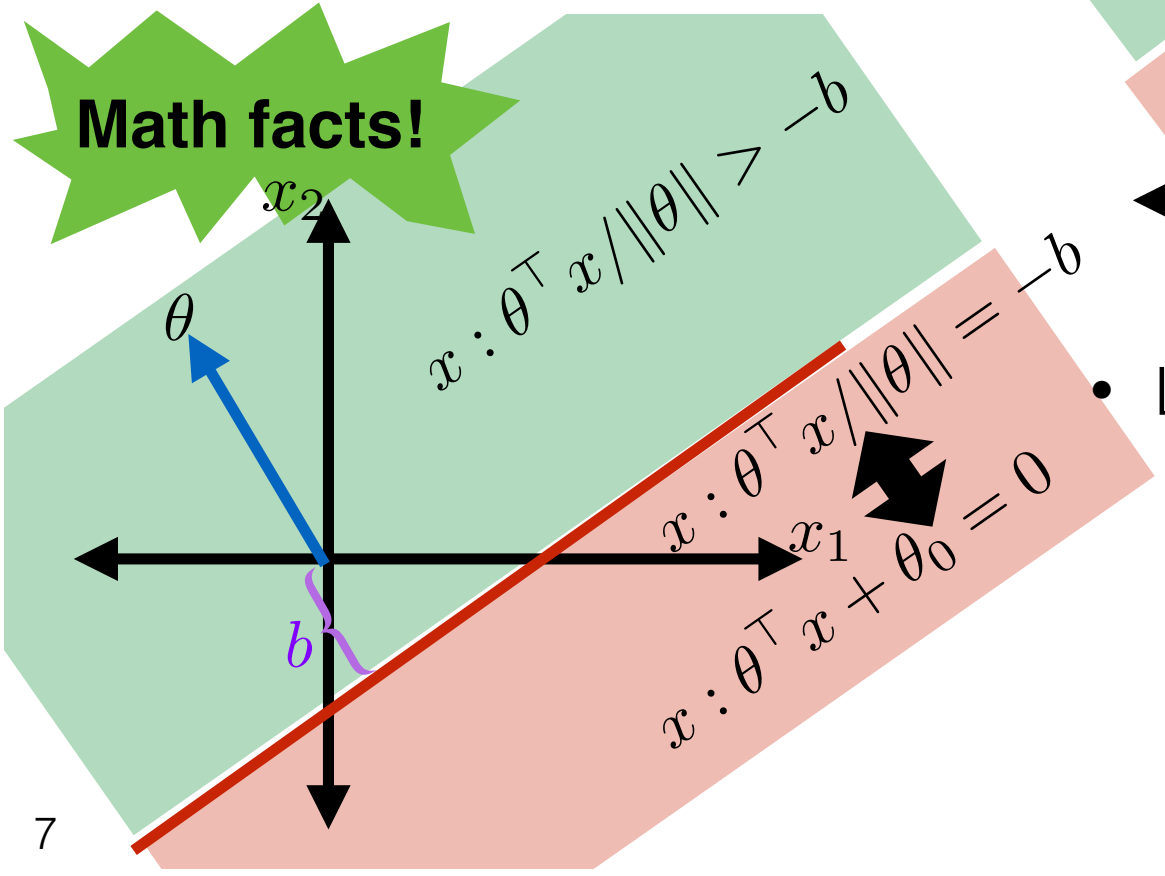

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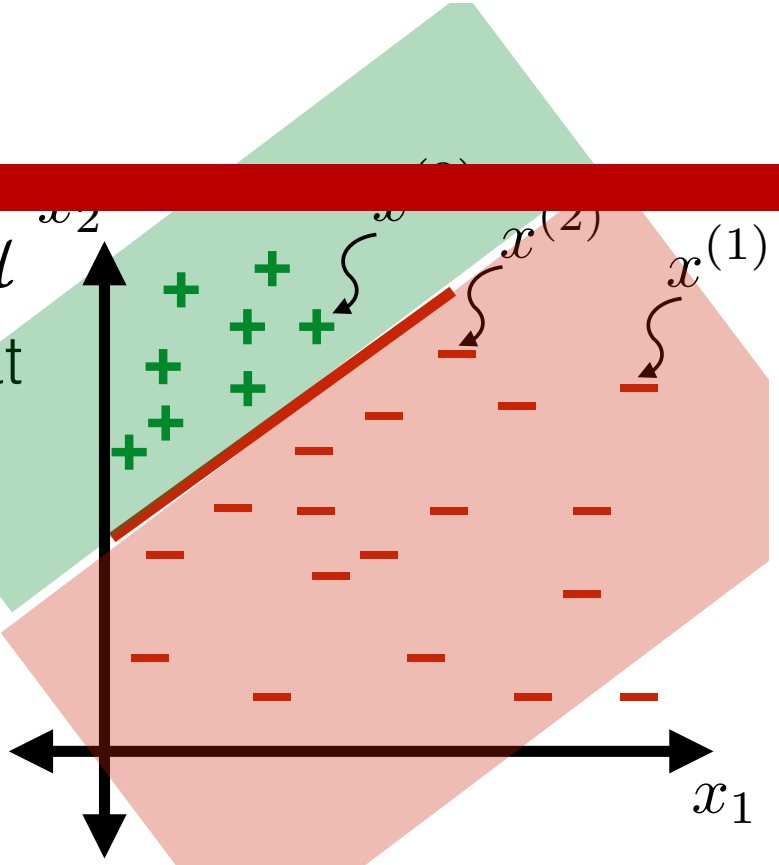

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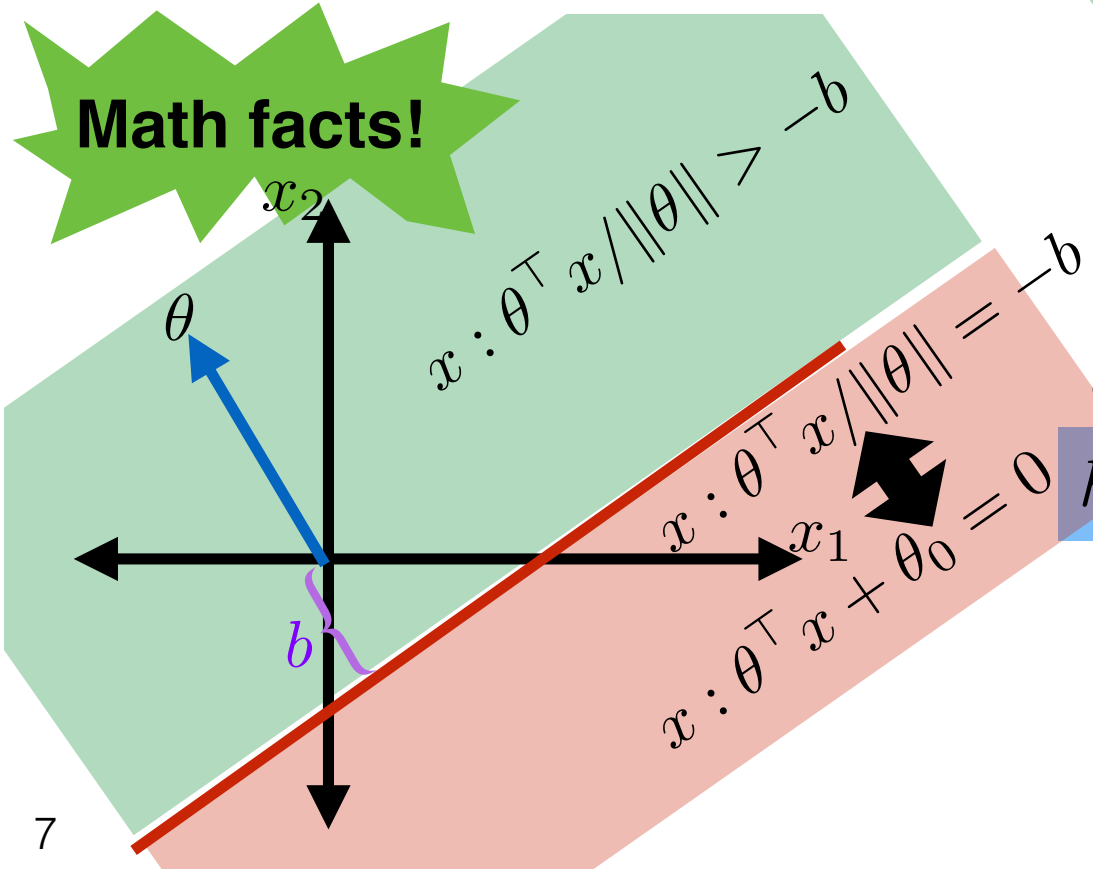

- Linear classifier:

$$h(x; \theta, \theta_0) = \text{sign}(\theta^\top x + \theta_0)$$

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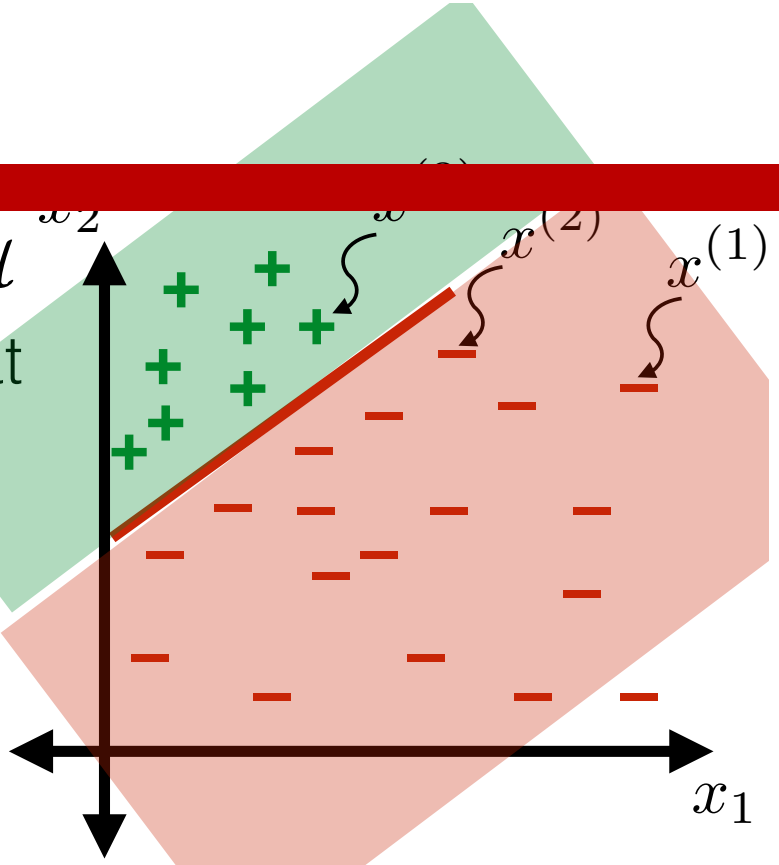

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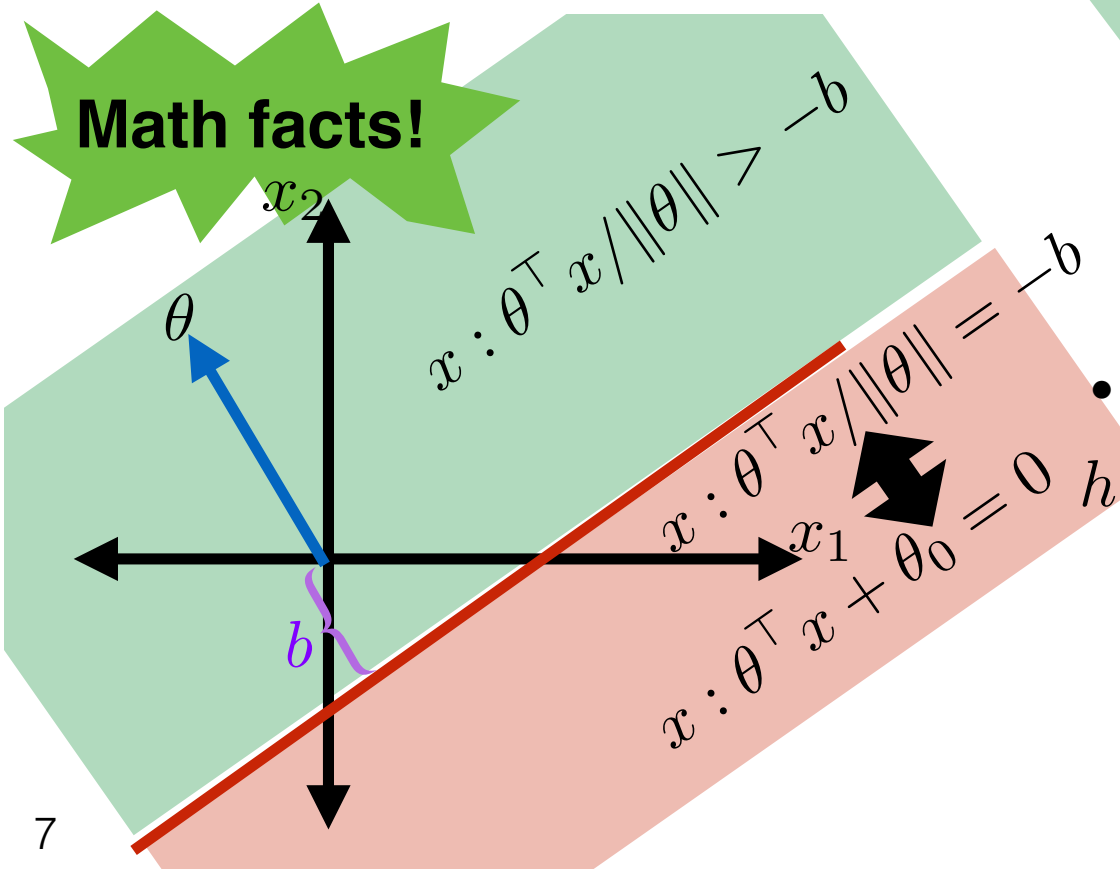

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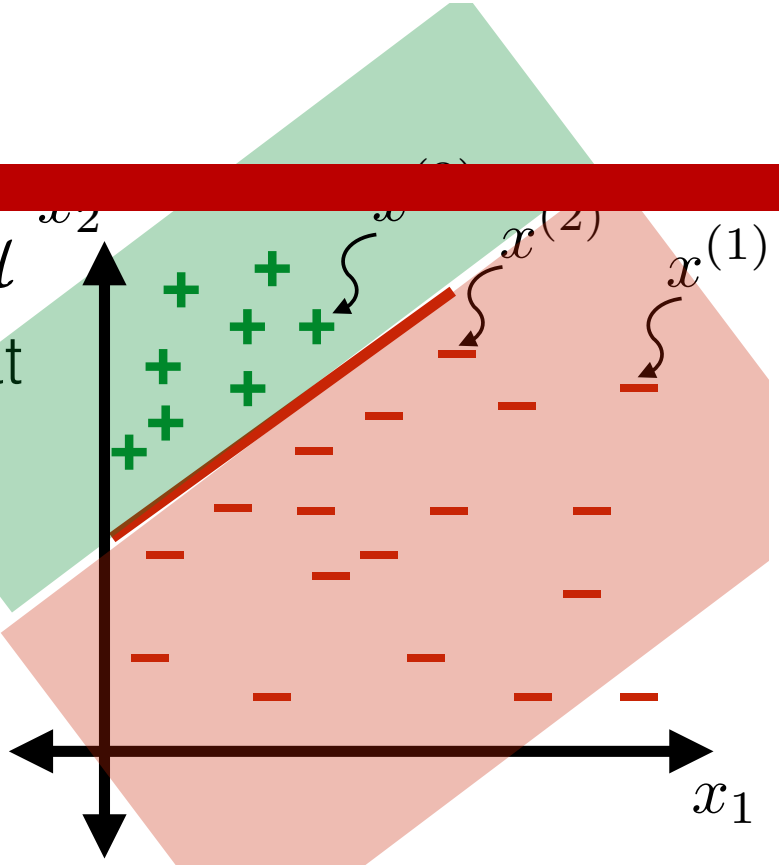

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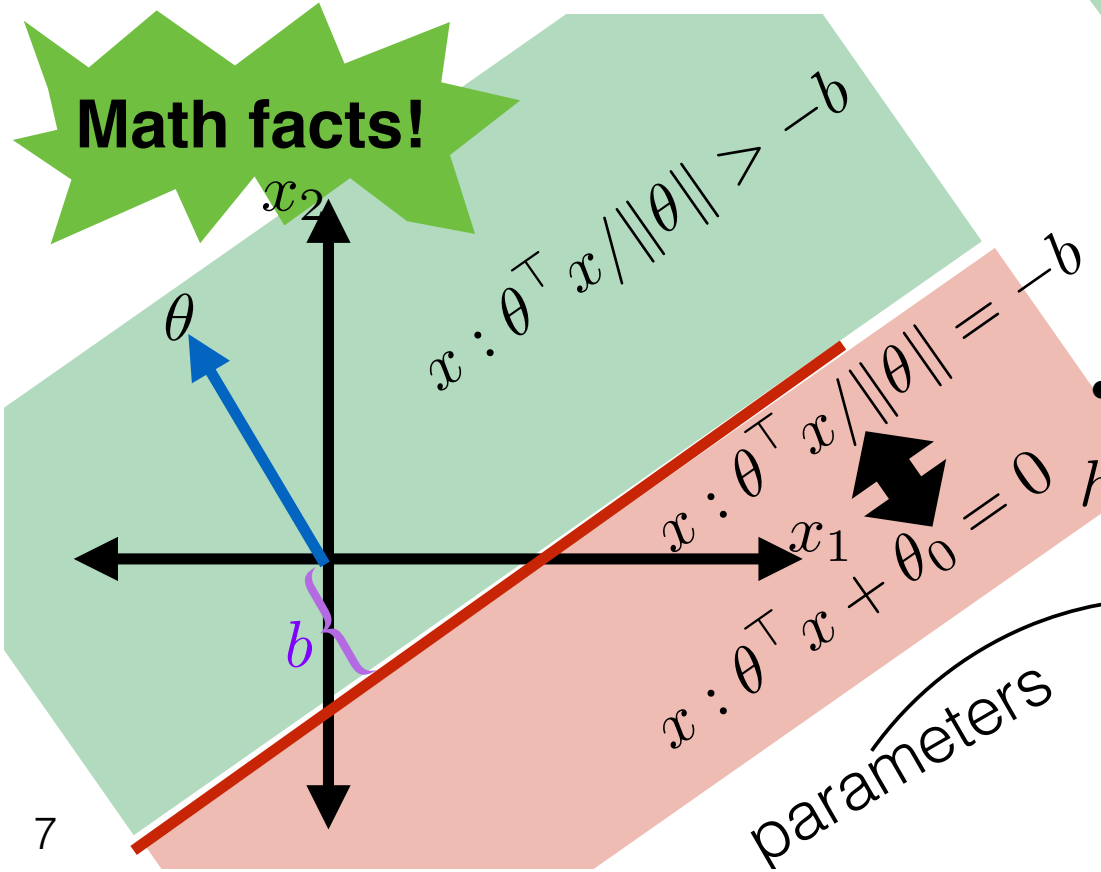

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parameters

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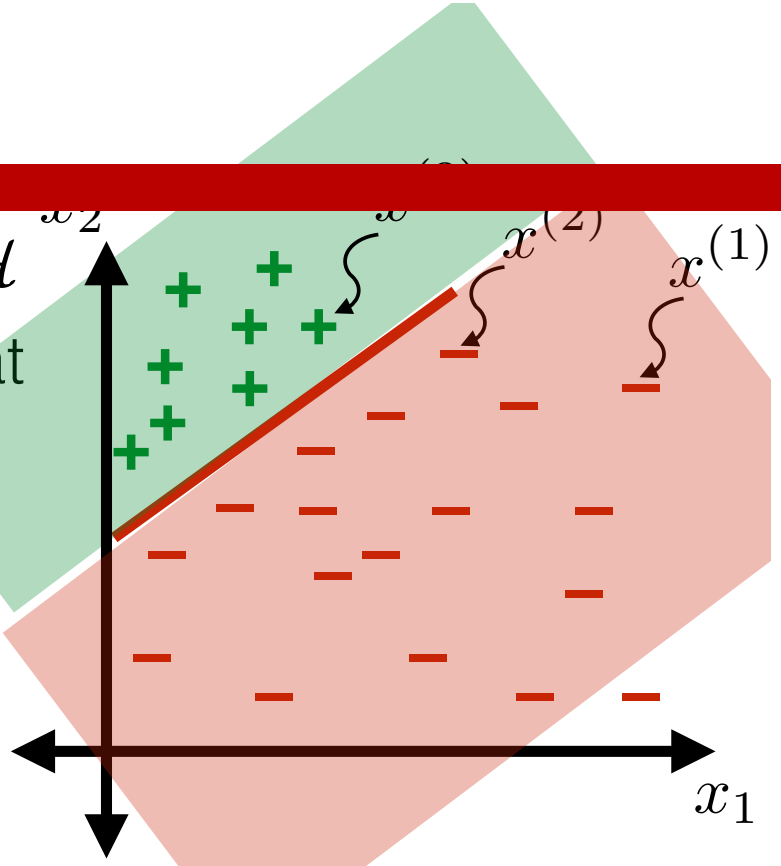

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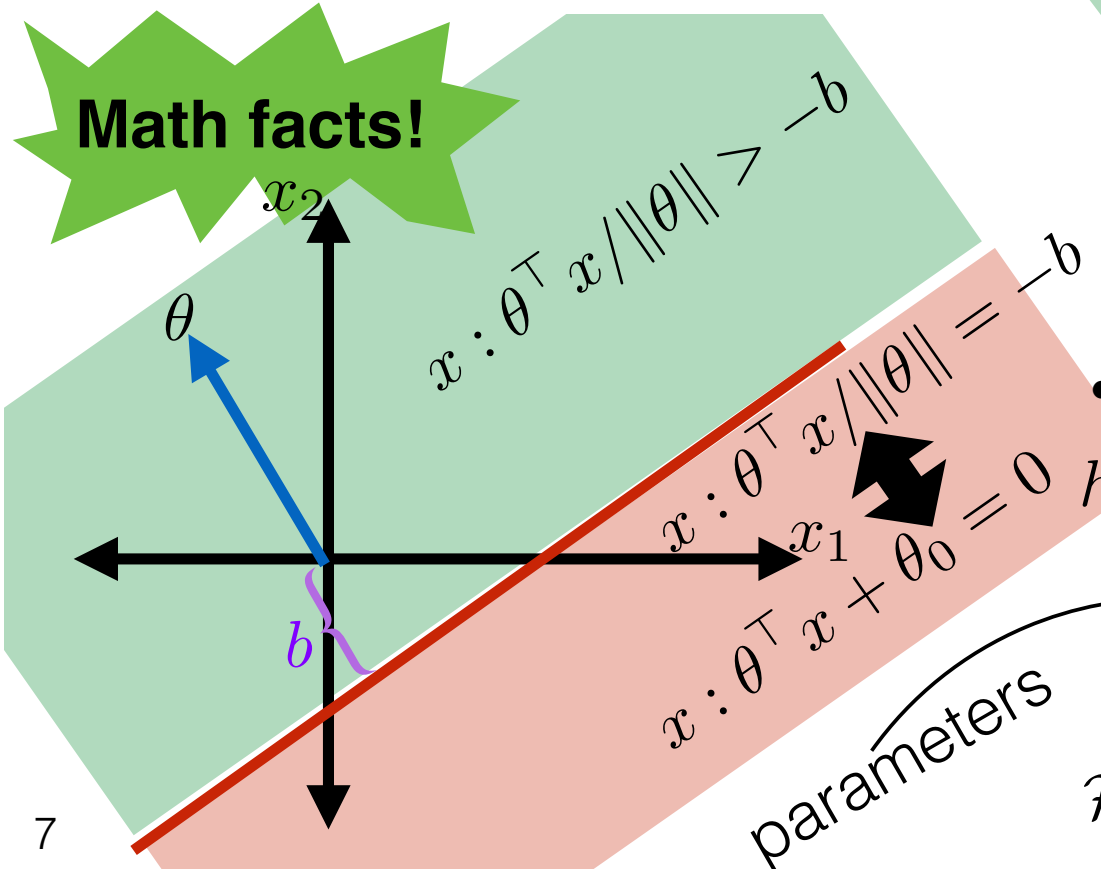

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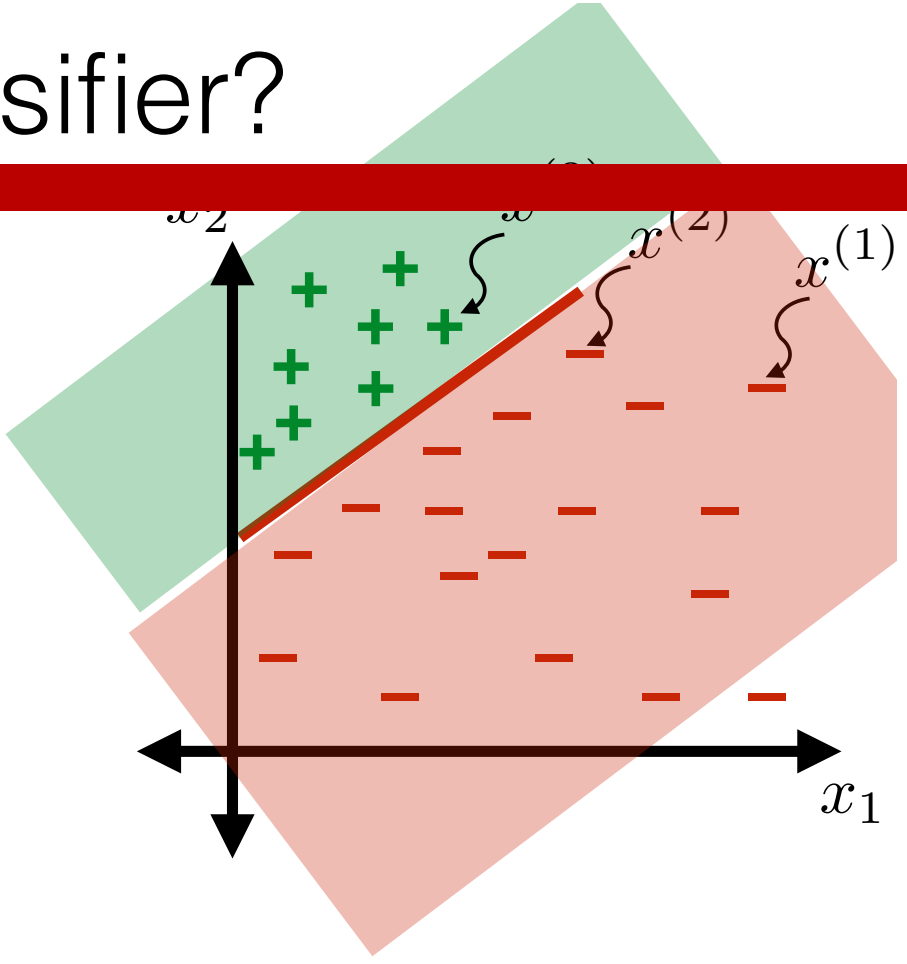
parameters

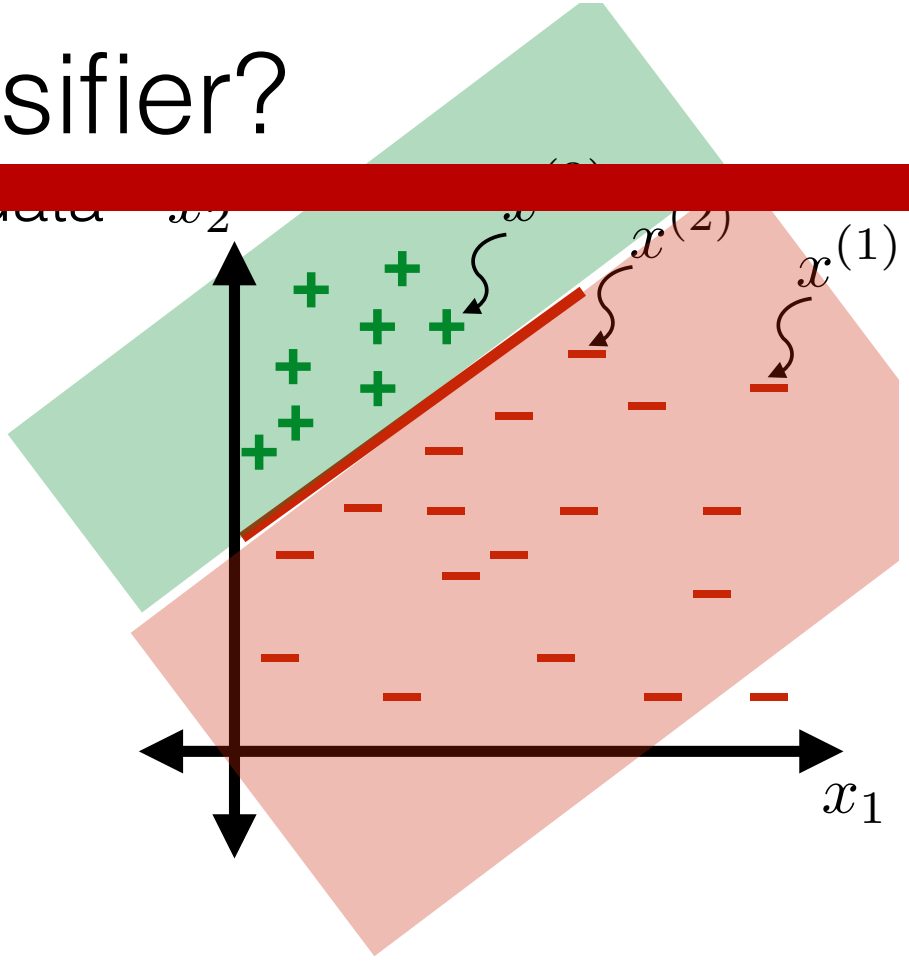
$$\mathcal{H} = \text{set of all such } h$$

How good is a classifier?



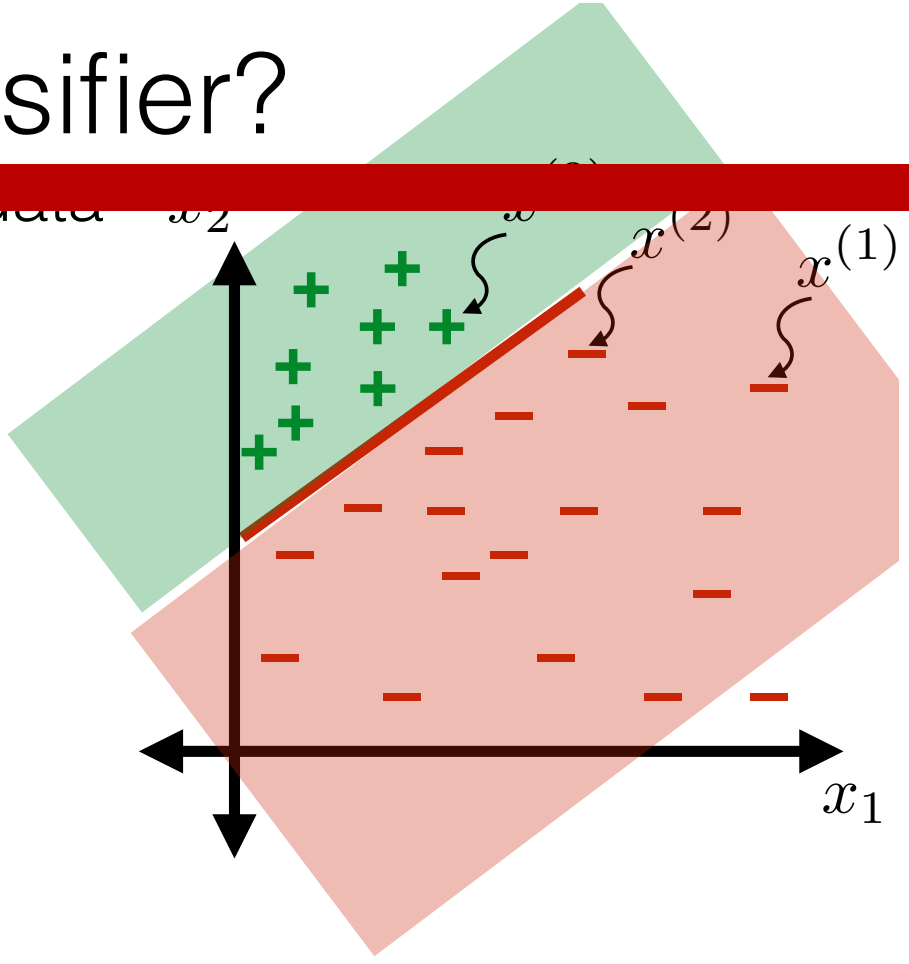
How good is a classifier?

Should predict well on future data



How good is a classifier?

- Should predict well on future data
- How good is a classifier at a single point?



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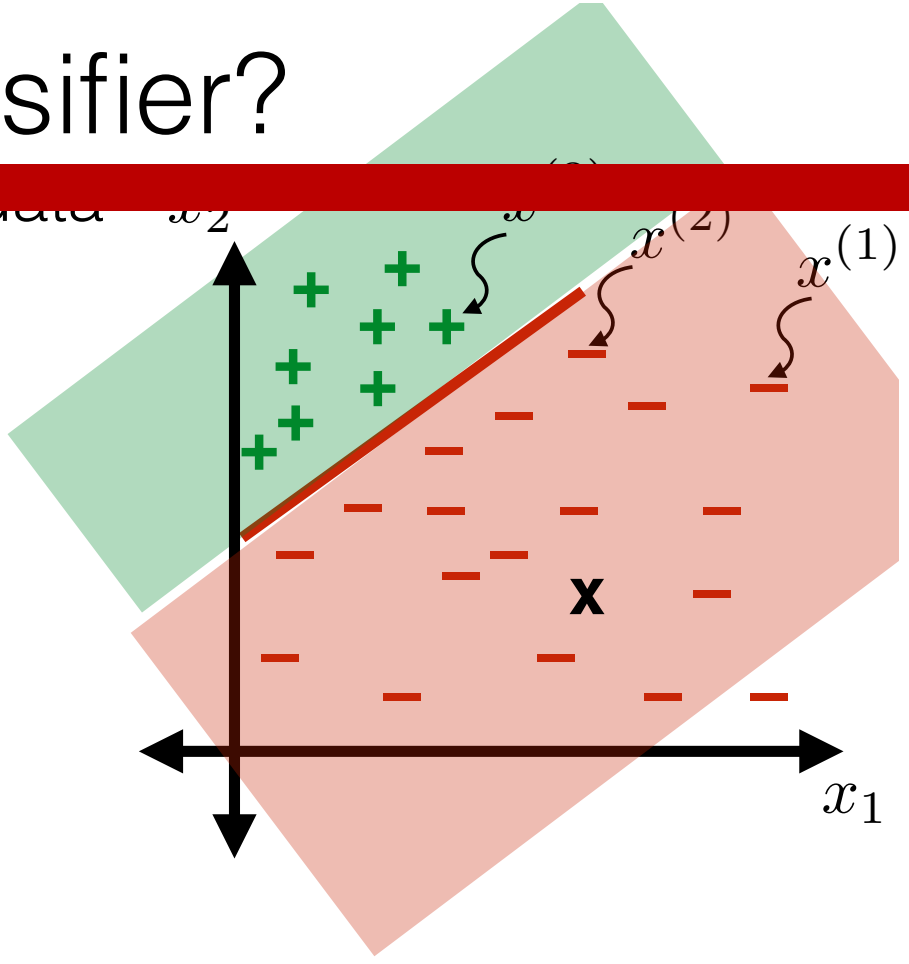

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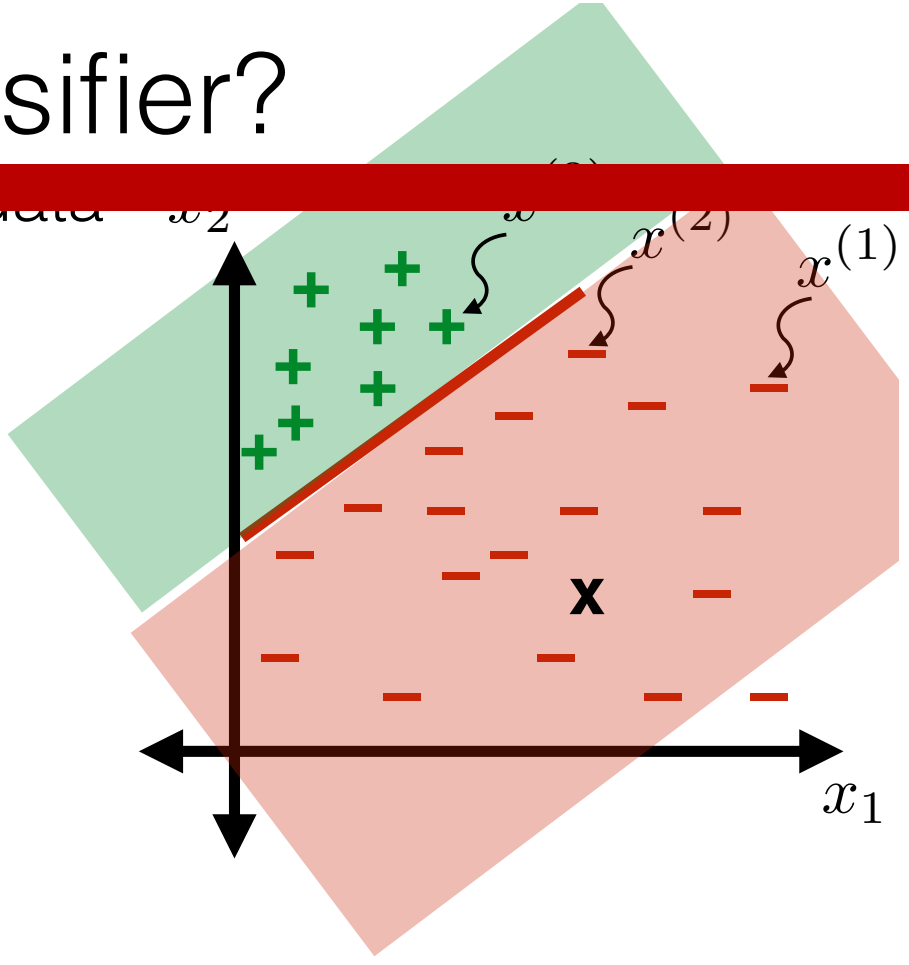

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- How good is a classifier at a single point? Loss $L(g, a)$

g: guess,
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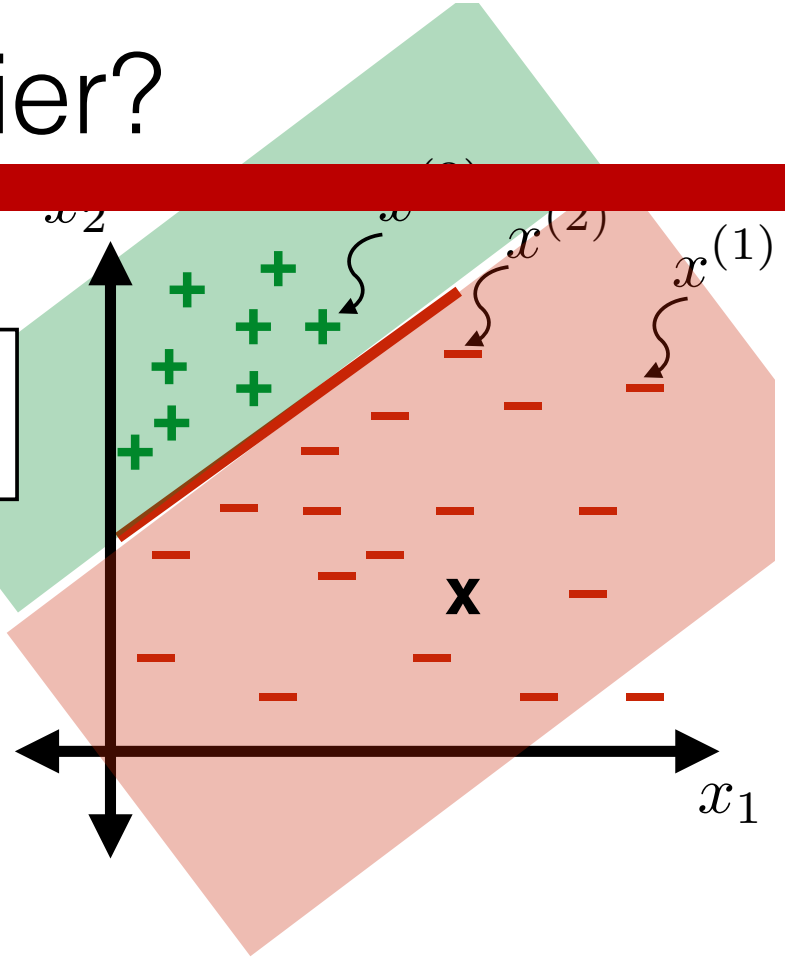

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- How good is a classifier at a single point? Loss $L(g, a)$
  - Example: 0-1 loss

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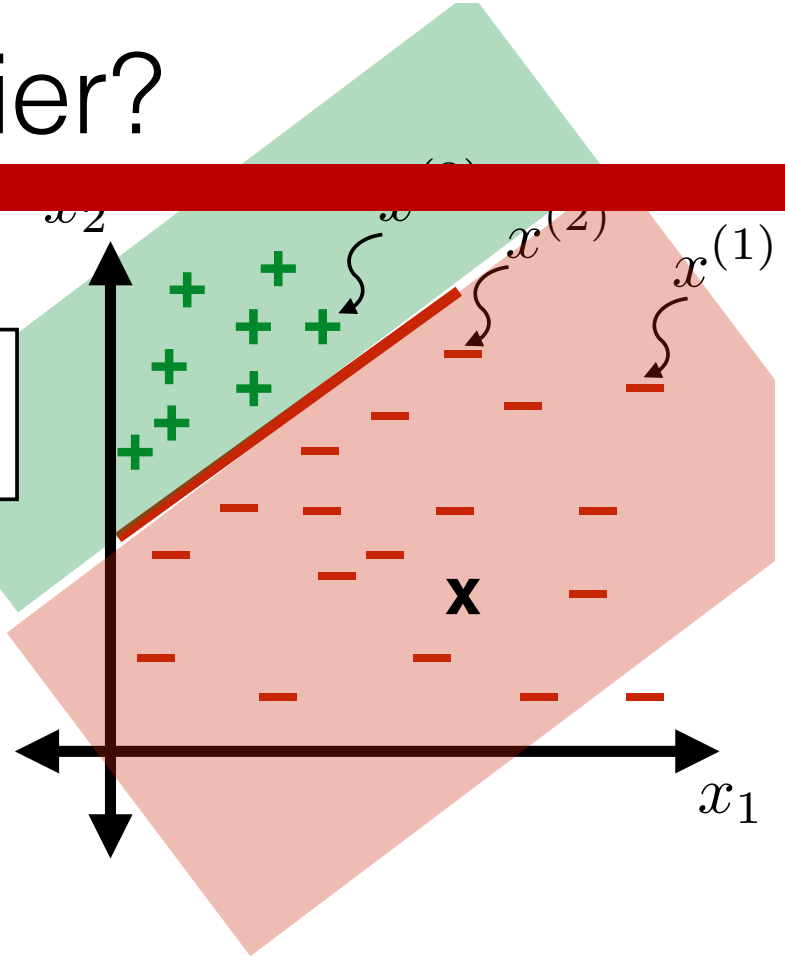

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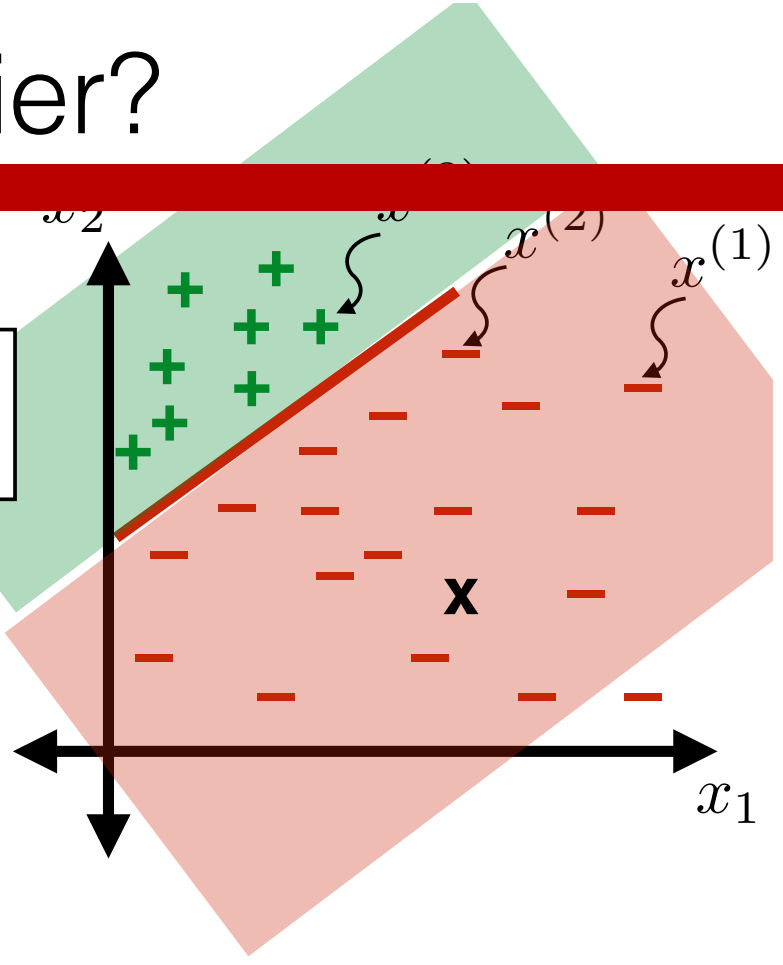

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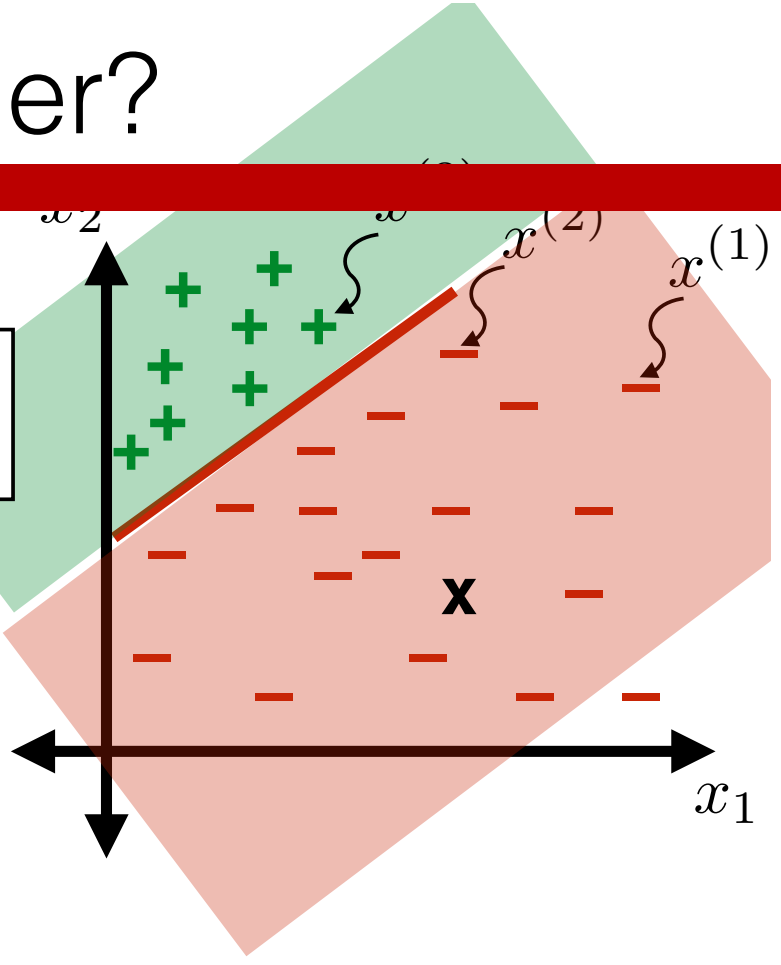

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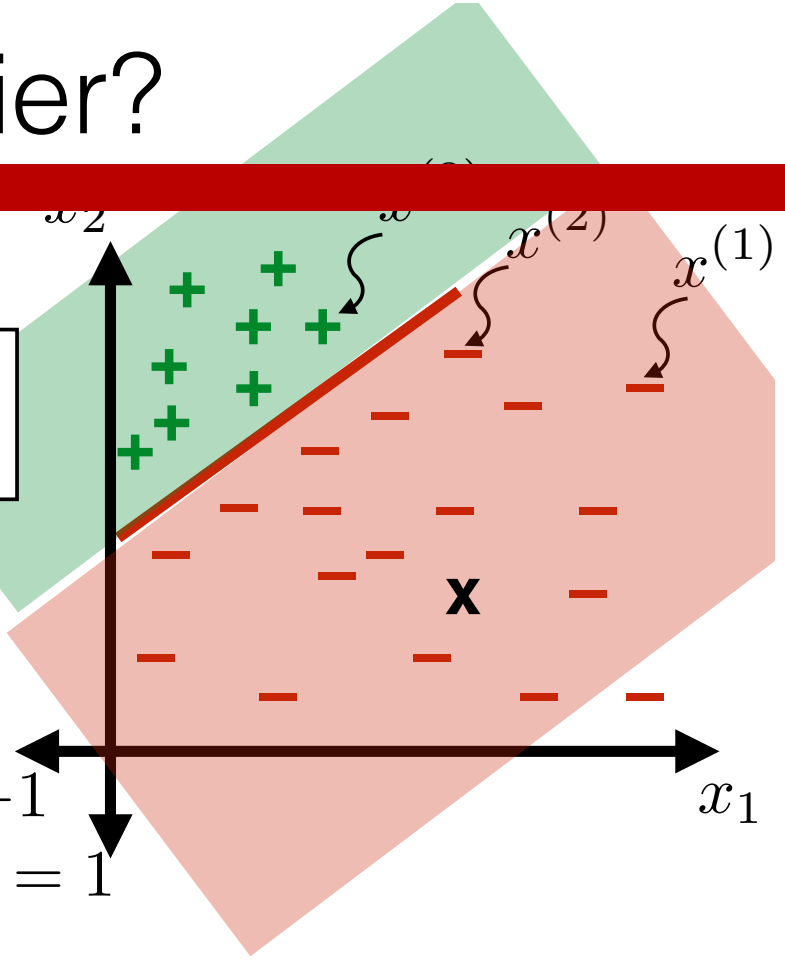

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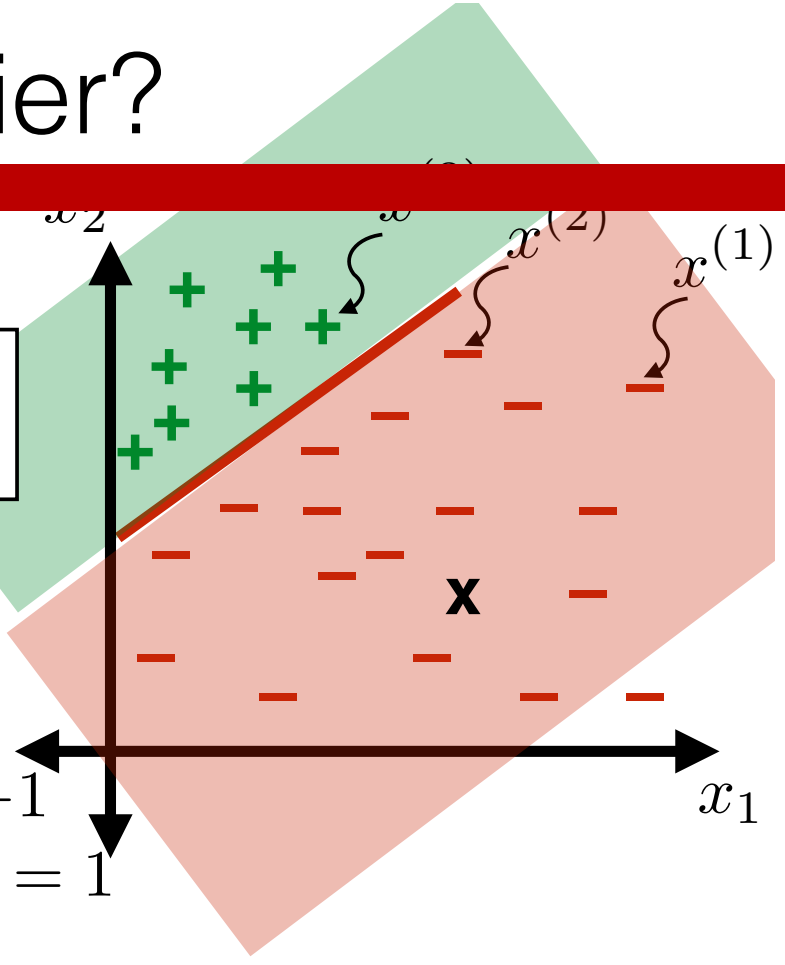
- Test error (*n'* new points):



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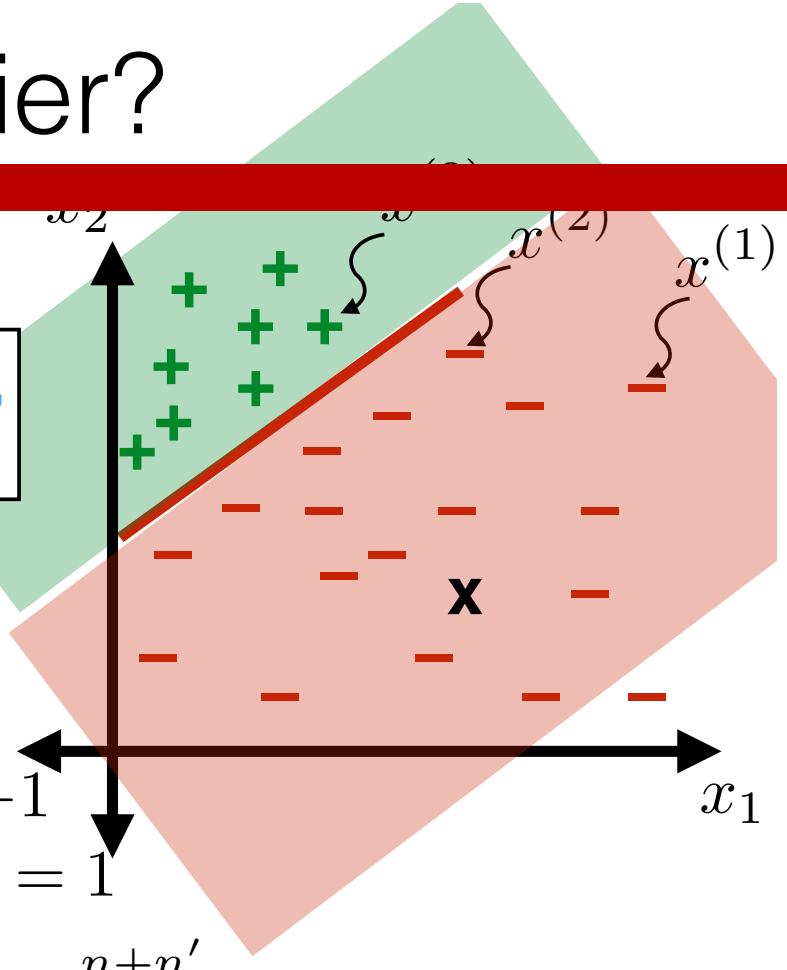

- Test error (n' new points): $\mathcal{E}(h) = \frac{1}{n'} \sum_{i=n+1}^{n+n'} L(h(x^{(i)}), y^{(i)})$

How good is a classifier?

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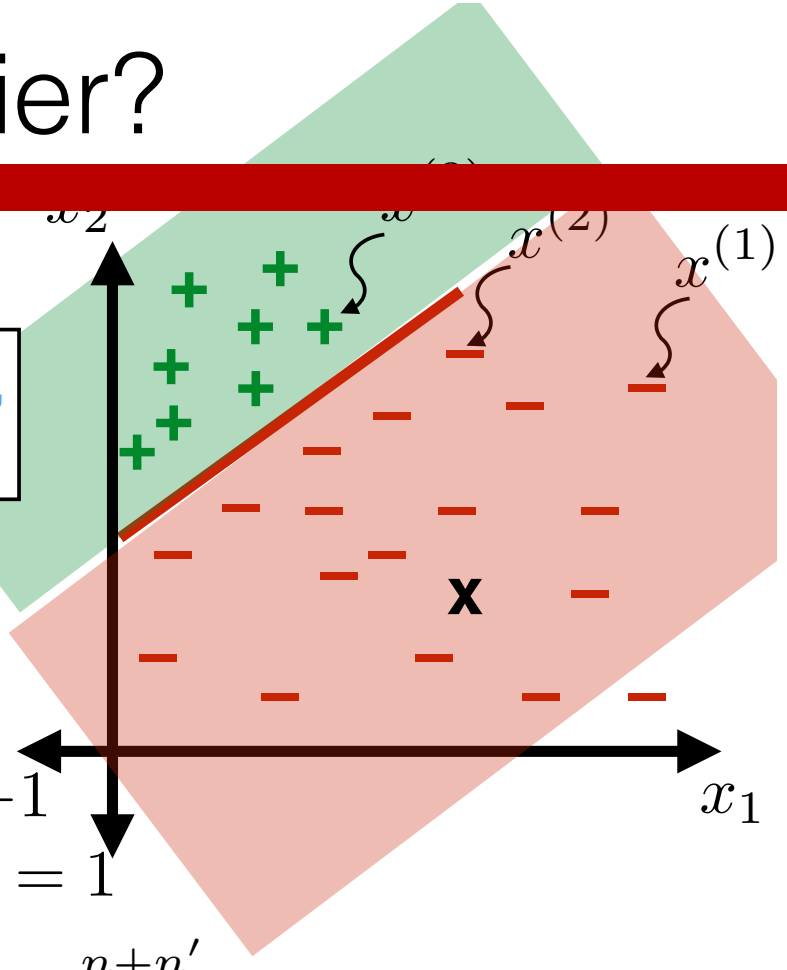

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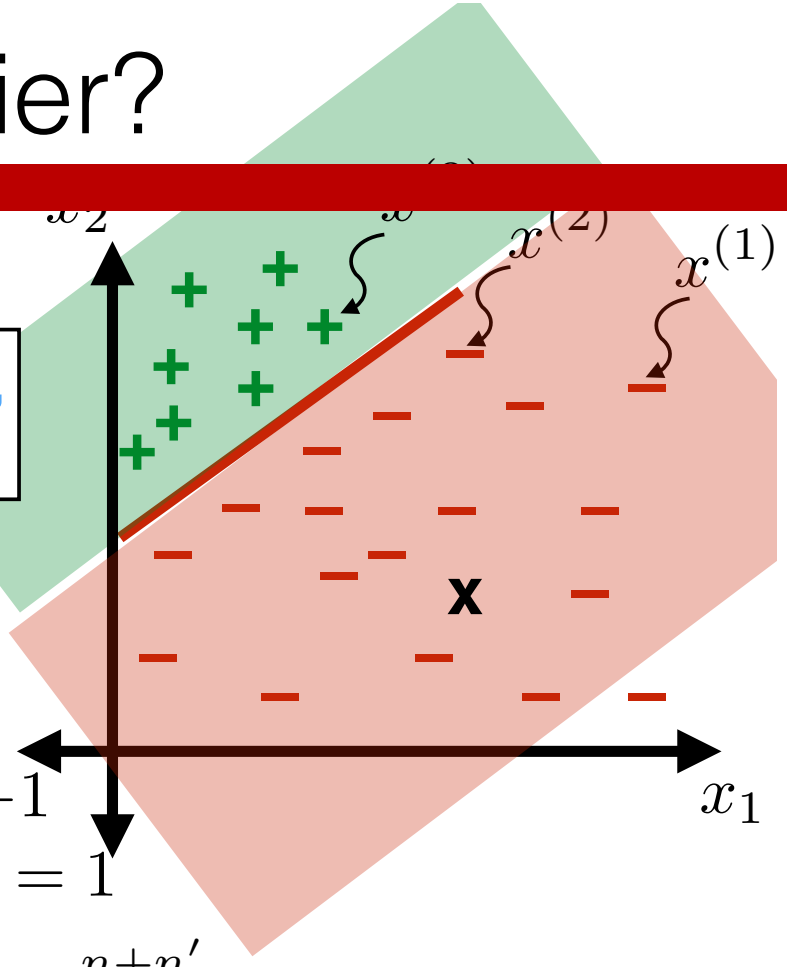
- Training error:

How good is a classifier?



- How good is a classifier at a single point? Loss $L(g, a)$ g: guess, a: actual
  - Example: 0-1 loss

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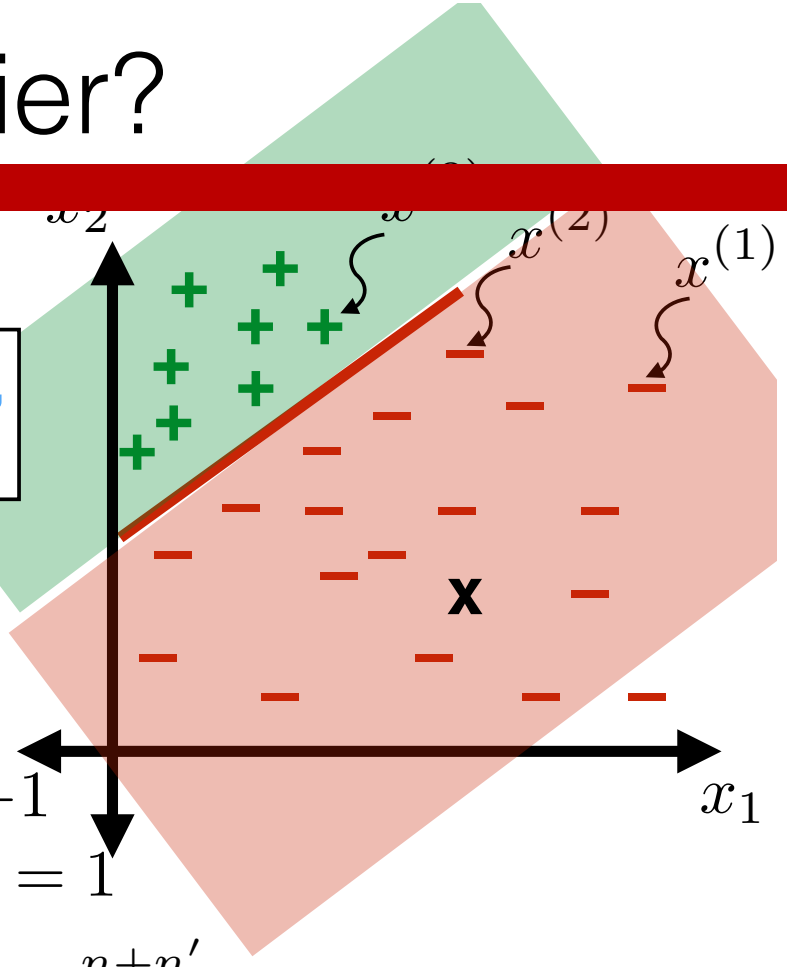

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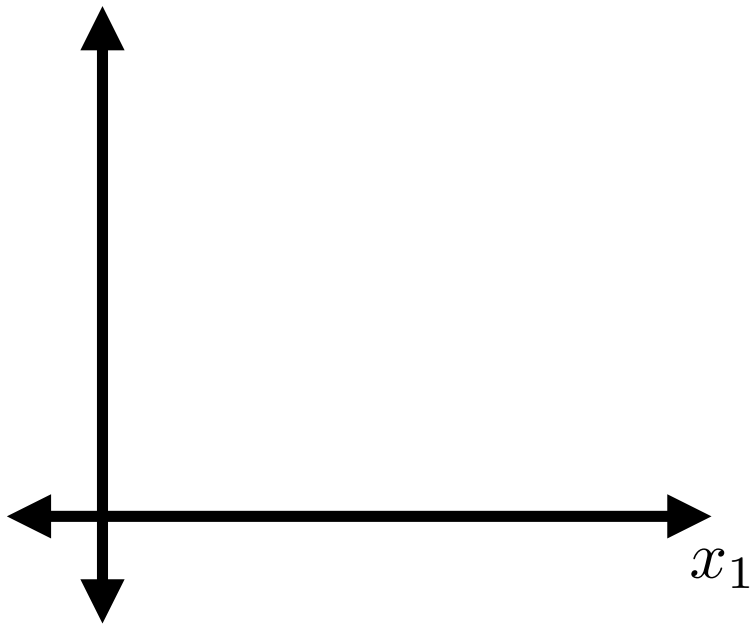
- Prefer h to $\tilde{h}$ if $\mathcal{E}_n(h) < \mathcal{E}_n(\tilde{h})$

Learning a classifier

Have data, have hypothesis class

- Want to choose a good classifier
  - Recall: $x \rightarrow \boxed{h} \rightarrow y$



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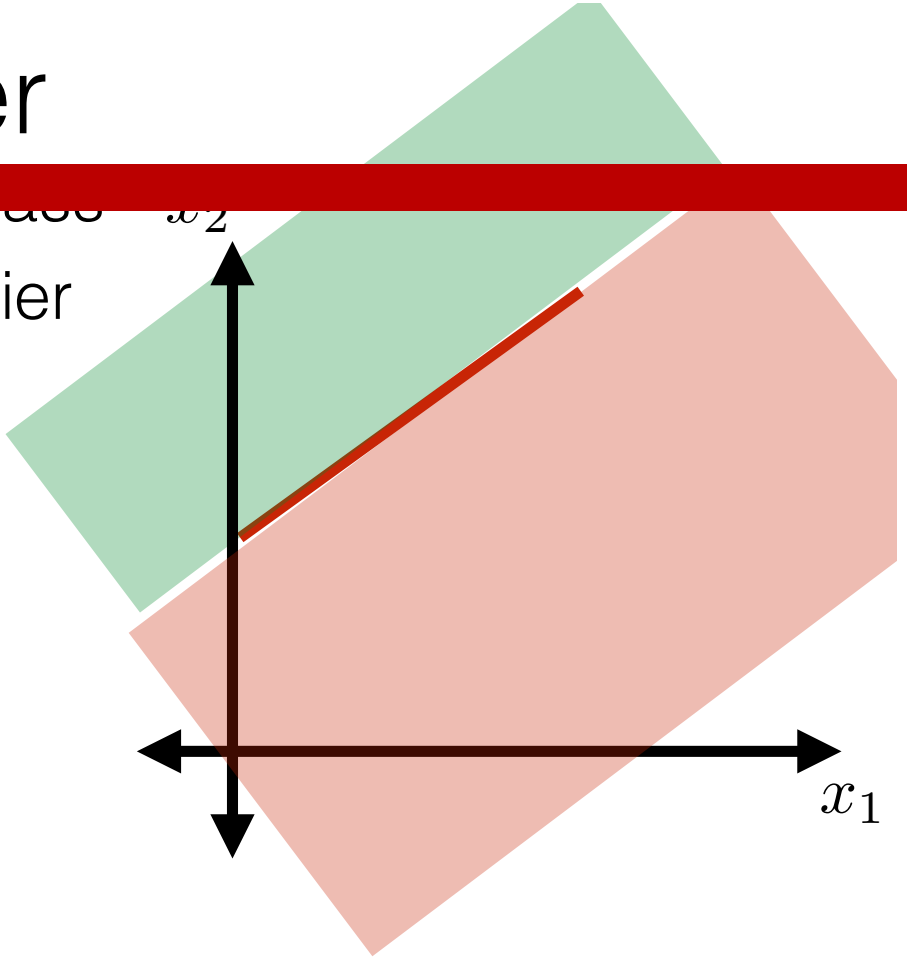

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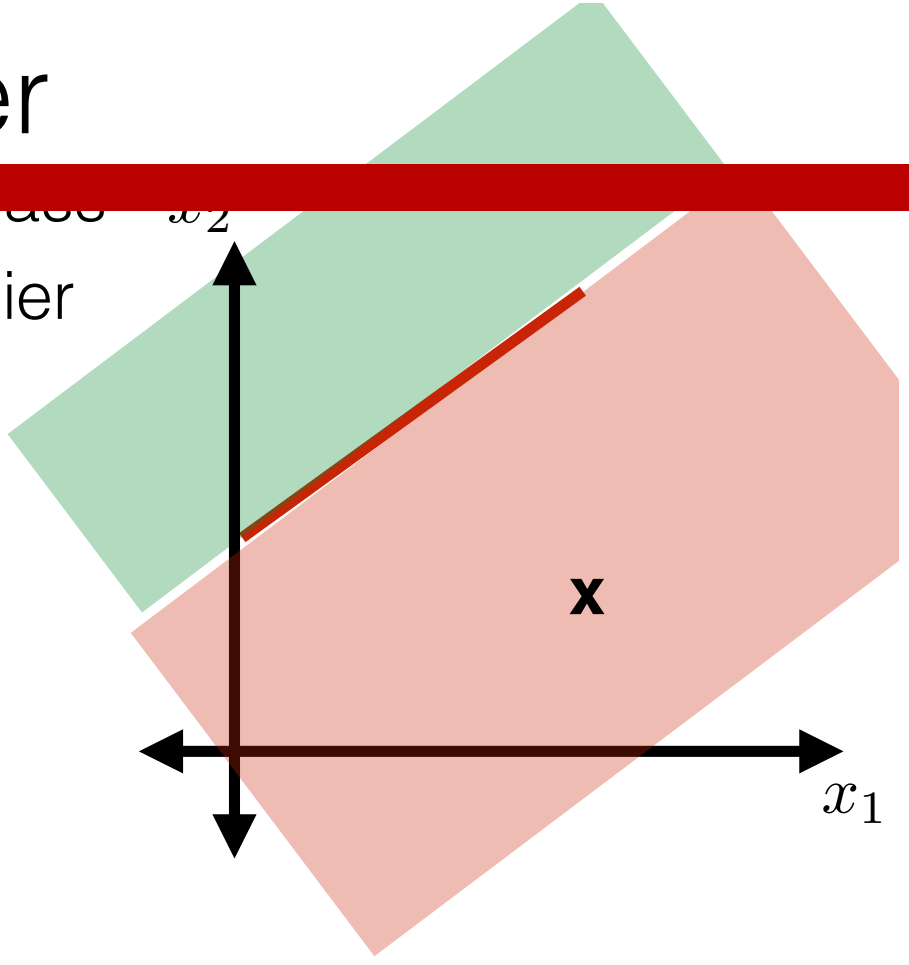

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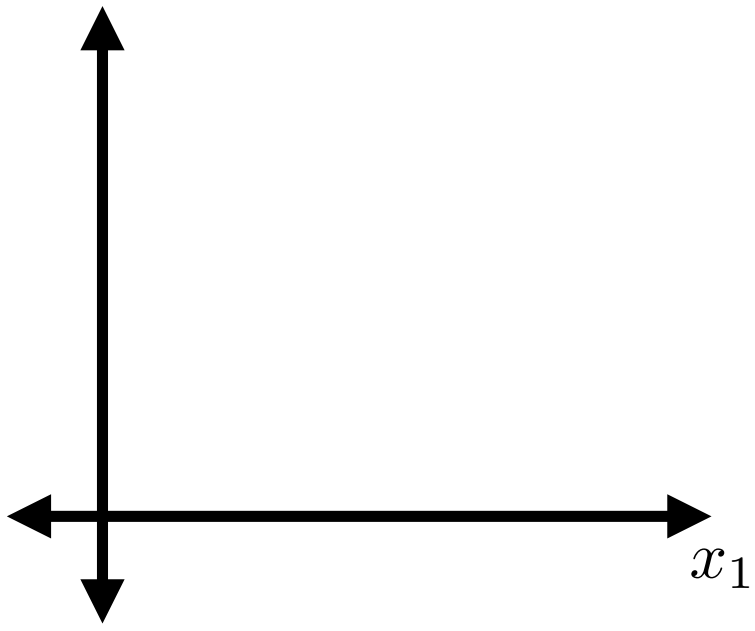

Learning a classifier

Have data, have hypothesis class

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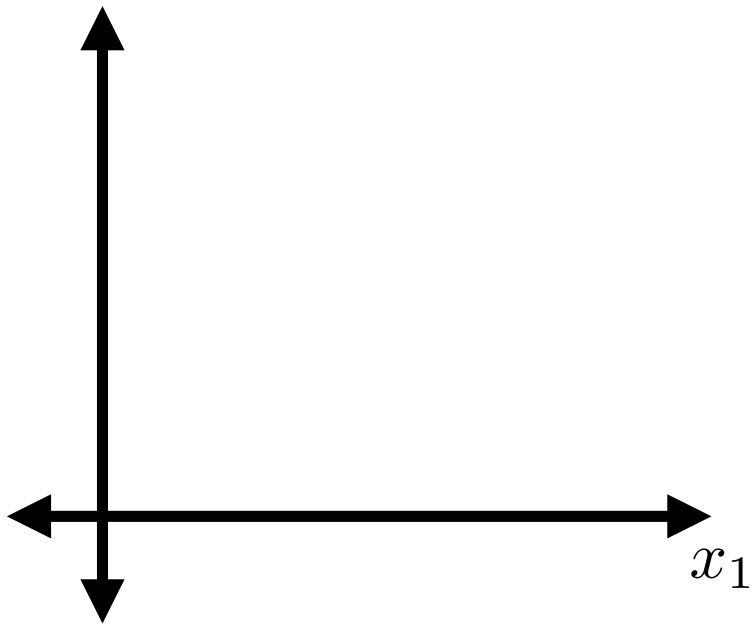
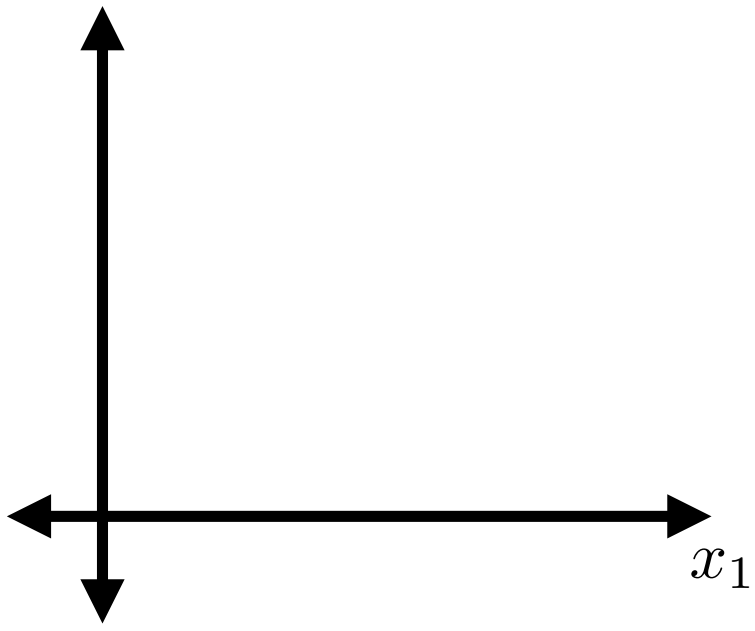
  - New:



Learning a classifier

Have data, have hypothesis class

- Want to choose a good classifier
  - Recall: $x \rightarrow h \rightarrow y$
  - New: $\mathcal{D}_n \rightarrow$ learning algorithm $\rightarrow h$



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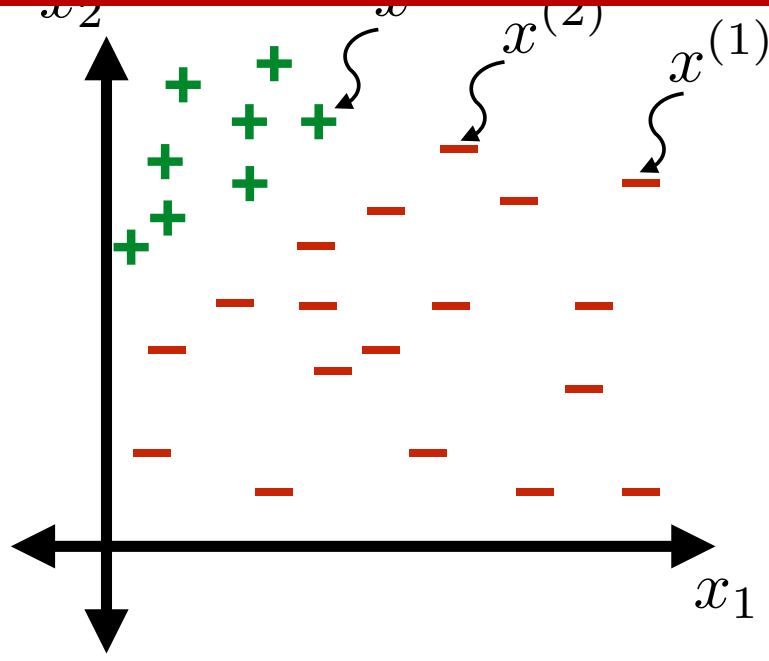

Learning a classifier

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- Want to choose a good classifier
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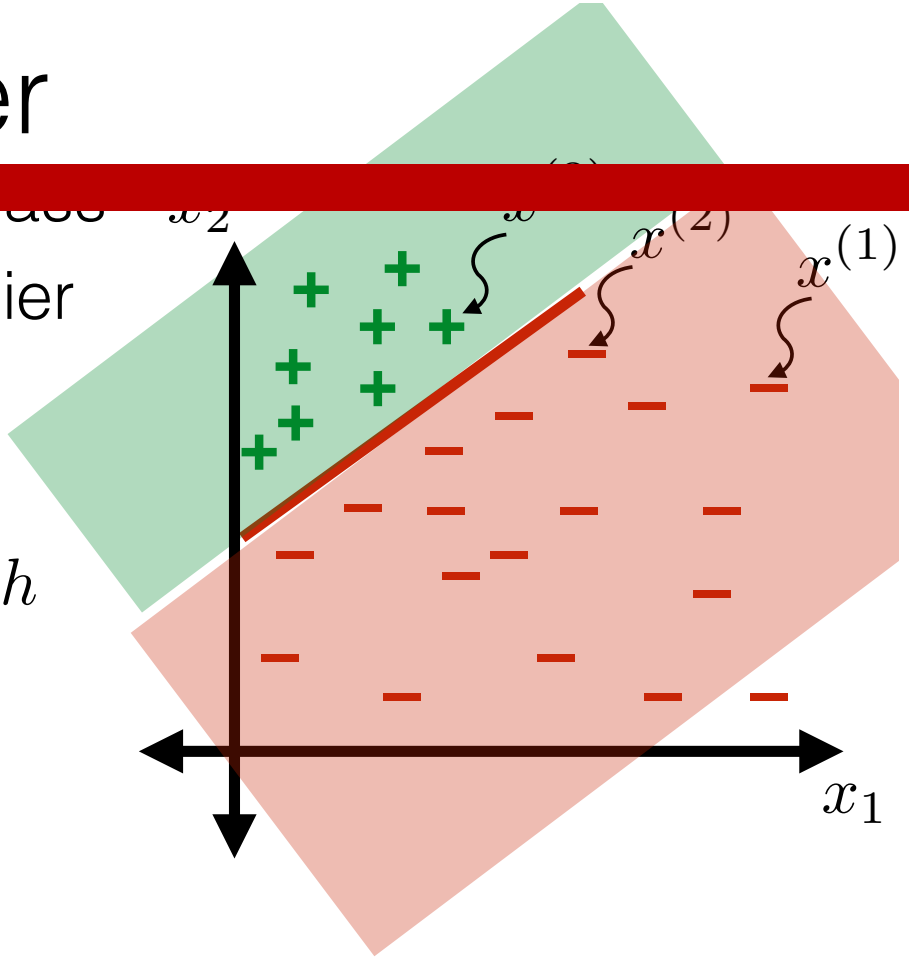

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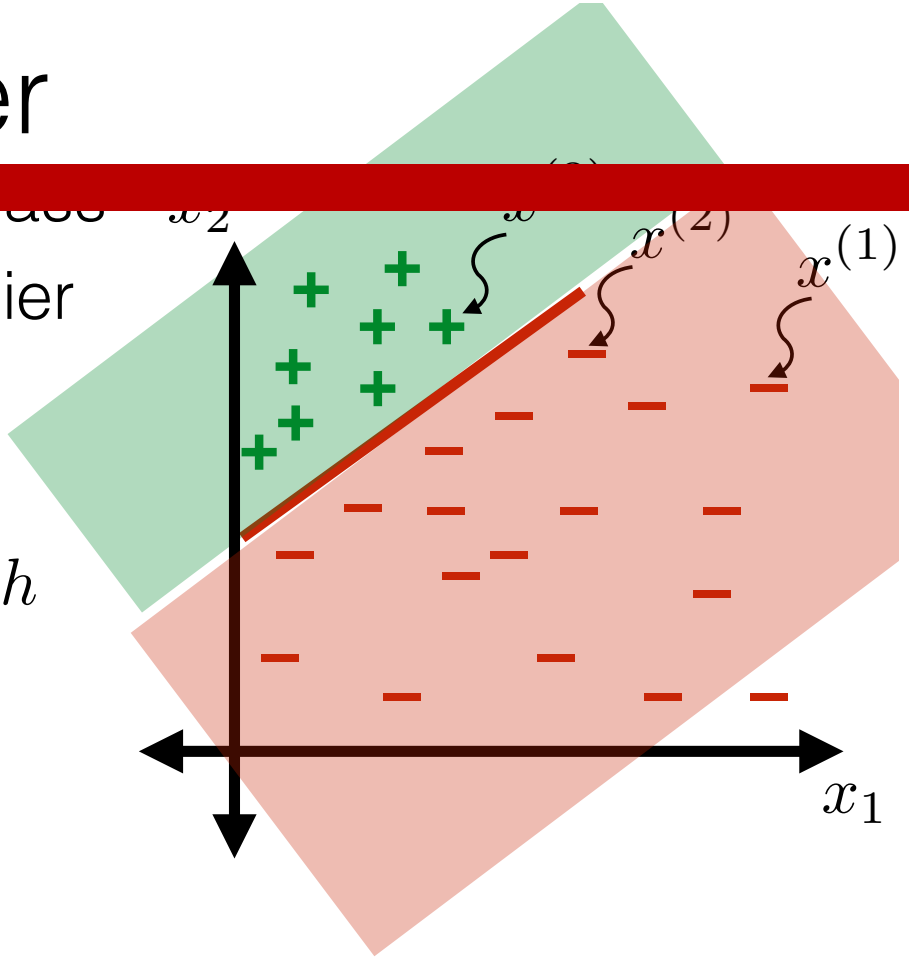

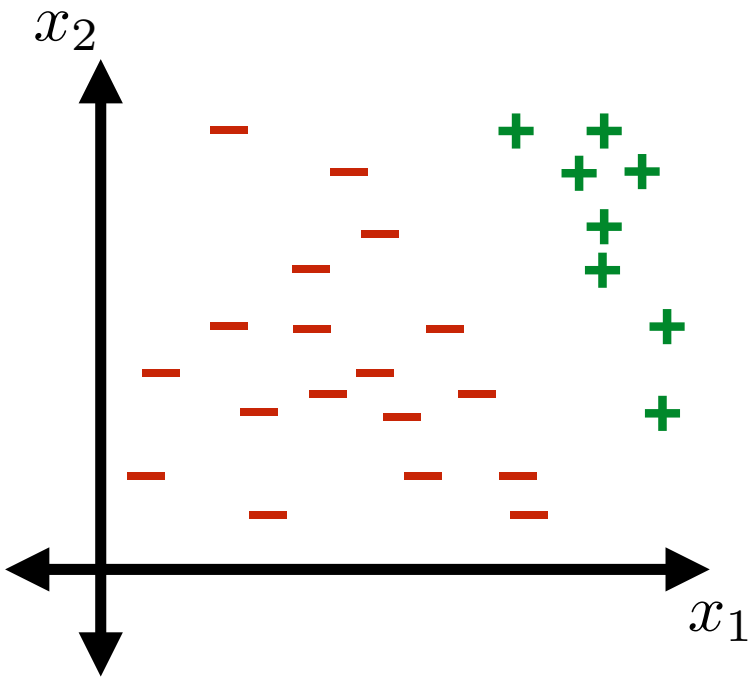

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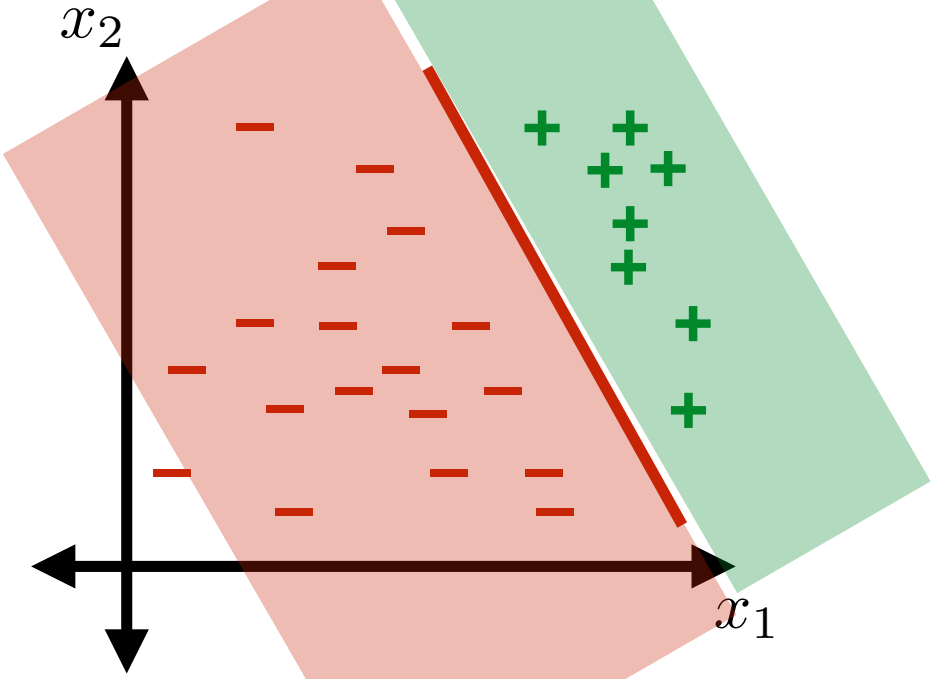

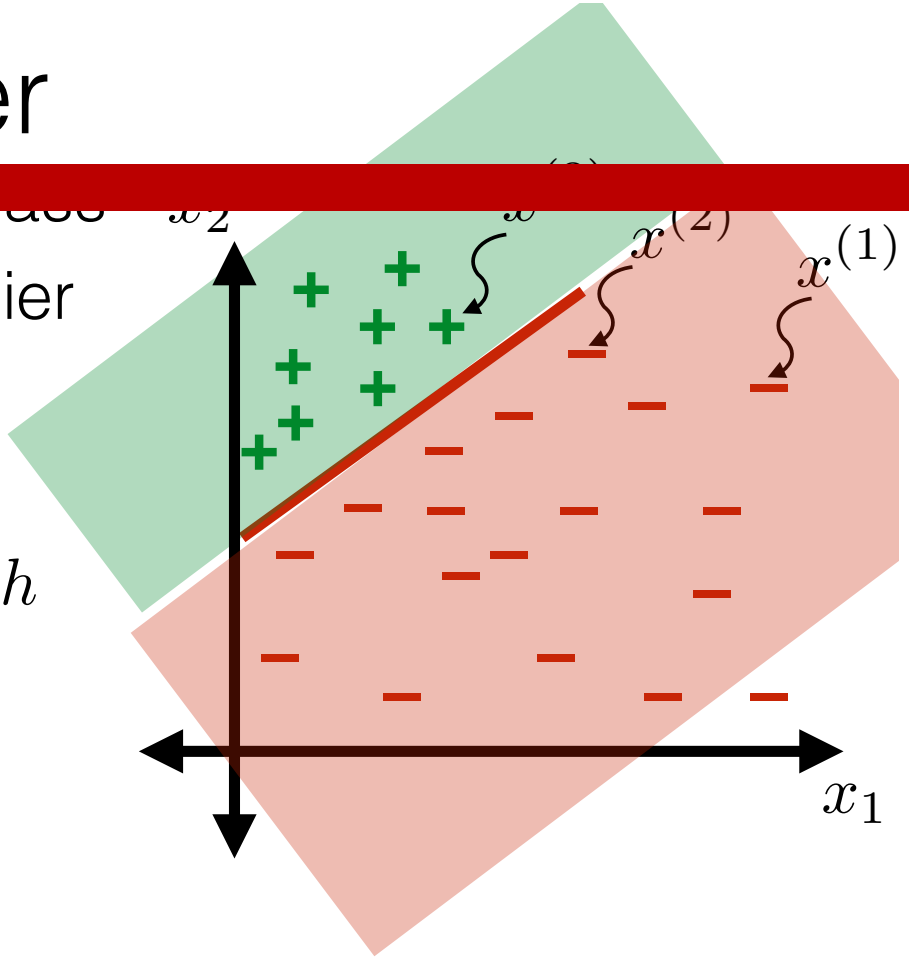

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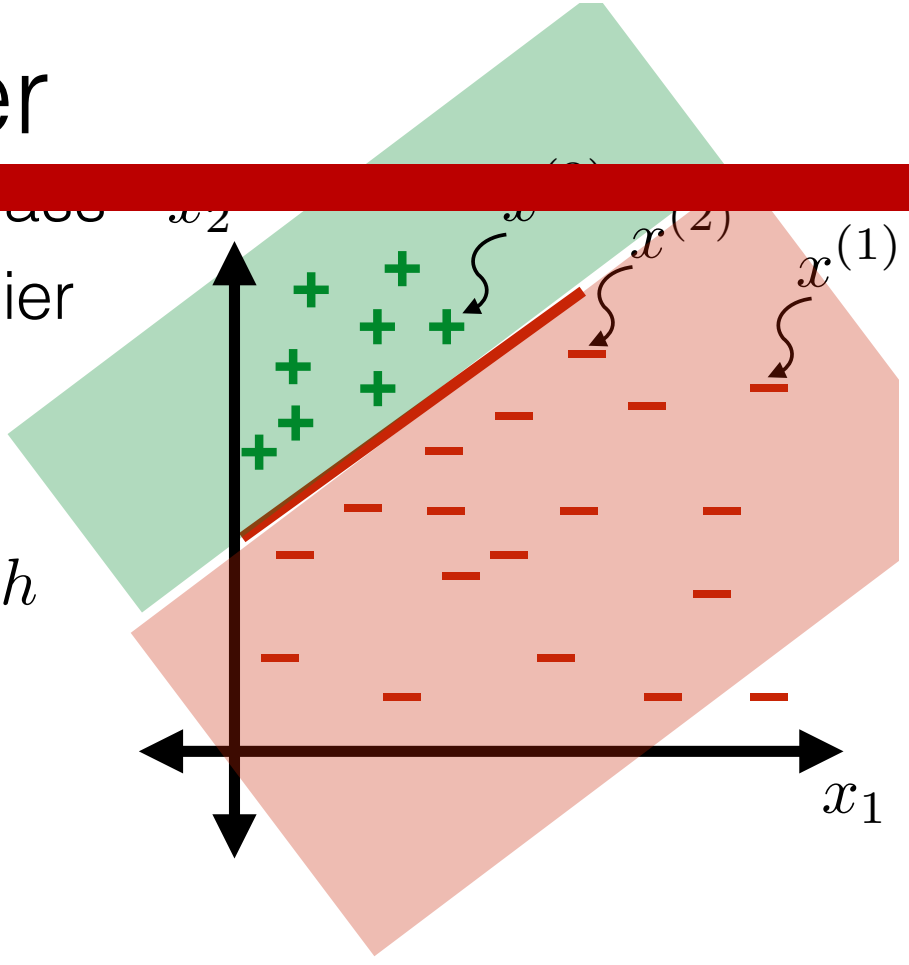

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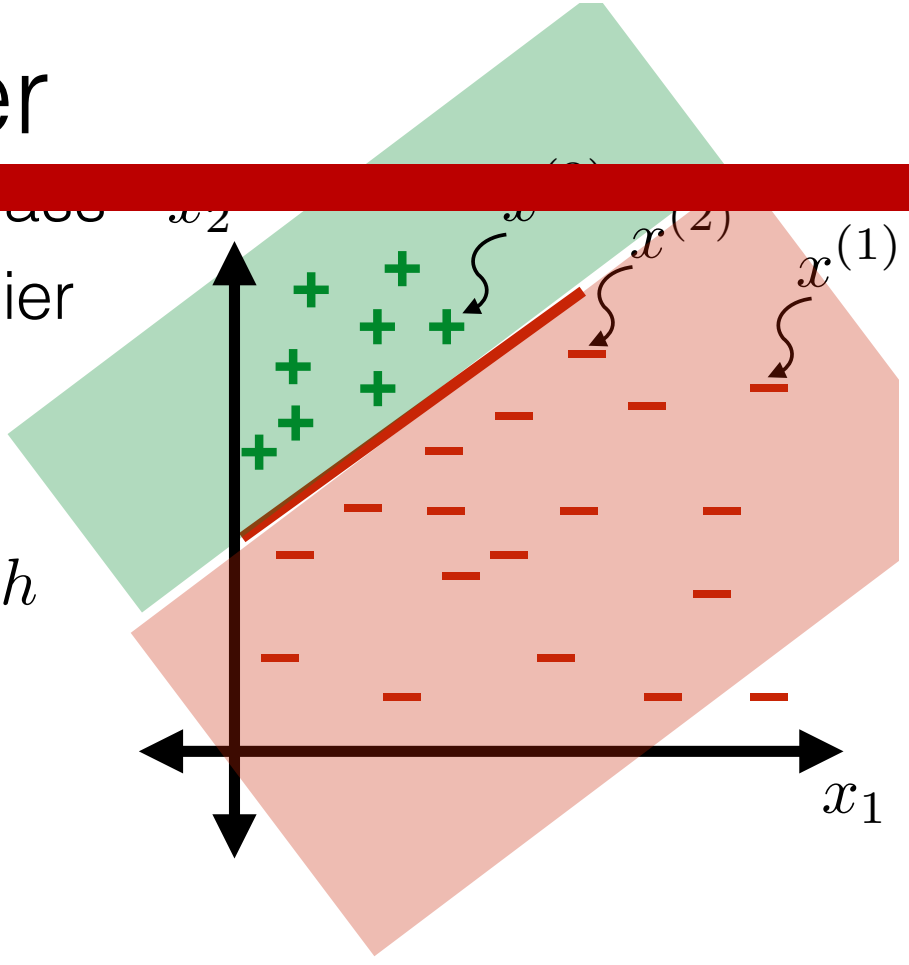

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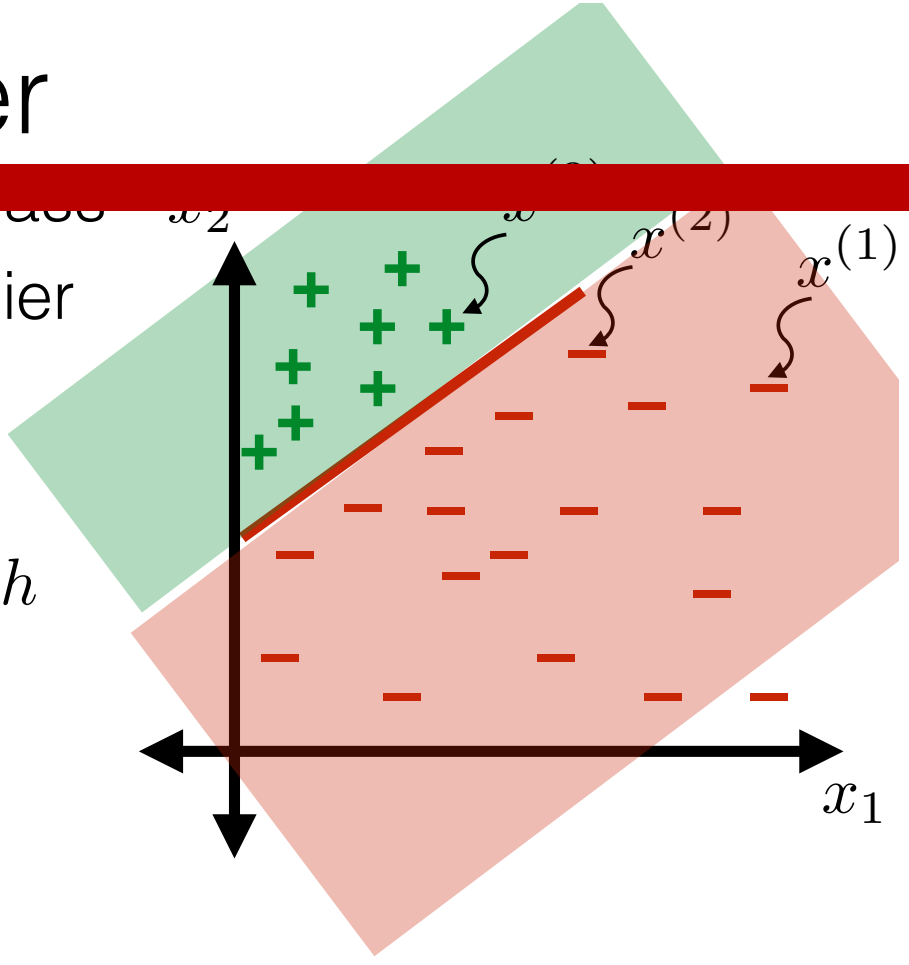

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- Example:

```
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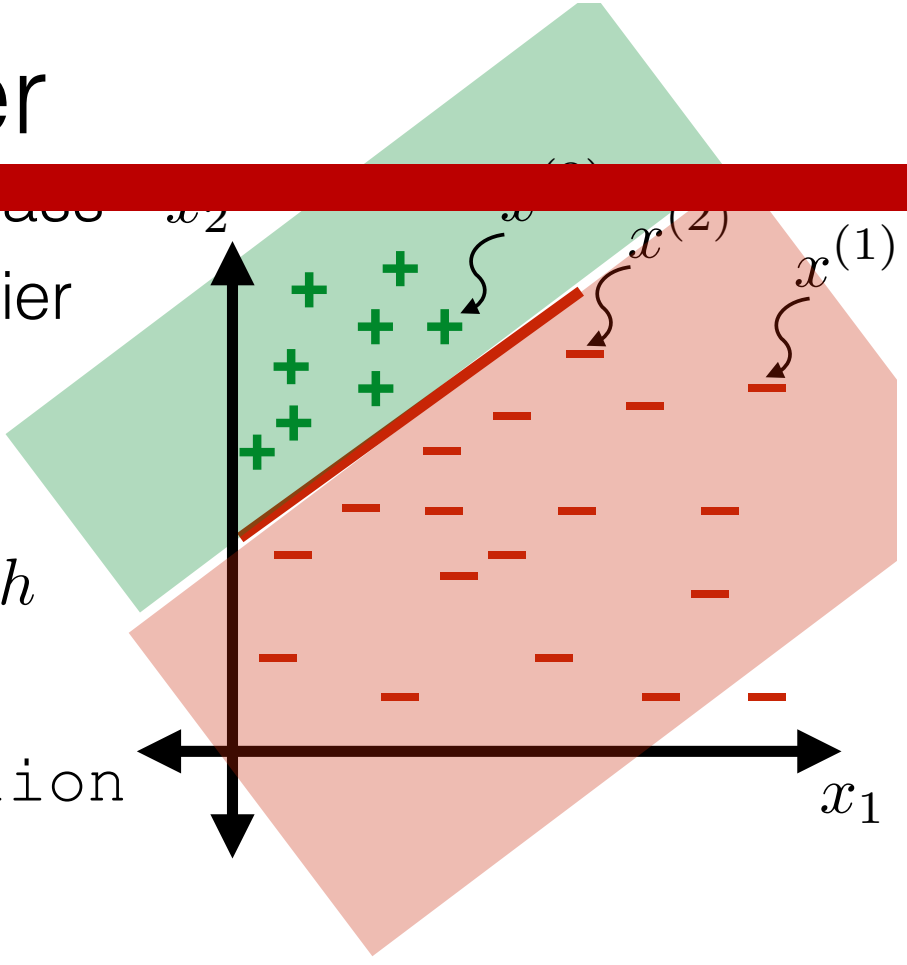
```



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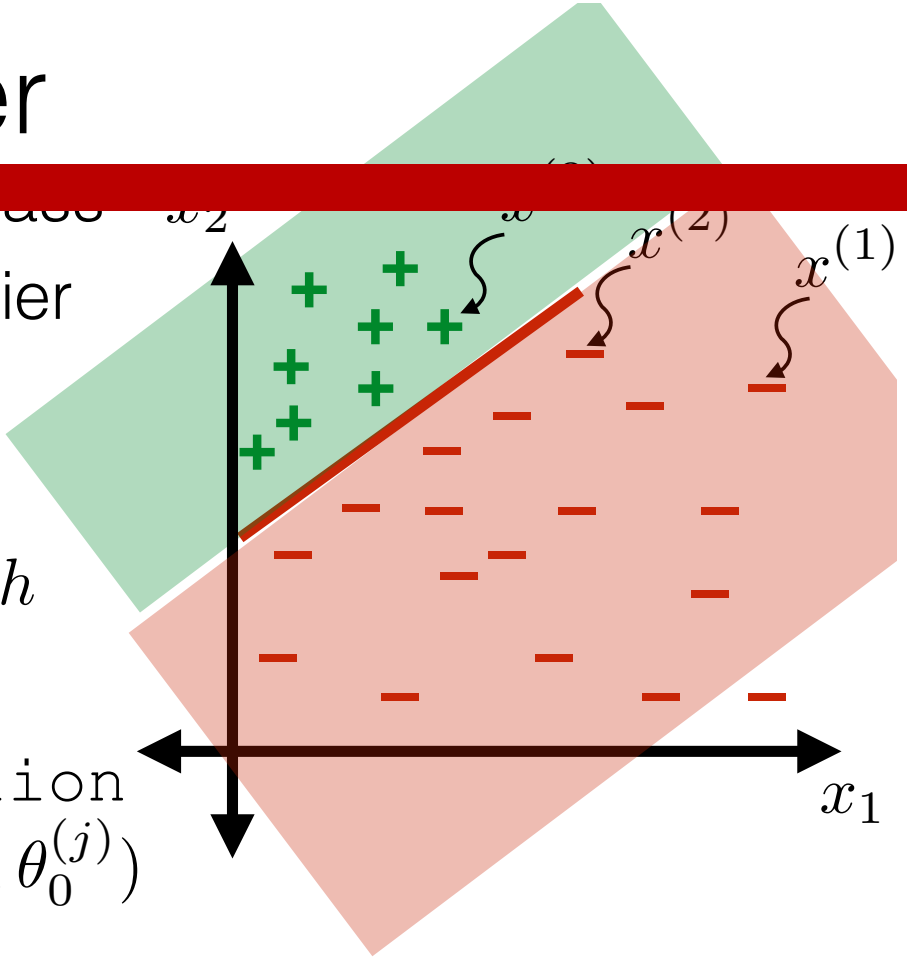

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- Example:

```
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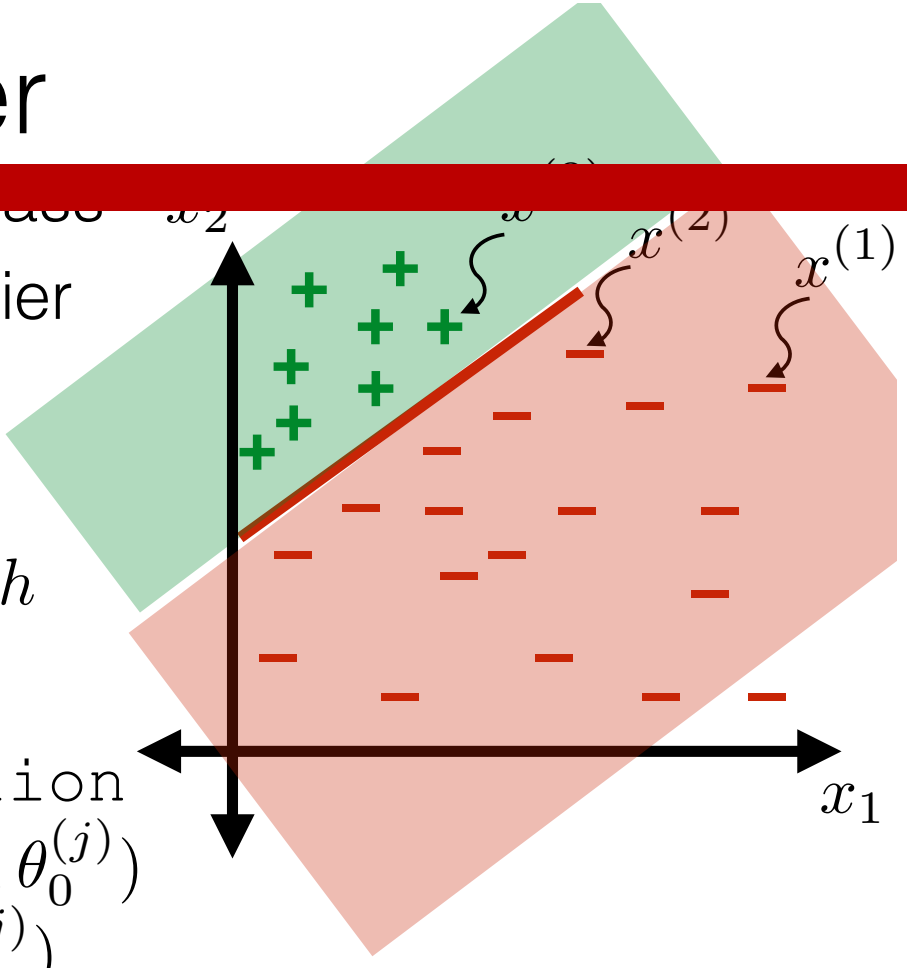
  Set h^(j)(x) = h(x; θ^(j), θ_0^(j))
```



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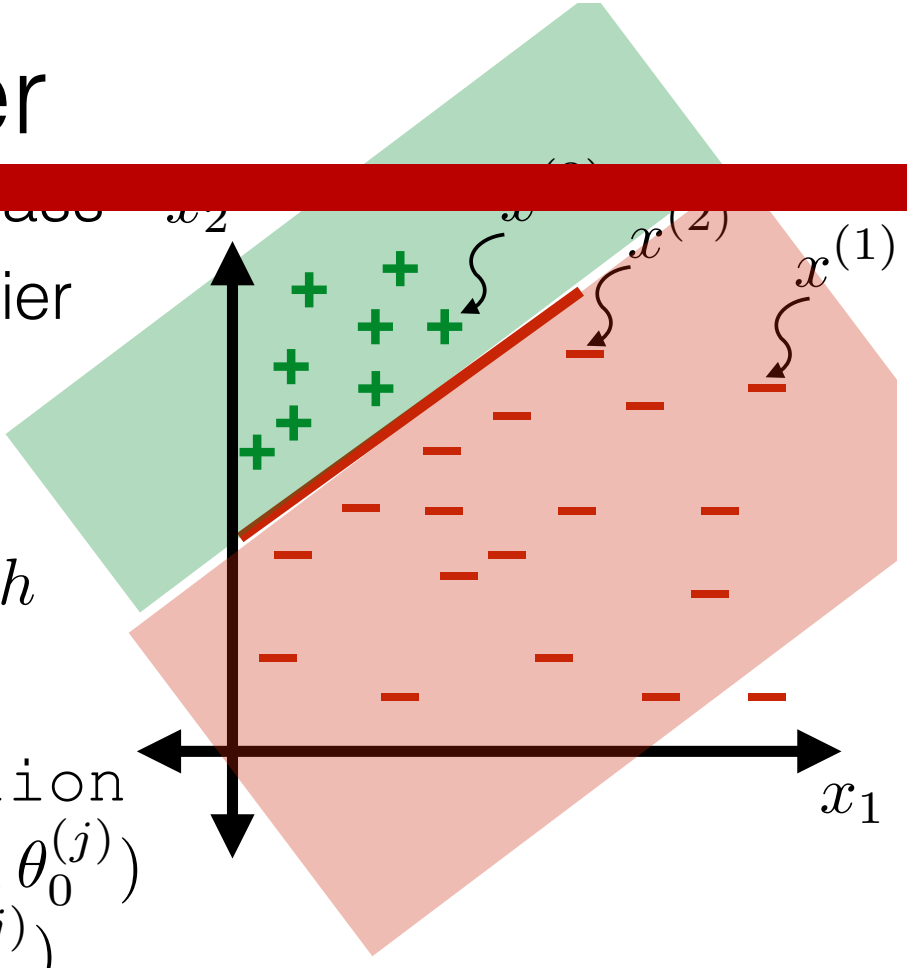

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for j = 1, …, 1 trillion
  Randomly sample (θ^(j), θ_0^(j))
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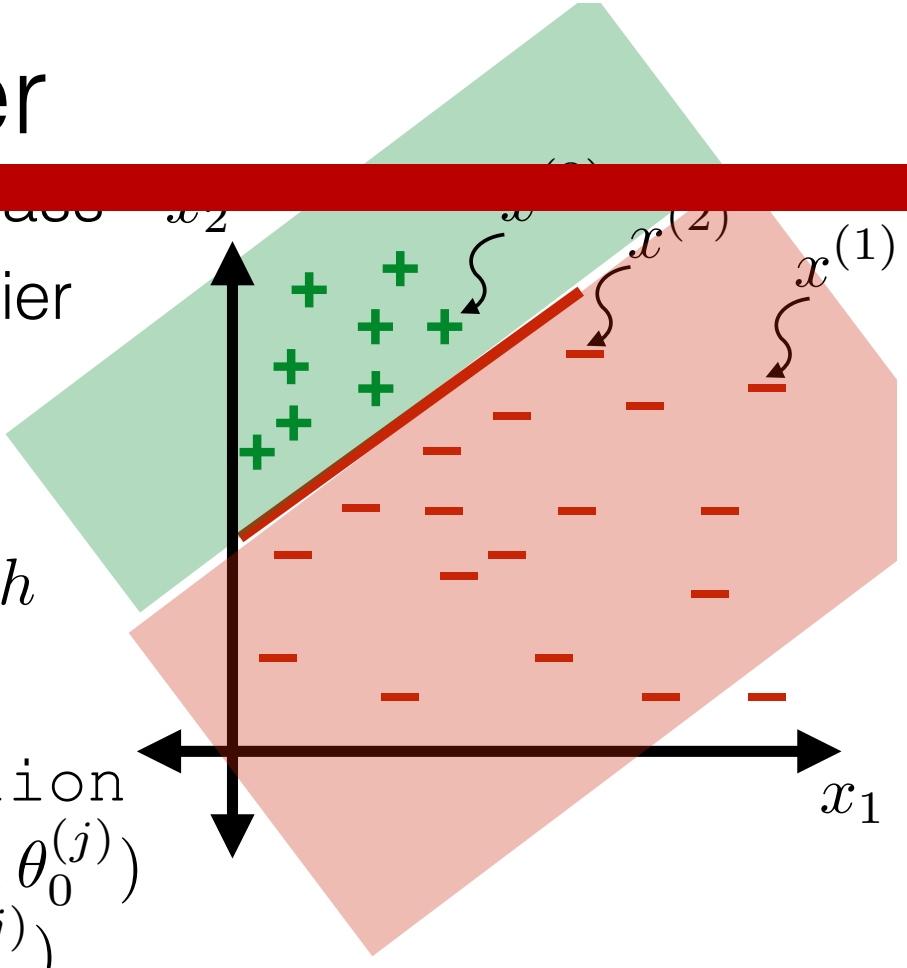
Ex_learning_alg
```



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- Example:

```
for j = 1, …, 1 trillion
  Randomly sample (θ^(j), θ_0^(j))
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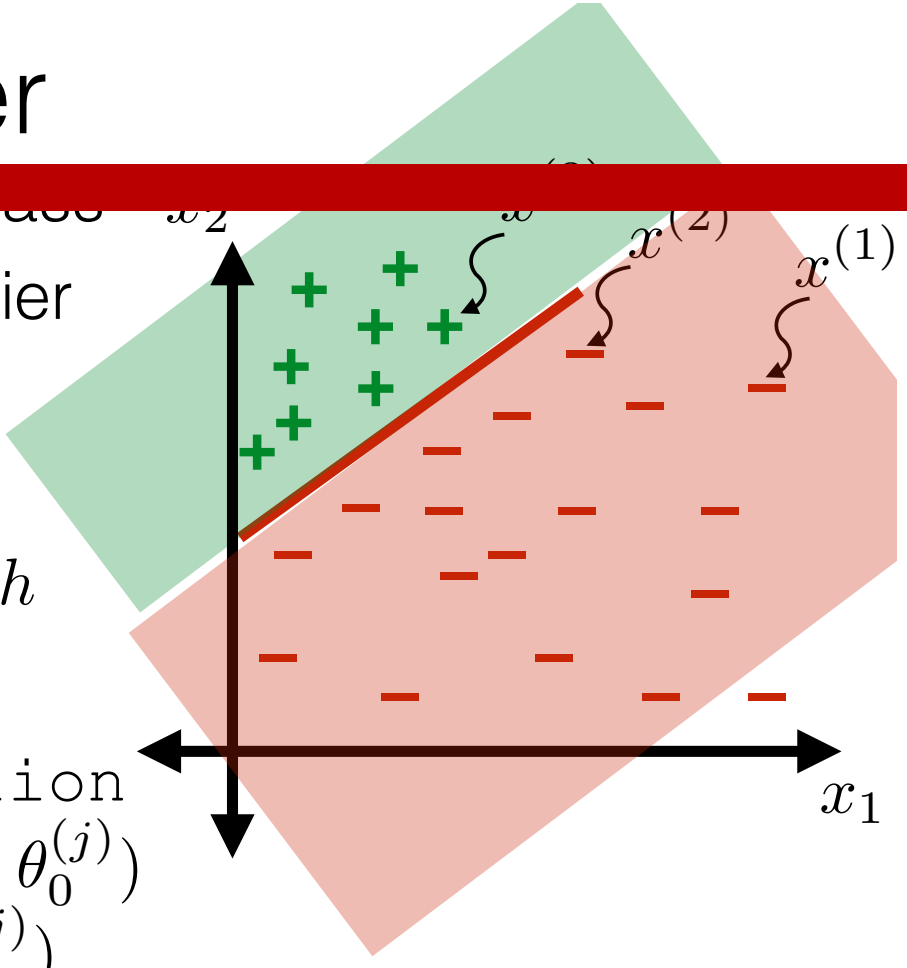
Ex_learning_alg( D_n )
```



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- Example:

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for j = 1, …, 1 trillion
    Randomly sample (θ^(j), θ_0^(j))
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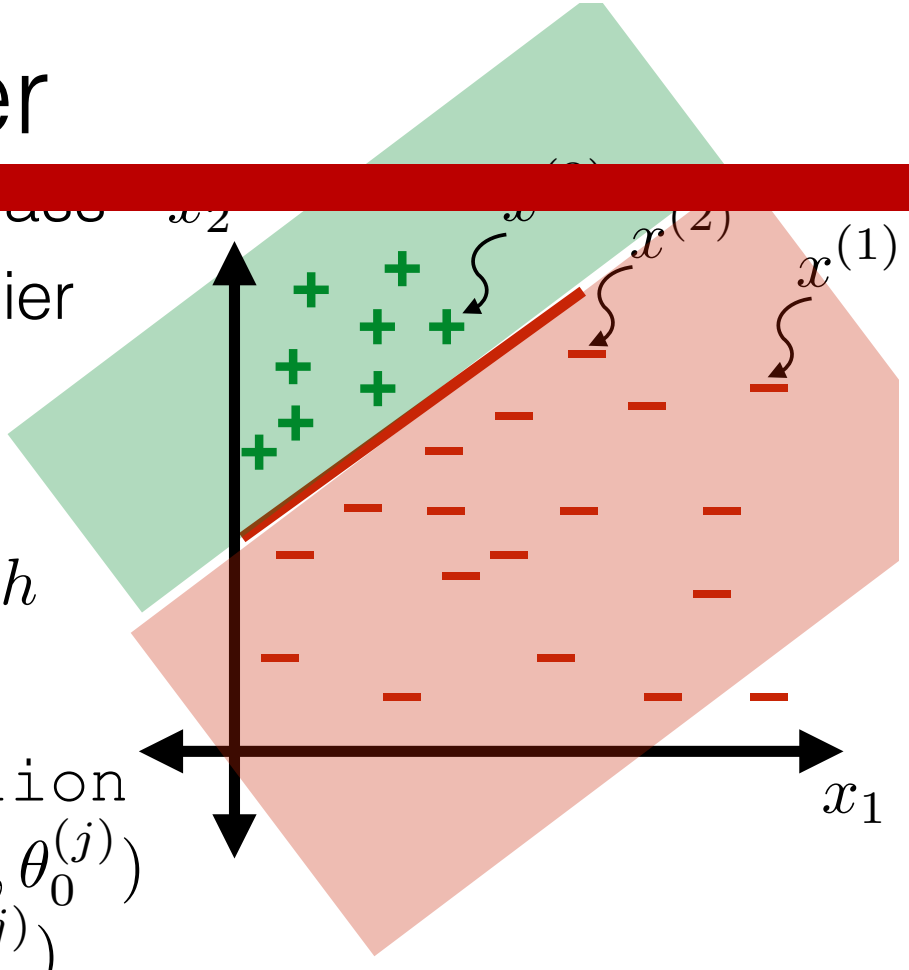
Ex_learning_alg( D_n ; k < 1 trillion)
```



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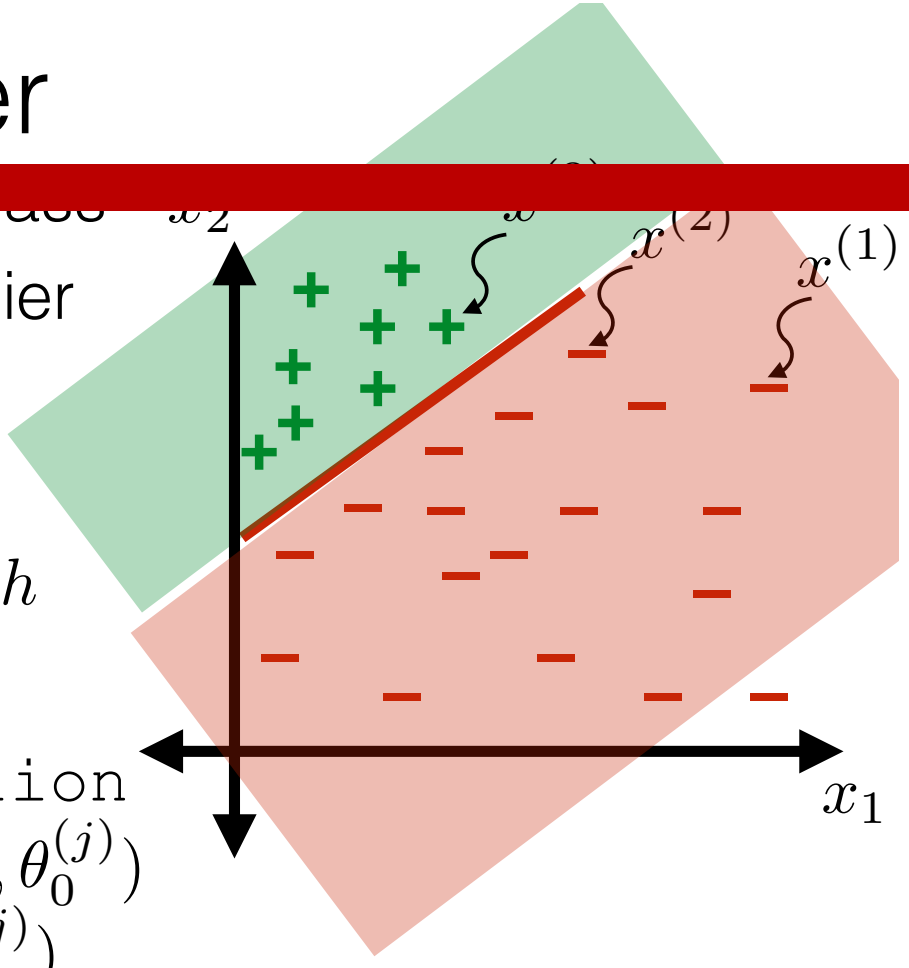
hyperparameter



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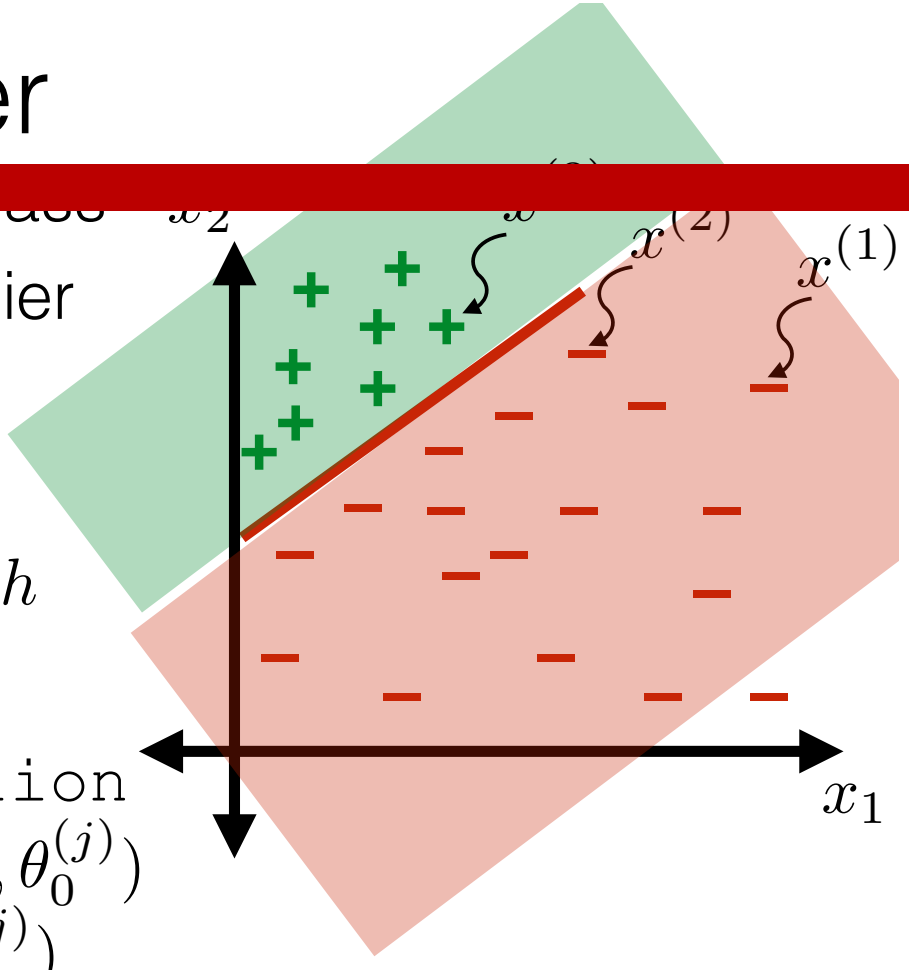
    Randomly sample (θ^(j), θ_0^(j))
    Set h^(j)(x) = h(x; θ^(j), θ_0^(j))
Ex_learning_alg( D_n ; k < 1 trillion)
    Set j* = argmin_{j∈{1,…,k}} E_n(h^(j))
```



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Ex_learning_alg( D_n ; k < 1 trillion)
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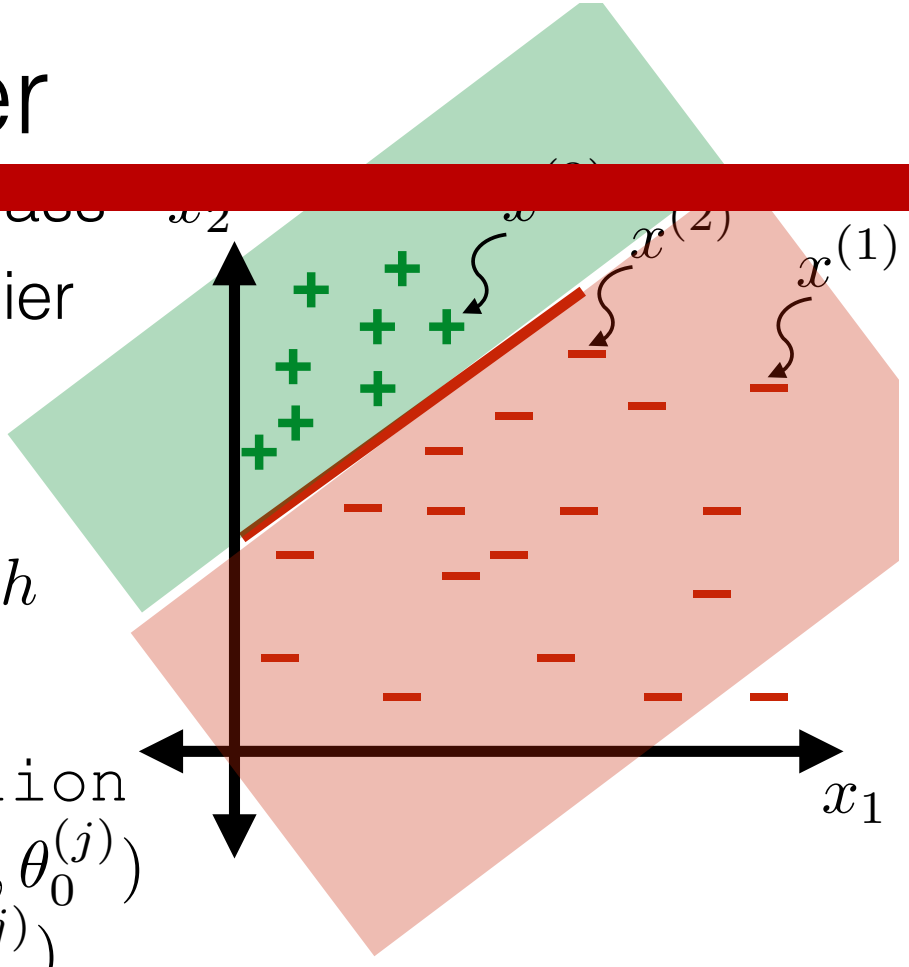
    Return h^(j*)
```

hyperparameter



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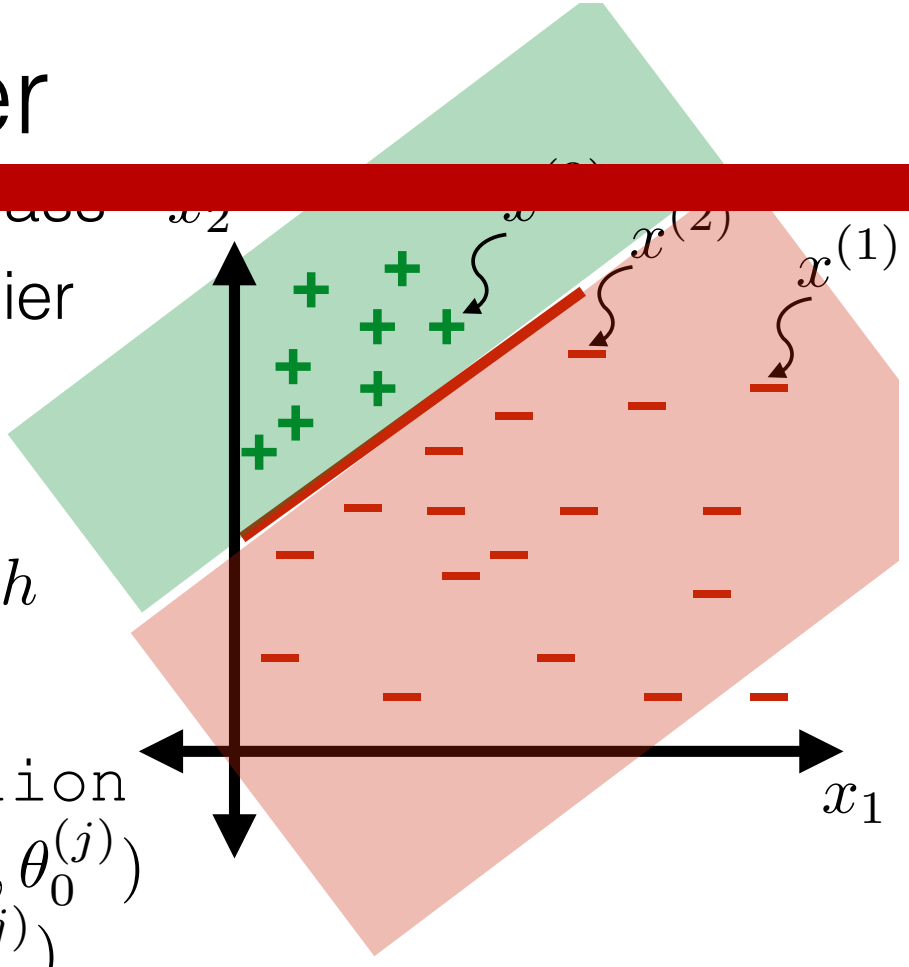

- How does training error of `Ex_learning_alg(`$\mathcal{D}_n$`;1)` compare to the training error of `Ex_learning_alg(`$\mathcal{D}_n$`;2)`?

Perceptron Algorithm

Perceptron Algorithm

Perceptron

Perceptron Algorithm

Perceptron($\mathcal{D}_n$; τ)

Perceptron Algorithm

Perceptron($\mathcal{D}_n$; τ)

  Initialize $\theta = [0 \ 0 \ \dots \ 0]^\top$

  Initialize $\theta_0 = 0$

Perceptron Algorithm

`Perceptron(` $\mathcal{D}_n$ `;` τ `)`

`Initialize` $\theta = [0 \;\; 0 \;\; \ldots \;\; 0]^\top$ [How many 0s?]

`Initialize` $\theta_0 = 0$

Perceptron Algorithm

Perceptron($\mathcal{D}_n$; τ)

  Initialize $\theta = [0 \ 0 \ \ldots \ 0]^\top$ [How many 0s?]

  Initialize $\theta_0 = 0$

  for t = 1 to τ

Perceptron Algorithm

Perceptron($\mathcal{D}_n$; τ)

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  for t = 1 to τ

    changed = False

Perceptron Algorithm

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  Initialize $\theta = [0 \ 0 \ \ldots \ 0]^\top$ [How many 0s?]

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  for t = 1 to τ

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  Initialize $\theta = [0 \ 0 \ \ldots \ 0]^\top$ [How many 0s?]

  Initialize $\theta_0 = 0$

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    for i = 1 to n

      if $y^{(i)}(\theta^\top x^{(i)} + \theta_0) \leq 0$

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[How many 0s?]

[i.e. True if either:

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Perceptron( D_n ; τ )
  Initialize  θ = [0  0  ...  0]^⊤
  Initialize  θ_0 = 0
  for t = 1 to τ
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    for i = 1 to n
      if  y^(i)(θ^⊤ x^(i) + θ_0) ≤ 0
```

[How many 0s?]

[i.e. True if either:

A. point is not on the line & prediction is wrong

Perceptron Algorithm

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  Initialize θ = [0 0 ... 0]^⊤
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```

[How many 0s?]

[i.e. True if either:

A. point is not on the line & prediction is wrong
B. point is on the line

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Perceptron($\mathcal{D}_n$; τ)

  Initialize $\theta = [0 \ 0 \ \ldots \ 0]^\top$

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[i.e. True if either:

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C. initial step]

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        Set $\theta = \theta + y^{(i)} x^{(i)}$

        Set $\theta_0 = \theta_0 + y^{(i)}$

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[i.e. True if either:

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        Set  θ = θ + y^(i) x^(i)
        Set  θ_0 = θ_0 + y^(i)
        changed = True
    if not changed
      break
```

[How many 0s?]

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        Set  θ_0 = θ_0 + y^(i)
        changed = True
    if not changed
      break
  Return  θ, θ_0
```

[How many 0s?]

[i.e. True if either:

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        Set $\theta_0 = \theta_0 + y^{(i)}$

        changed = True

    if not changed

      break

  Return θ, θ_0

[How many 0s?]

[i.e. True if either:

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C. initial step]

Perceptron Algorithm

```
Perceptron( D_n ; τ )
  Initialize  θ = [0 0 ... 0]^T
  Initialize  θ_0 = 0
  for t = 1 to τ
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    for i = 1 to n
      if y^(i)(θ^T x^(i) + θ_0) ≤ 0
          Set  θ = θ + y^(i) x^(i)
          Set  θ_0 = θ_0 + y^(i)
          changed = True
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```

[How many 0s?]

[i.e. True if either:

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C. initial step]

What does an update do?

Perceptron Algorithm

```
Perceptron( D_n ; τ )
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  Initialize  θ_0 = 0
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      if y^(i)(θ^T x^(i) + θ_0) ≤ 0
        Set  θ = θ + y^(i) x^(i)
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    if not changed
      break
  Return  θ, θ_0
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[How many 0s?]

[i.e. True if either:

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What does an update do?

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        Set $\theta = \theta + y^{(i)} x^{(i)}$

        Set $\theta_0 = \theta_0 + y^{(i)}$

        changed = True

    if not changed

      break

  Return θ, θ_0

[How many 0s?]

[i.e. True if either:

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What does an update do?

$$y^{(i)}\left(\theta_{\text{updated}}^{\top} x^{(i)} + \theta_{0,\text{updated}}\right)$$

Perceptron Algorithm

Perceptron($\mathcal{D}_n$; τ)

  Initialize $\theta = [0 \;\; 0 \;\; \ldots \;\; 0]^\top$

  Initialize $\theta_0 = 0$

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      if $y^{(i)}(\theta^\top x^{(i)} + \theta_0) \leq 0$

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- *Definition*: A training set $\mathcal{D}_n$ is **linearly separable** if there exist θ, θ_0 such that, for every point index $i \in \{1, \ldots, n\}$, we have

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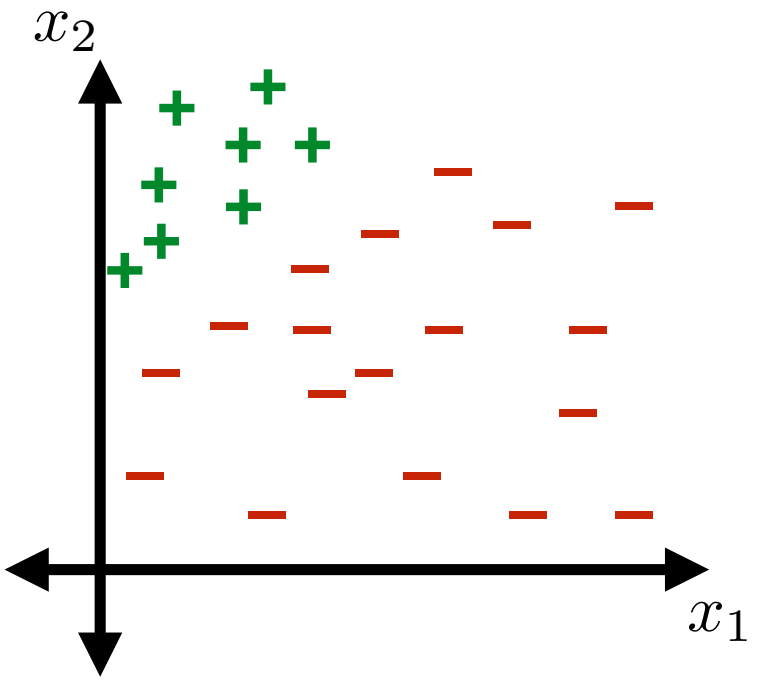

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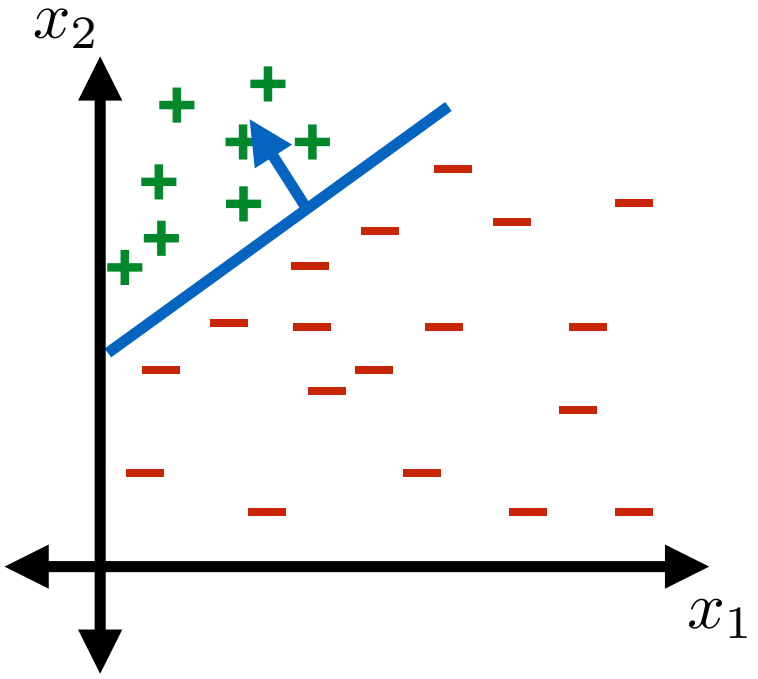

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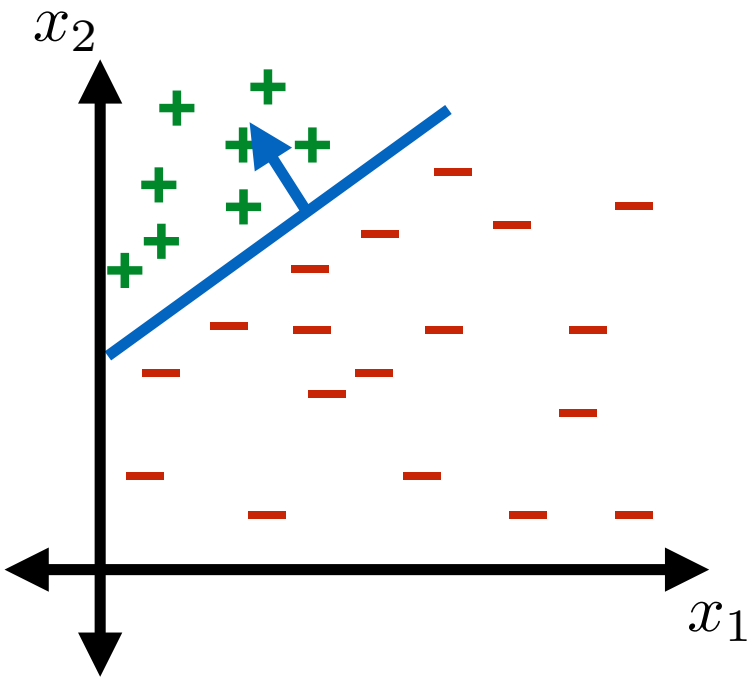

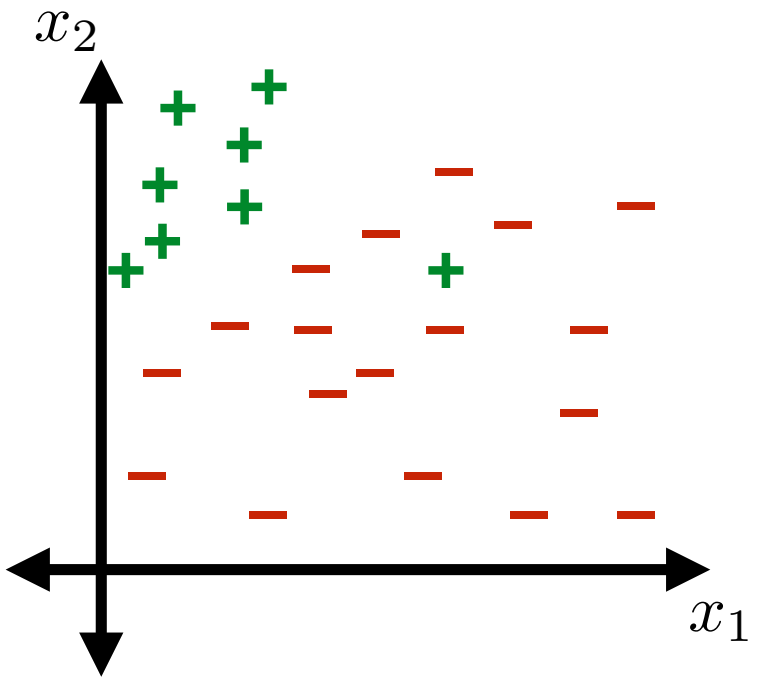

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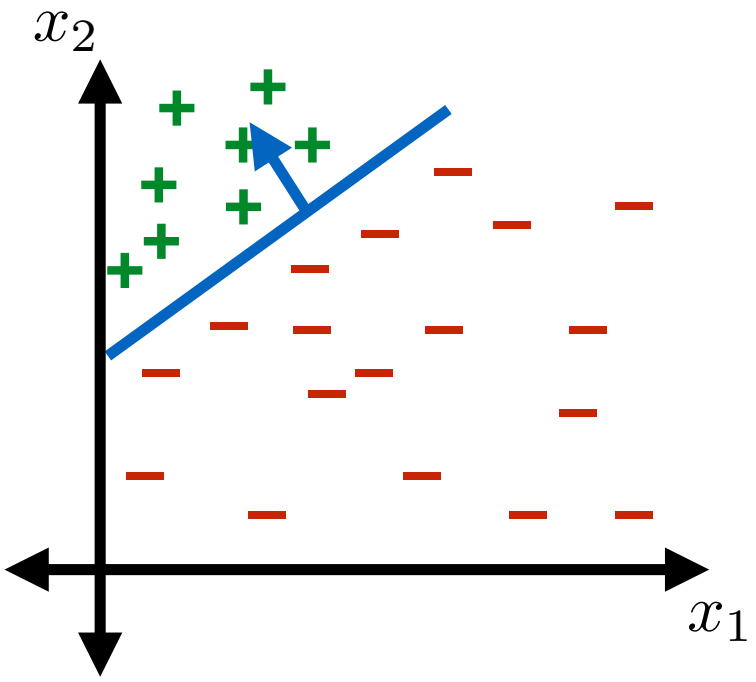

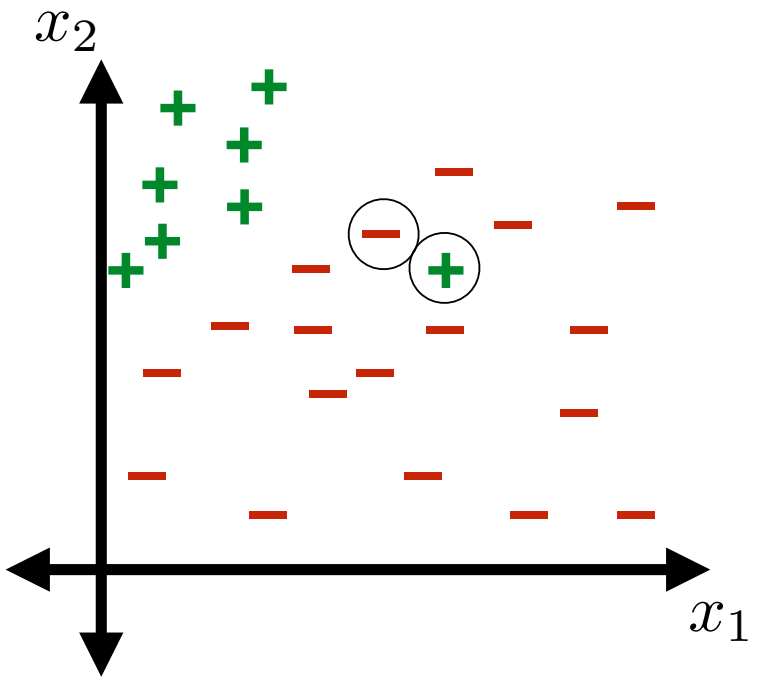

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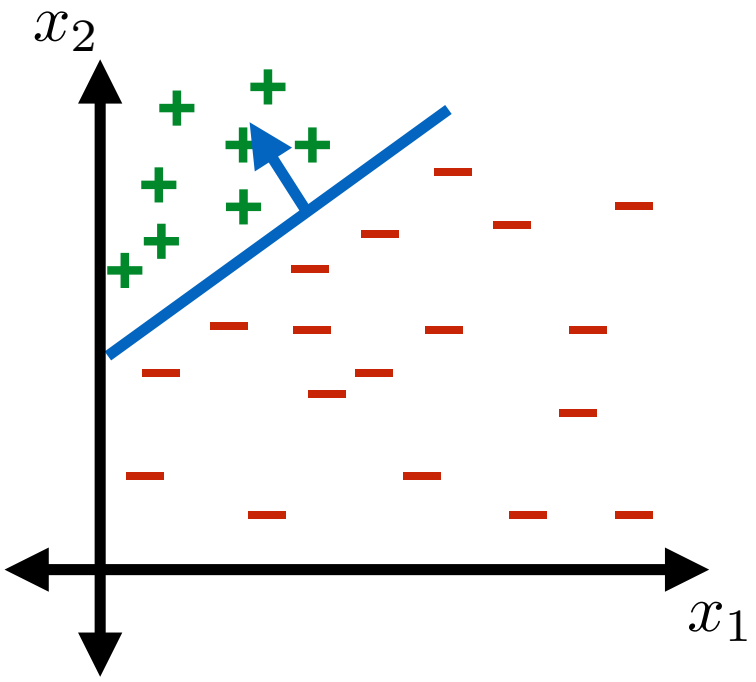
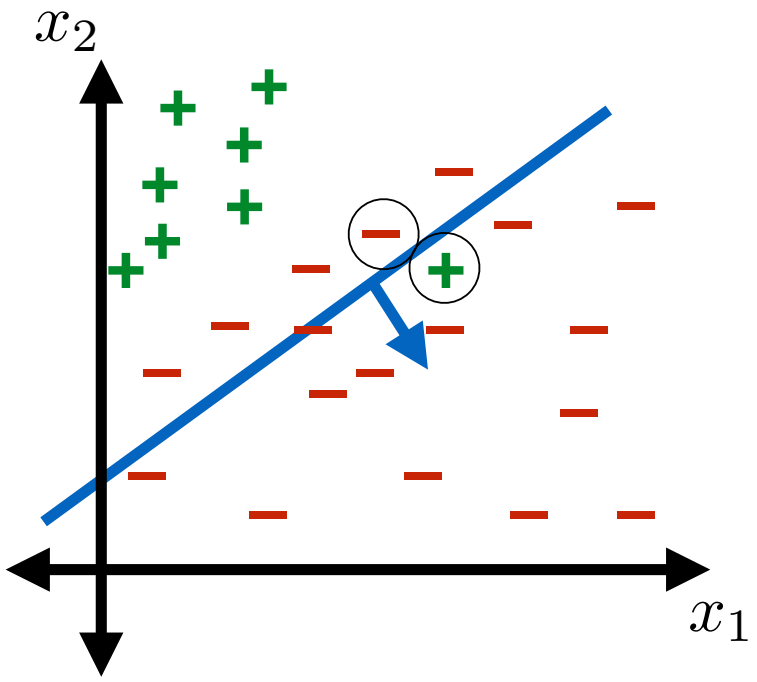
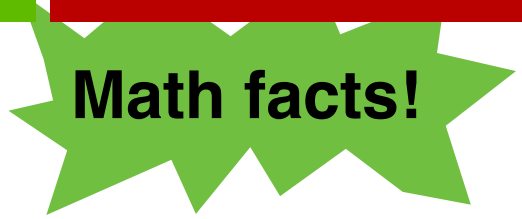




Let's Talk About Classifier Quality



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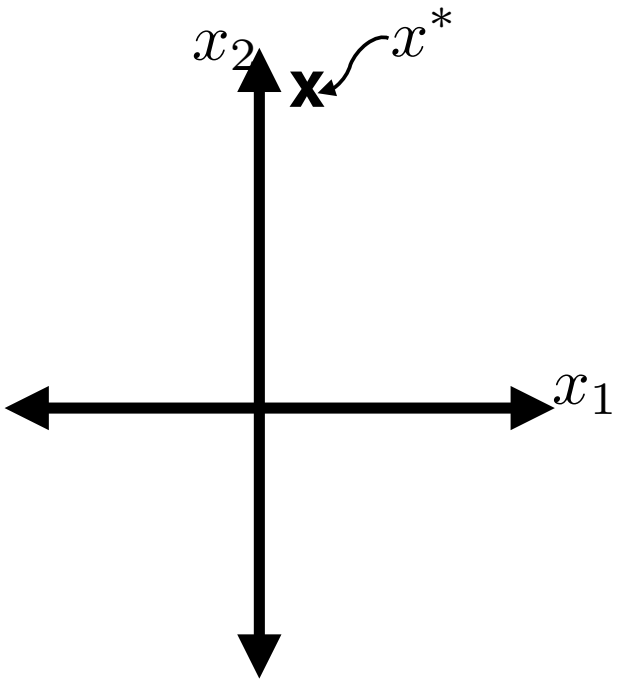

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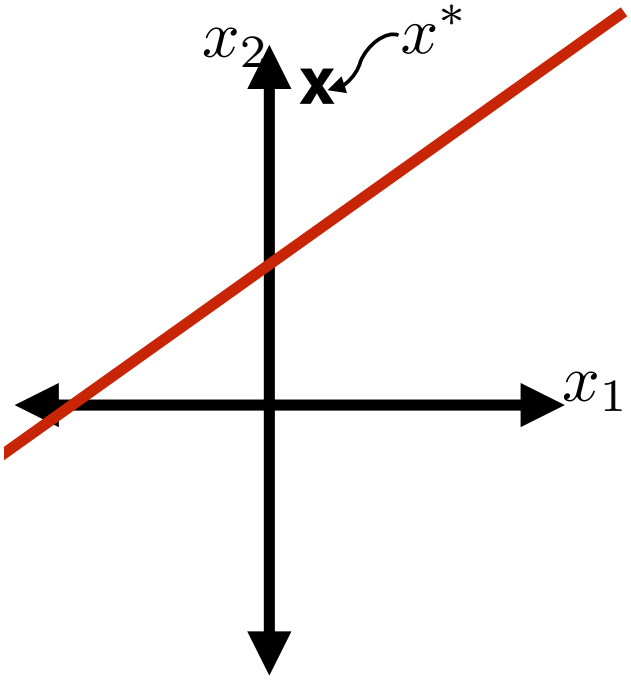

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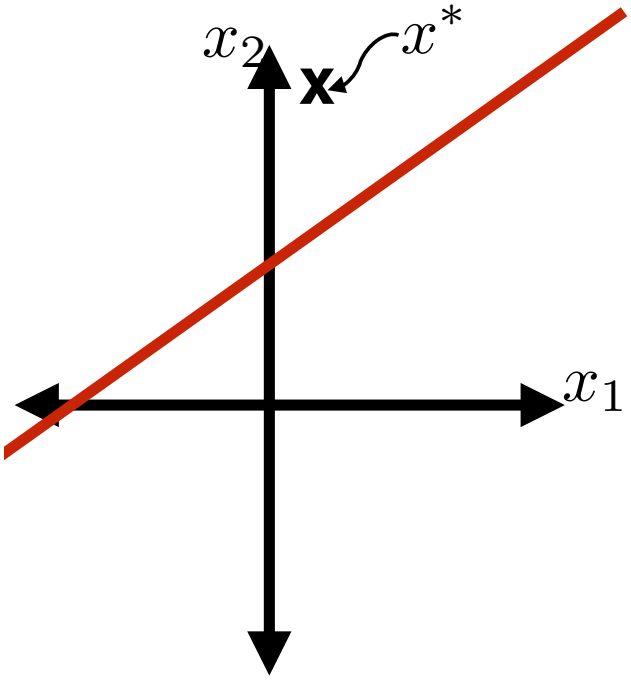

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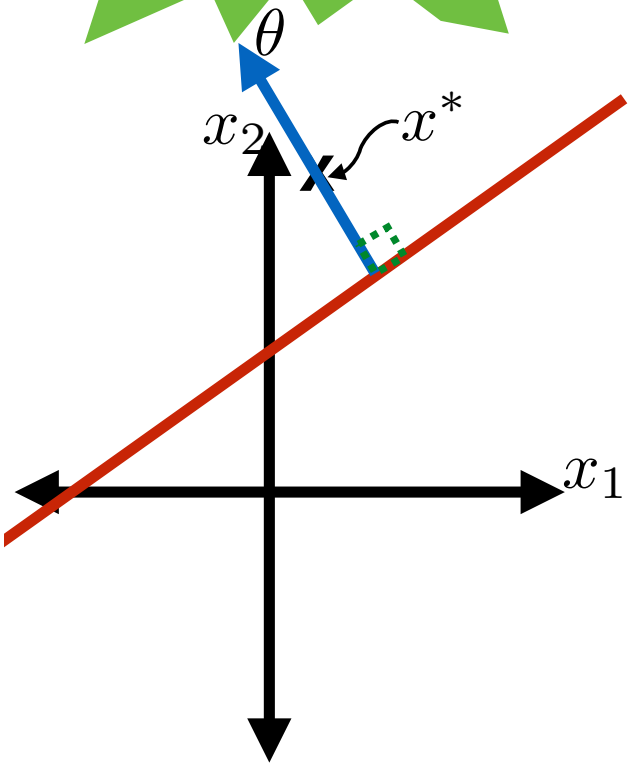

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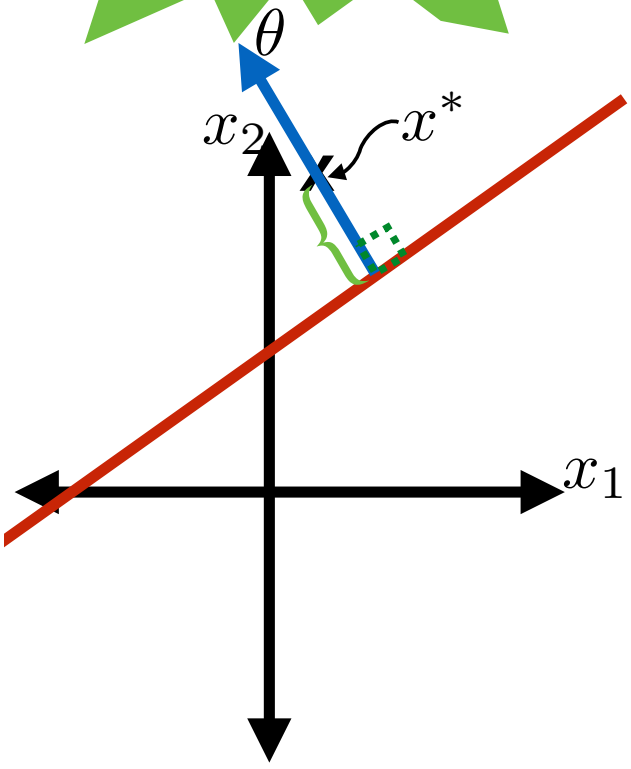

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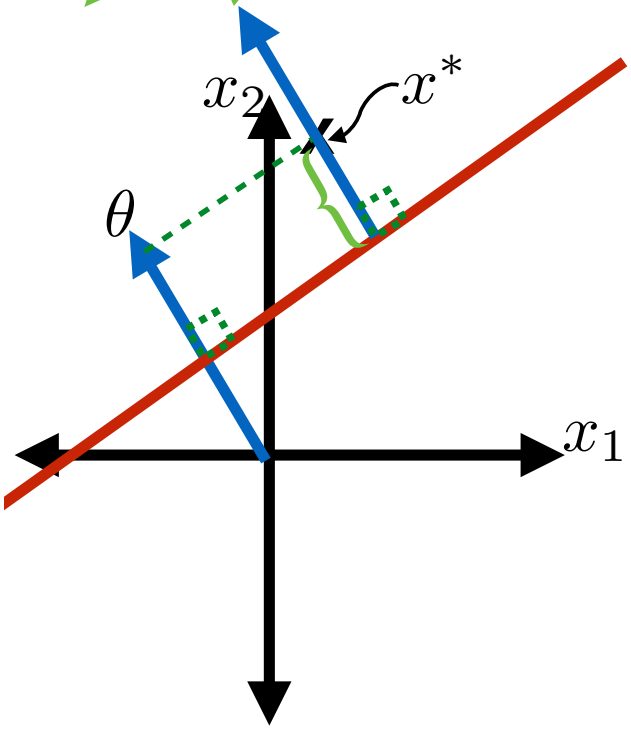

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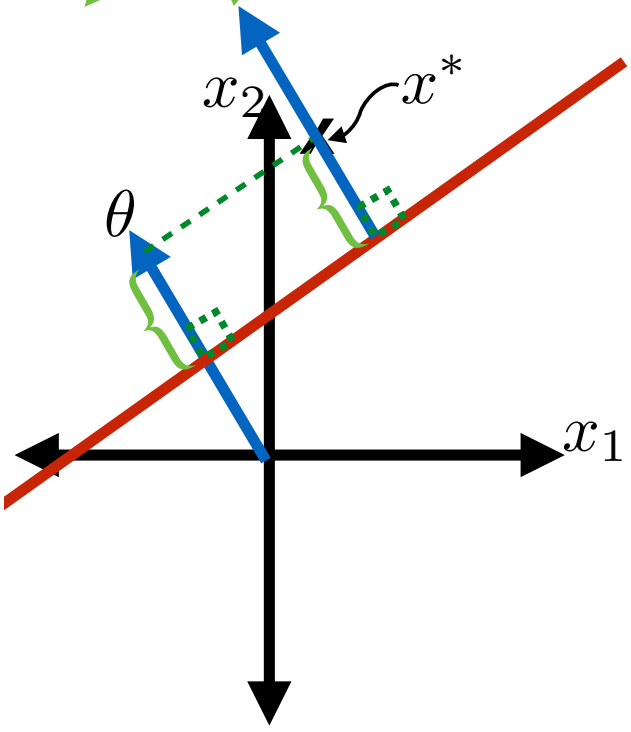

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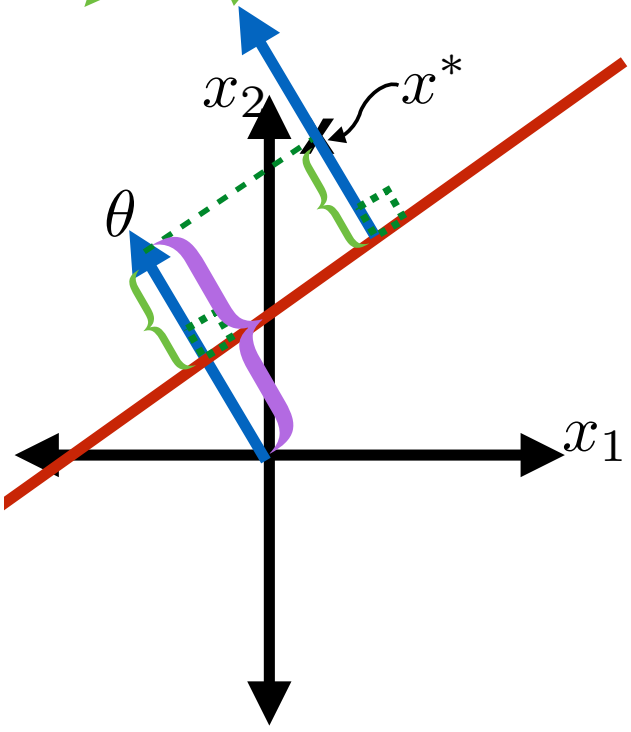

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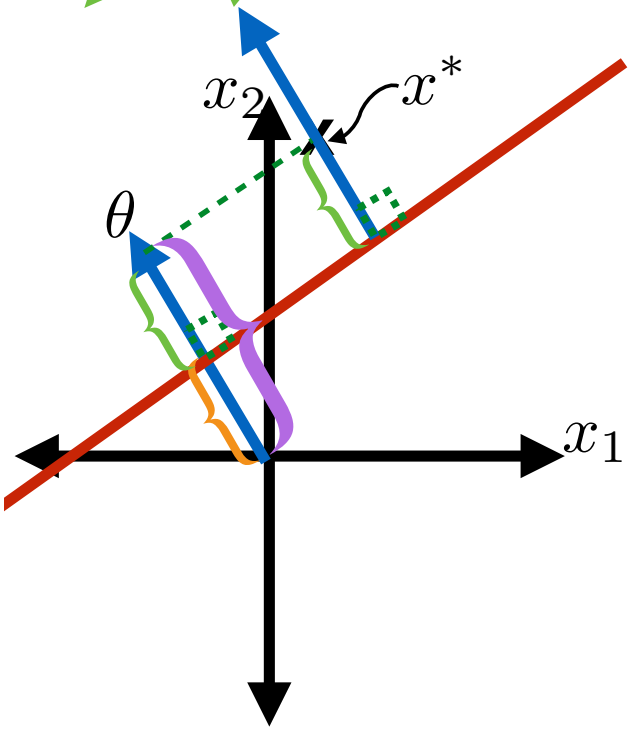

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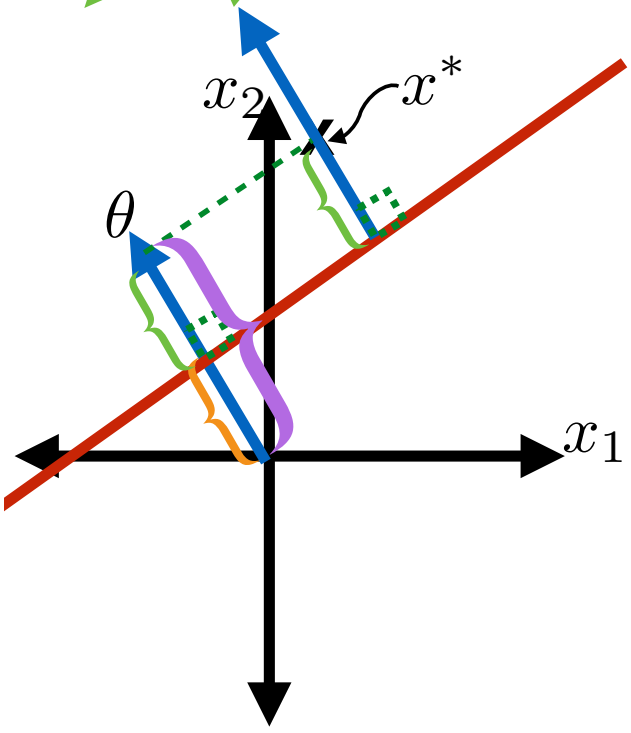

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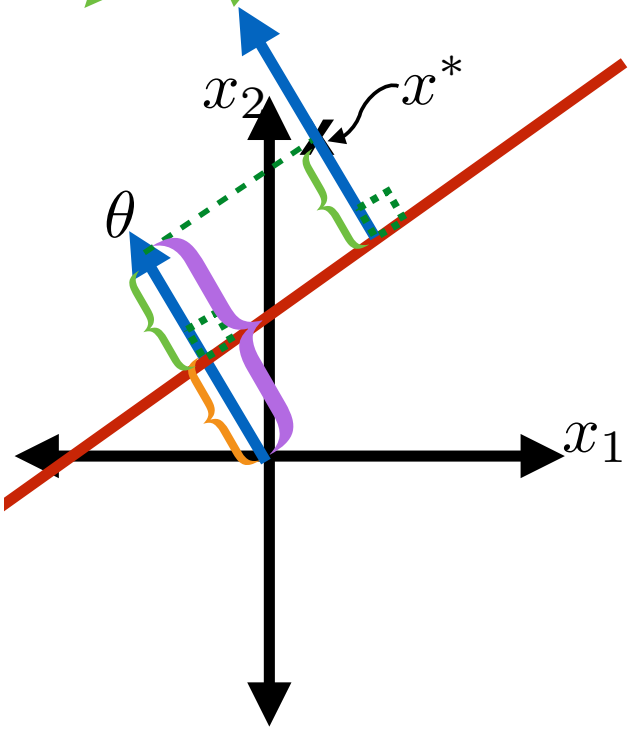

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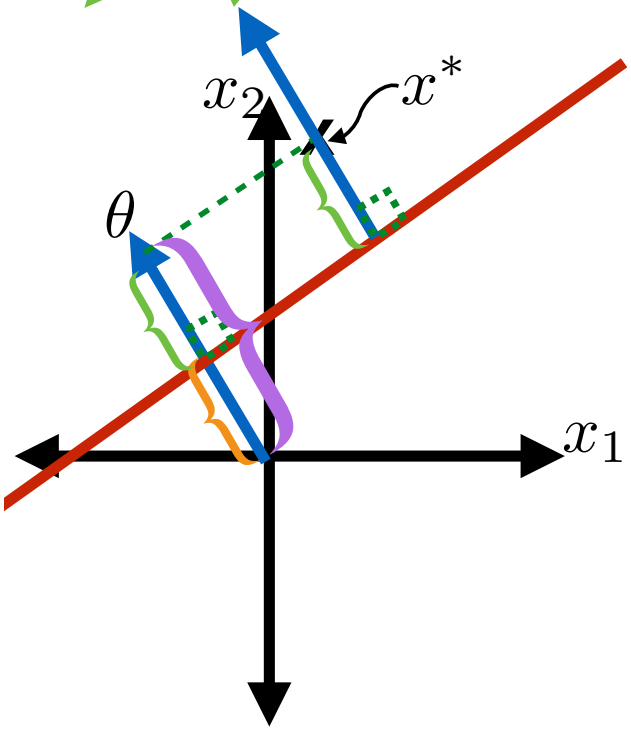

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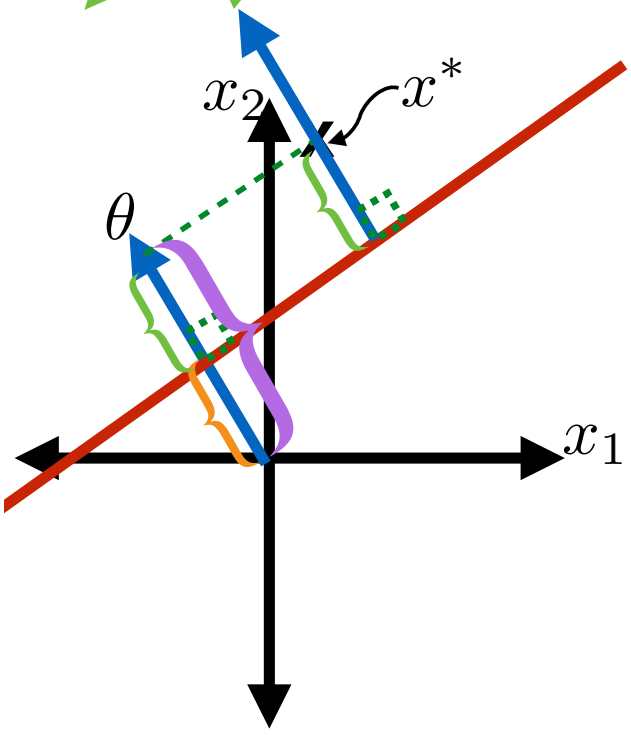

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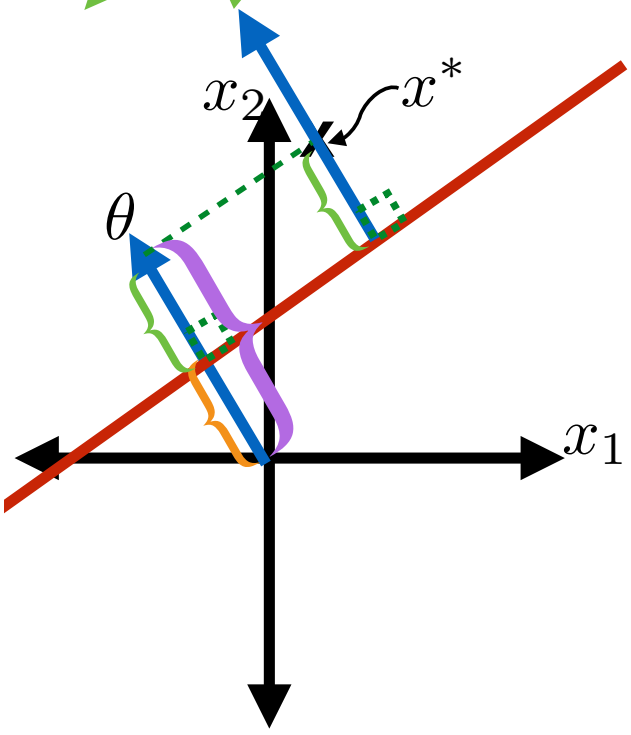

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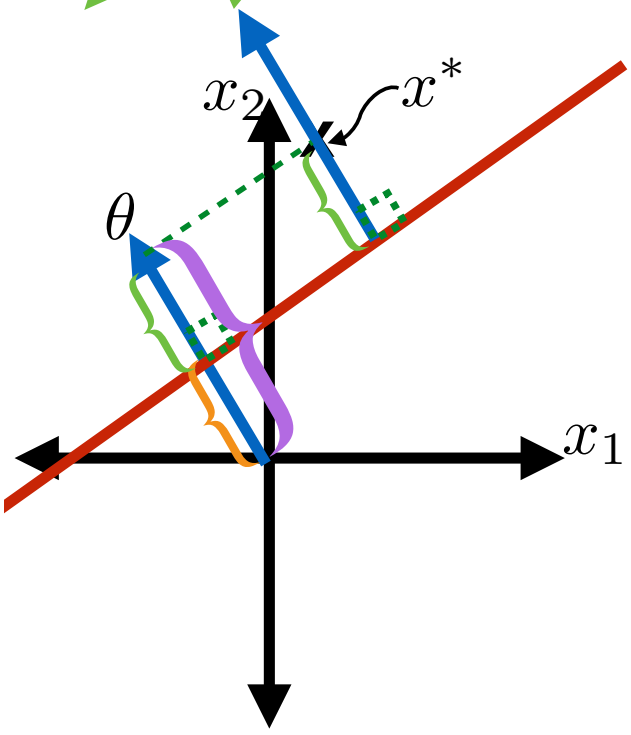

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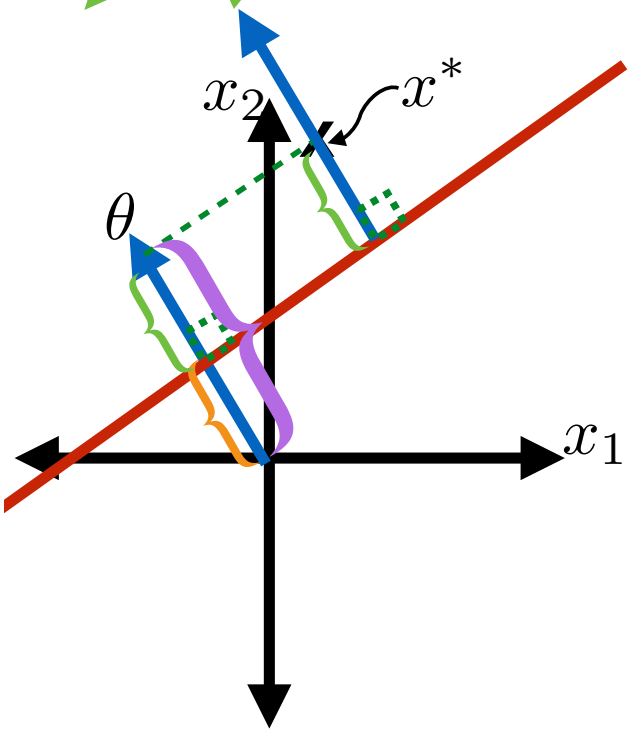

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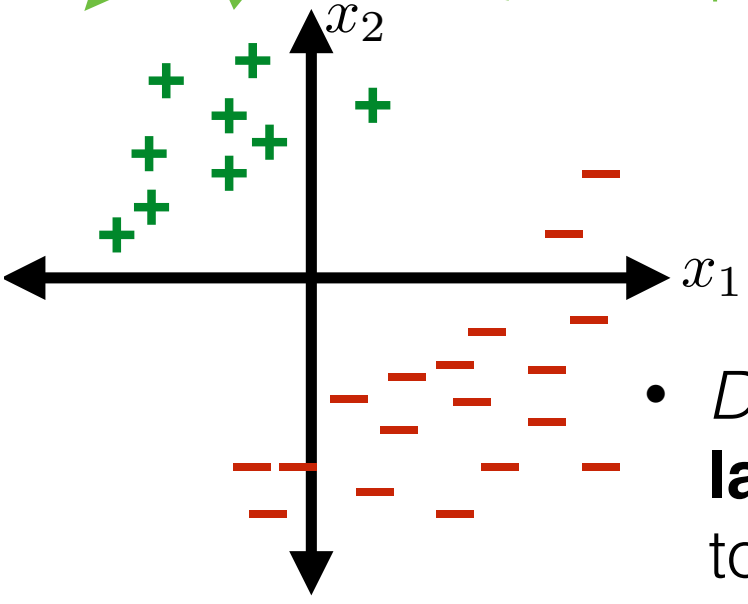

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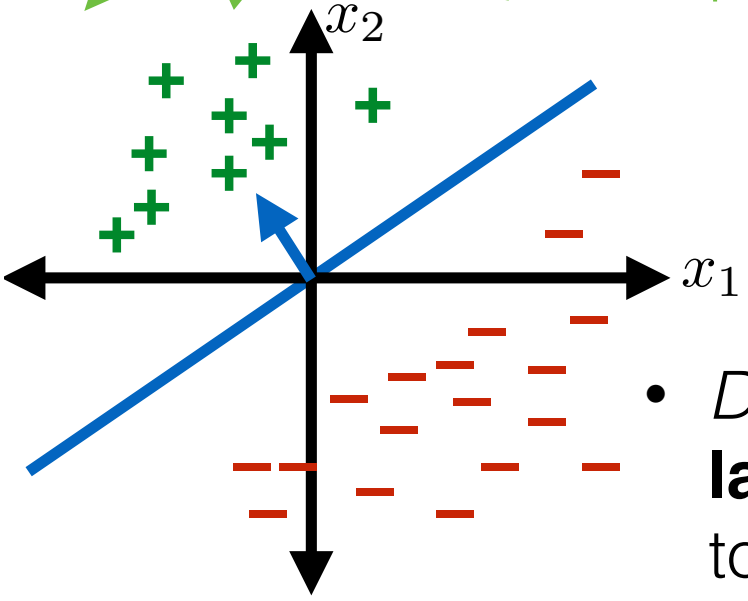

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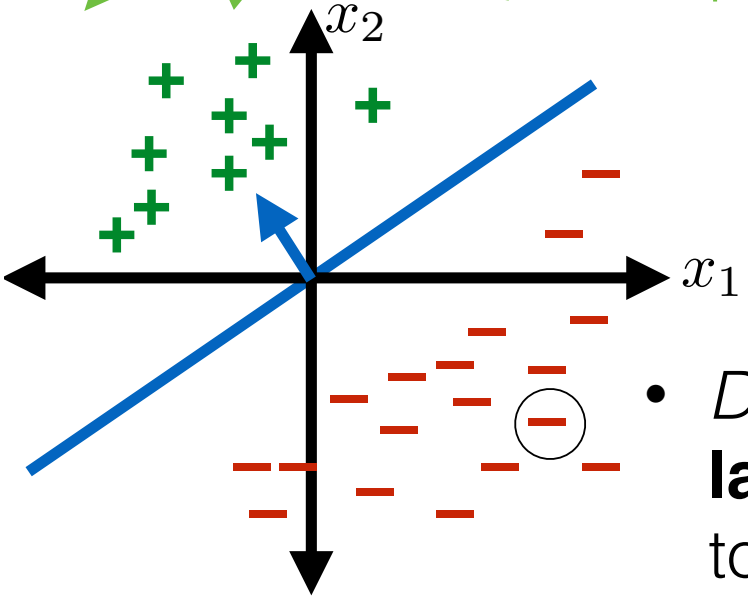

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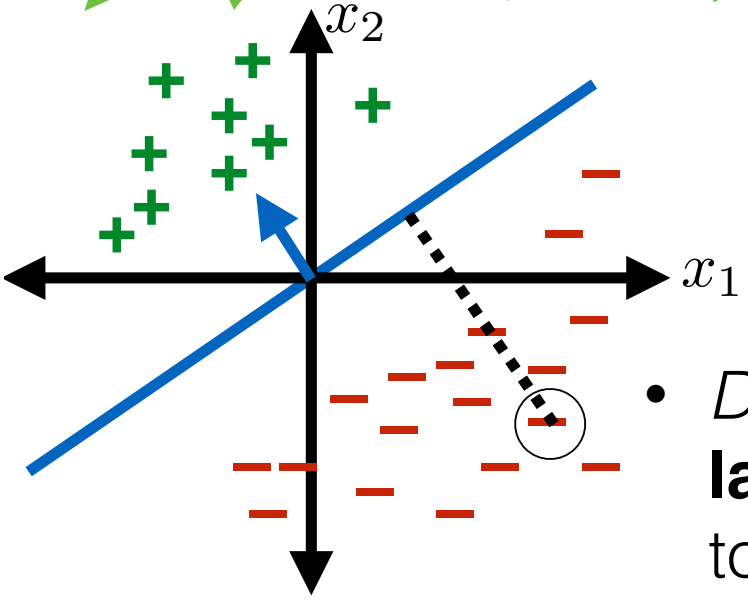

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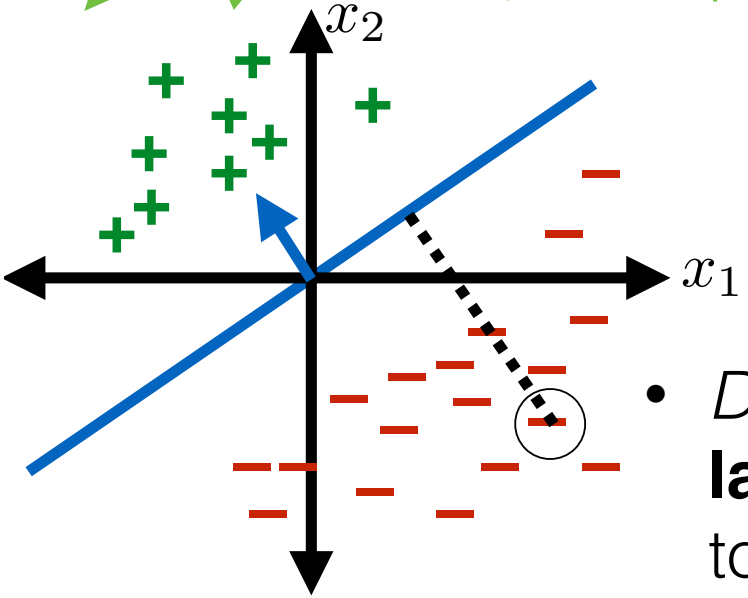

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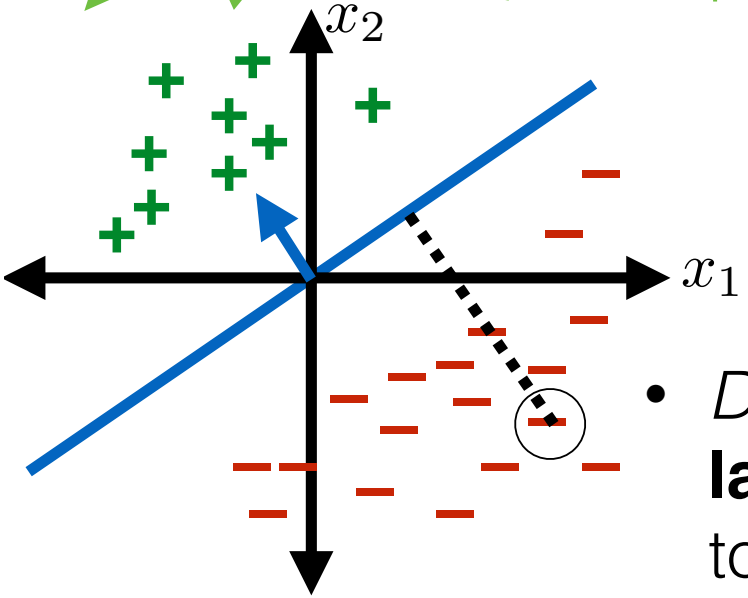

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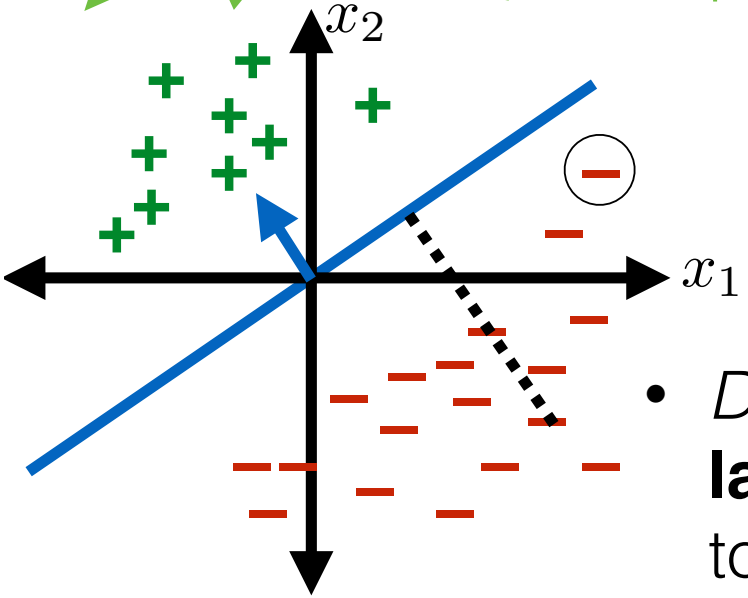

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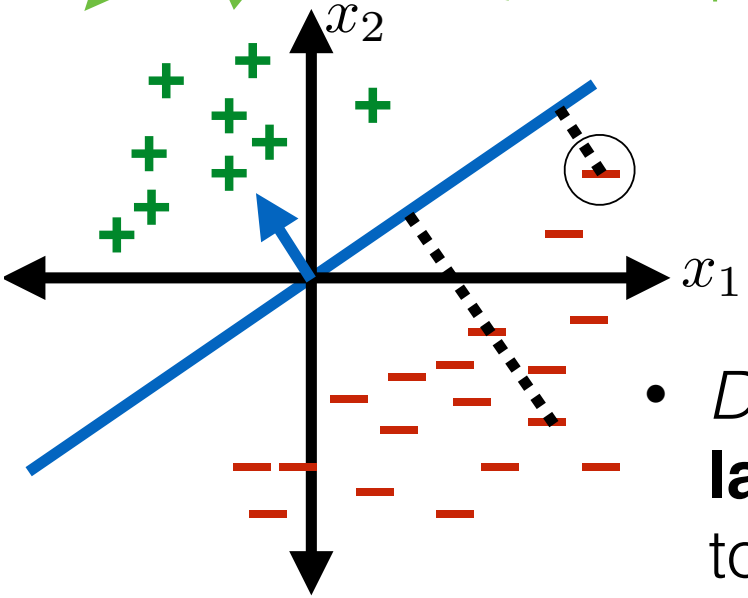

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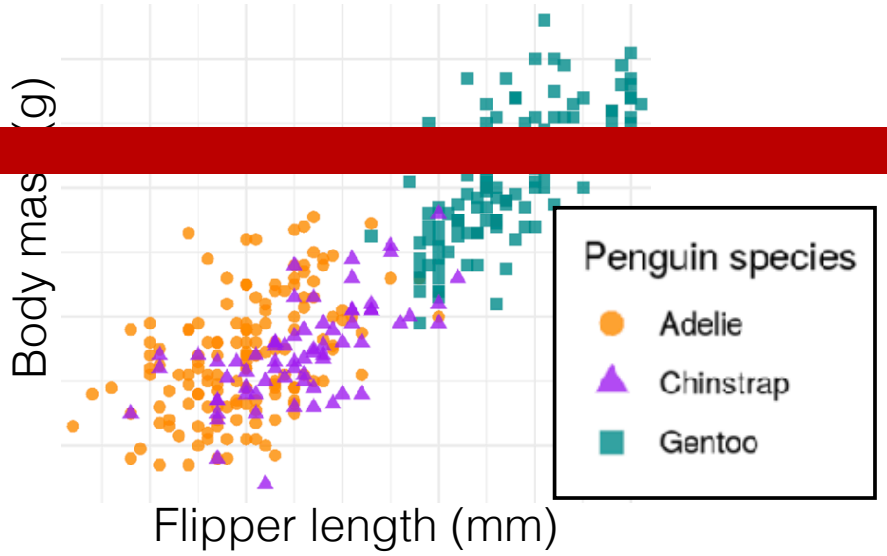
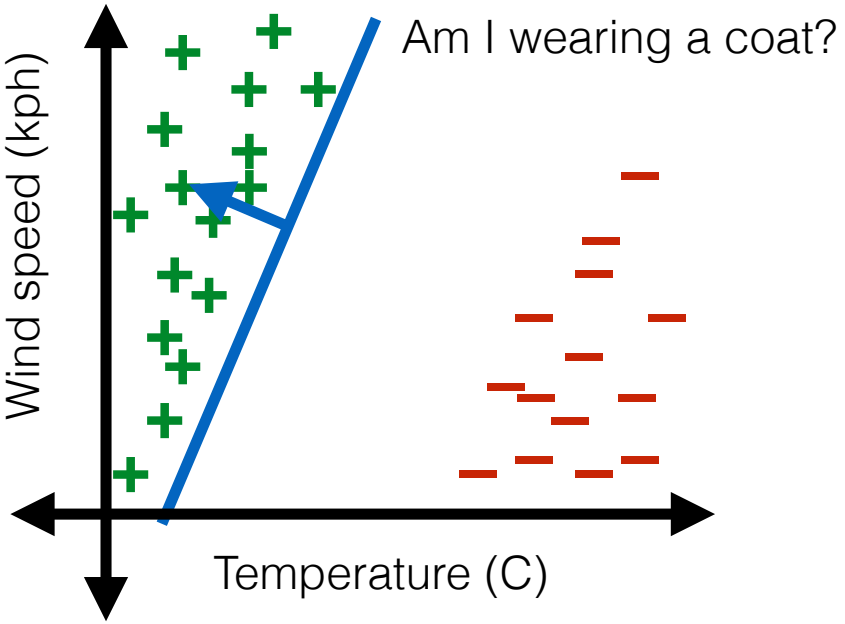
Recall

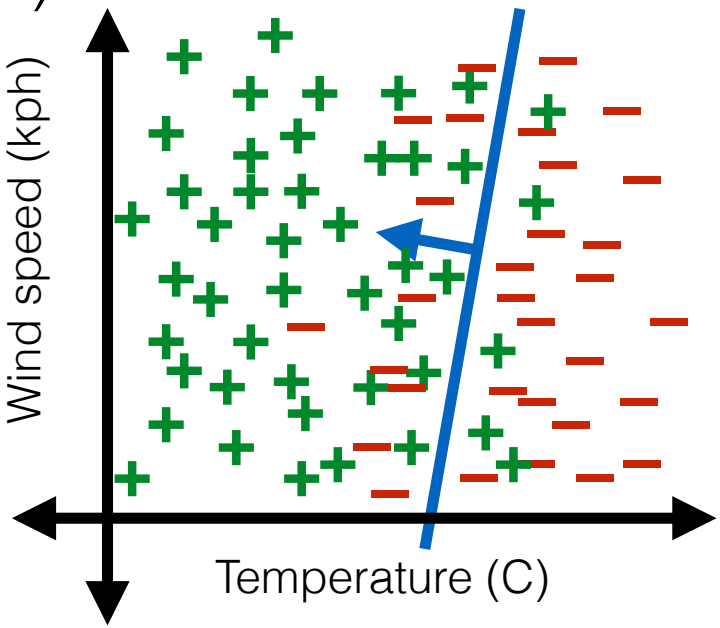
- Perceptron struggles with data that's not linearly separable



Notice

- Perceptron doesn't have a notion of uncertainty (how well do we know what we know?)





Capturing uncertainty

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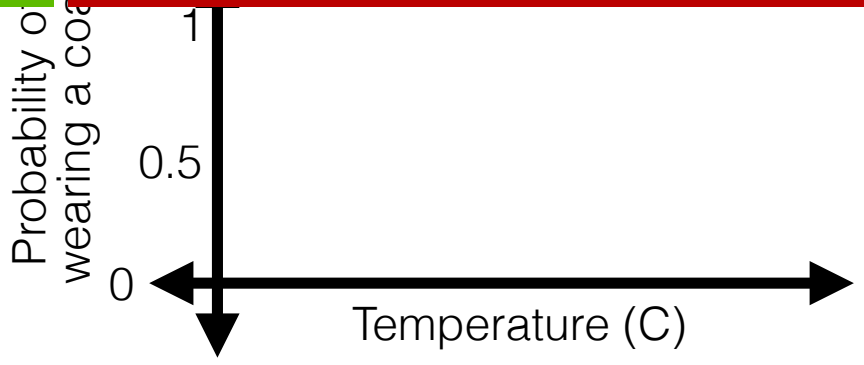

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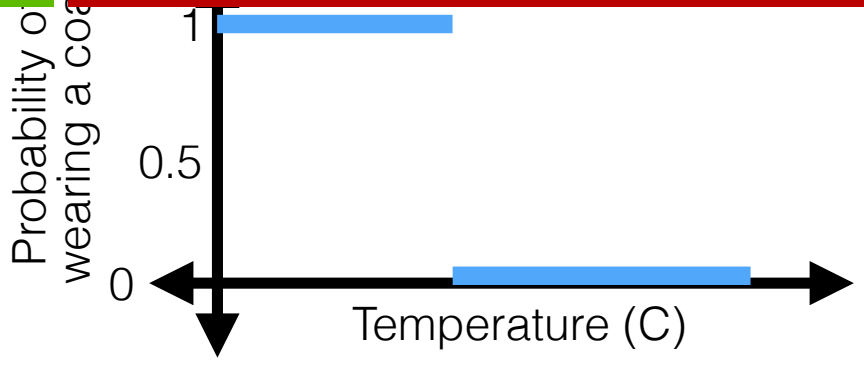

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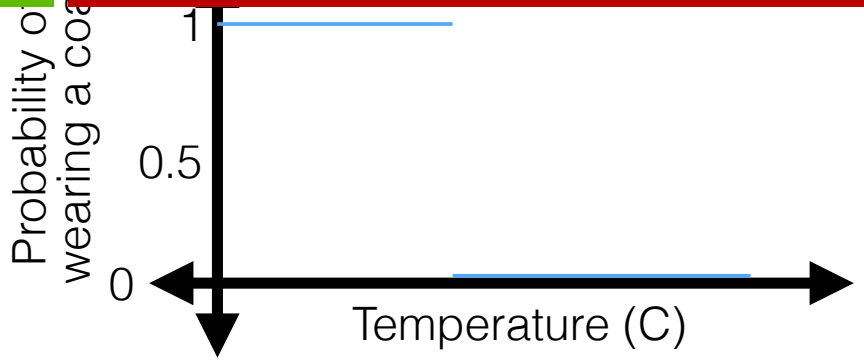

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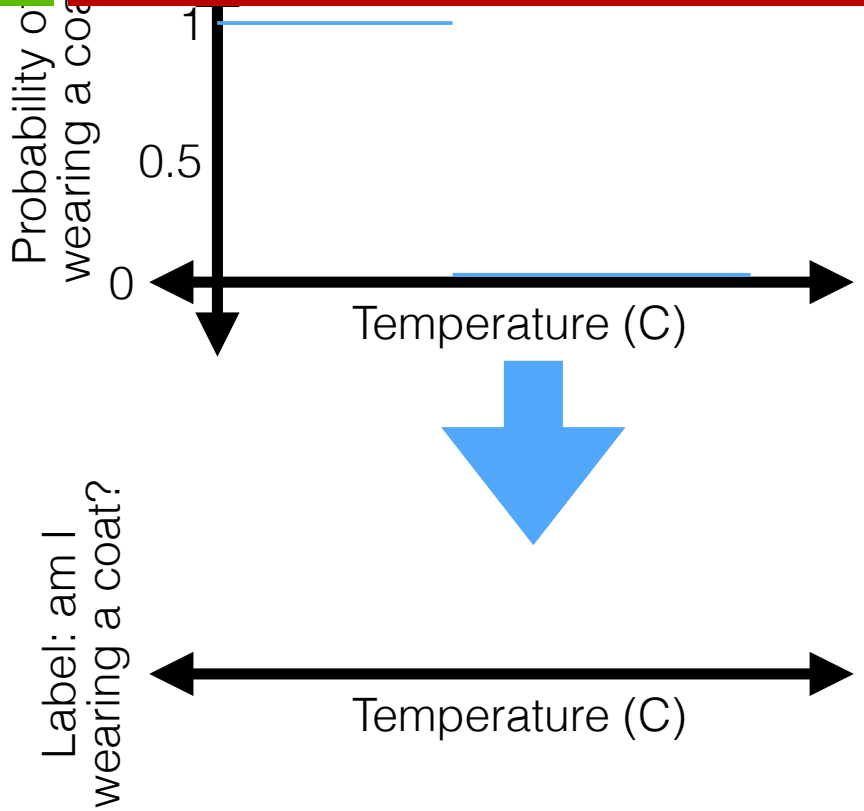

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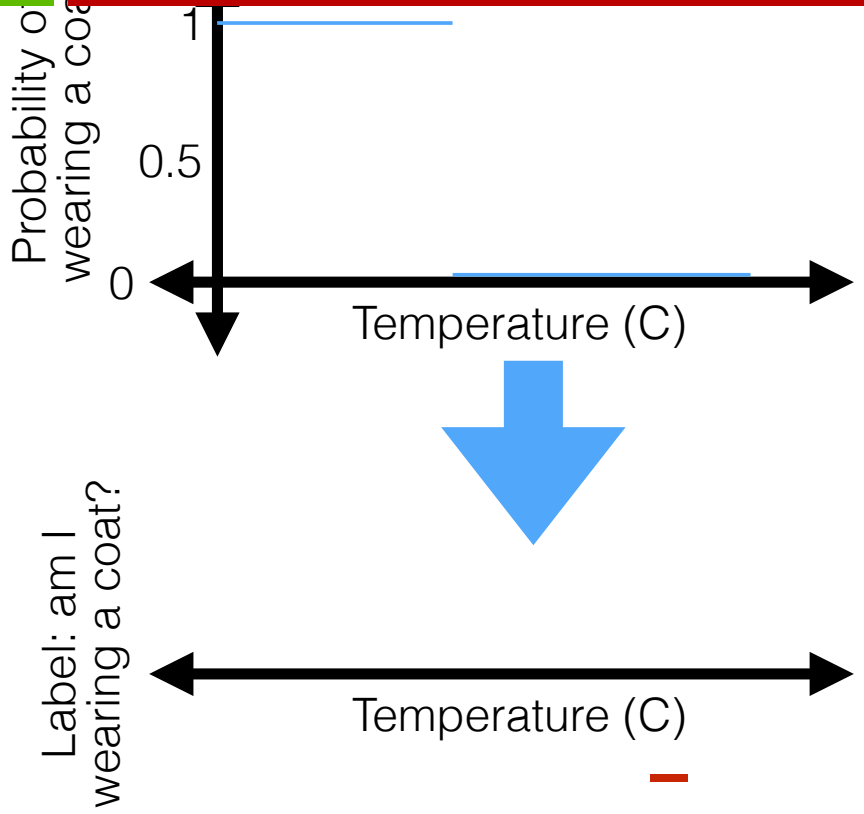

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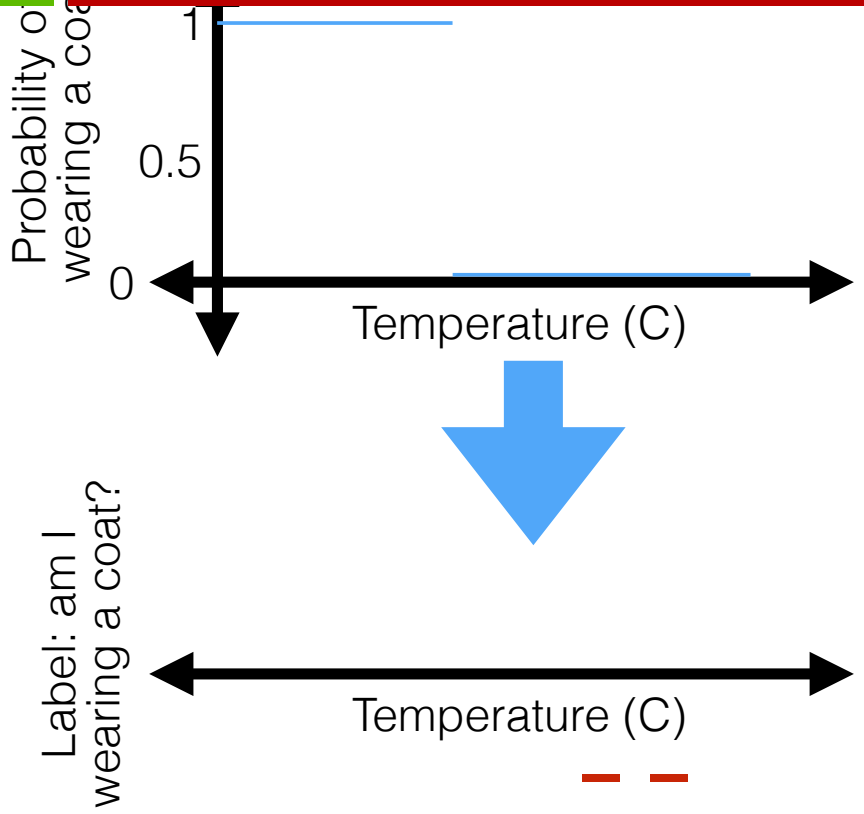

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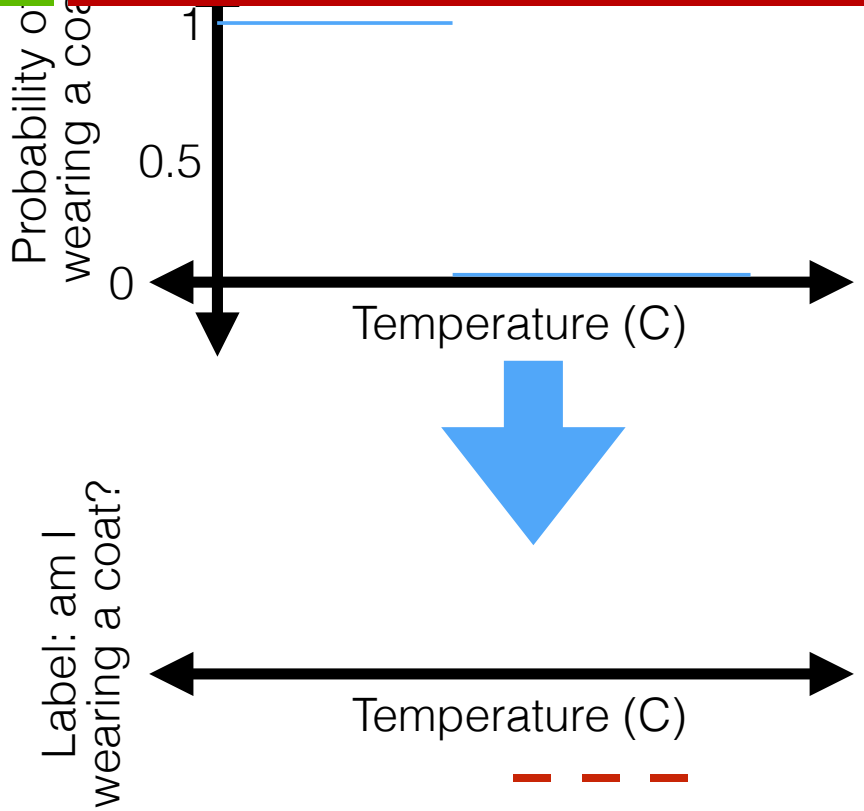

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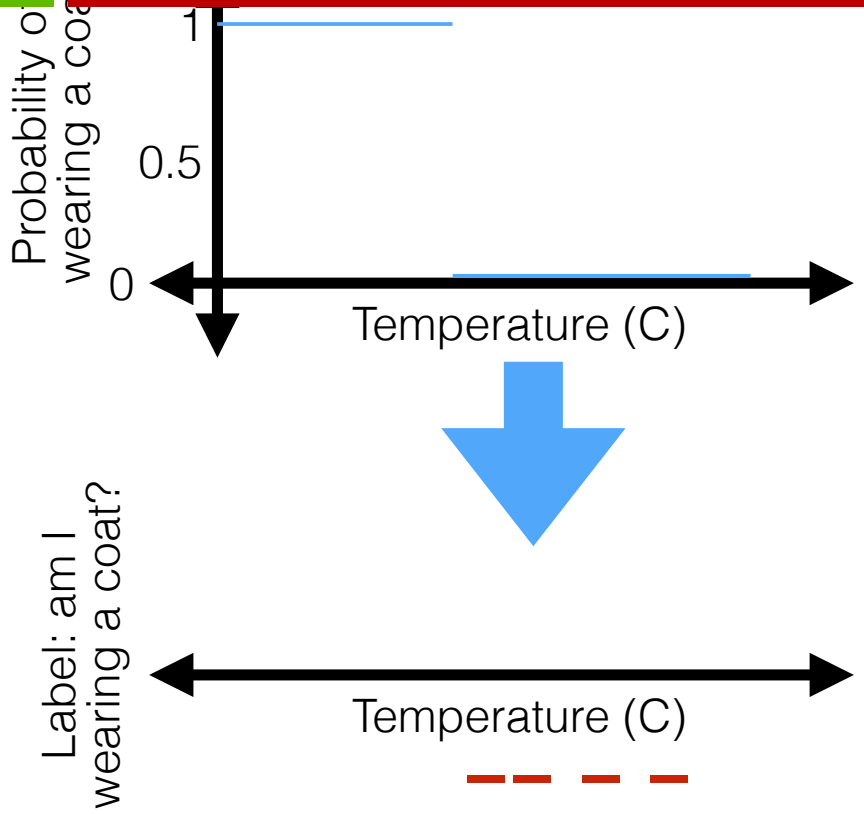

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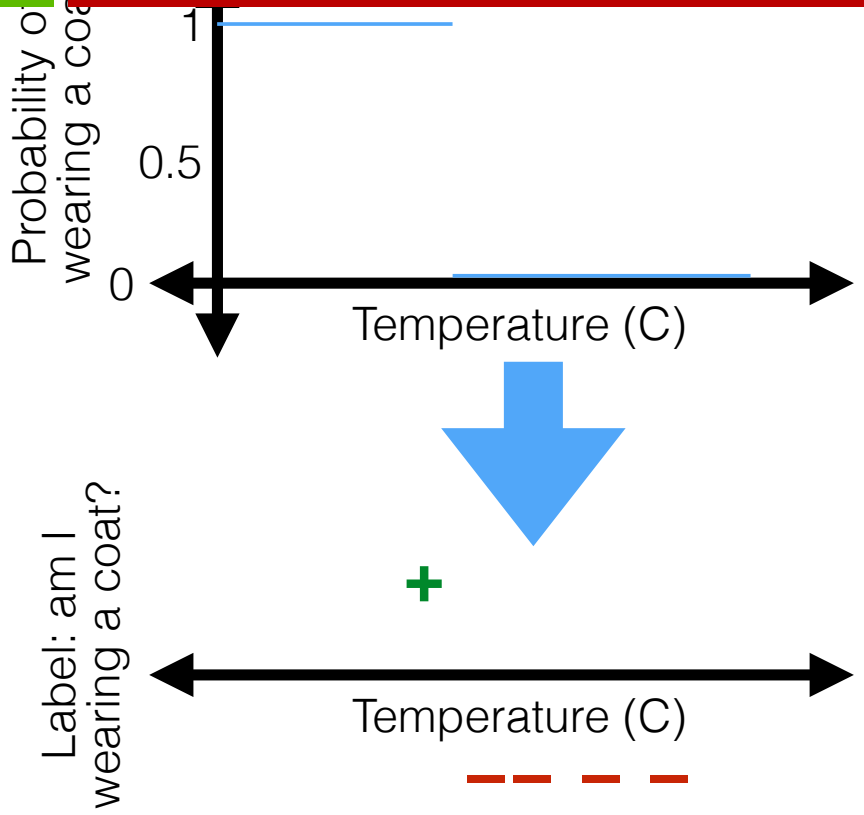

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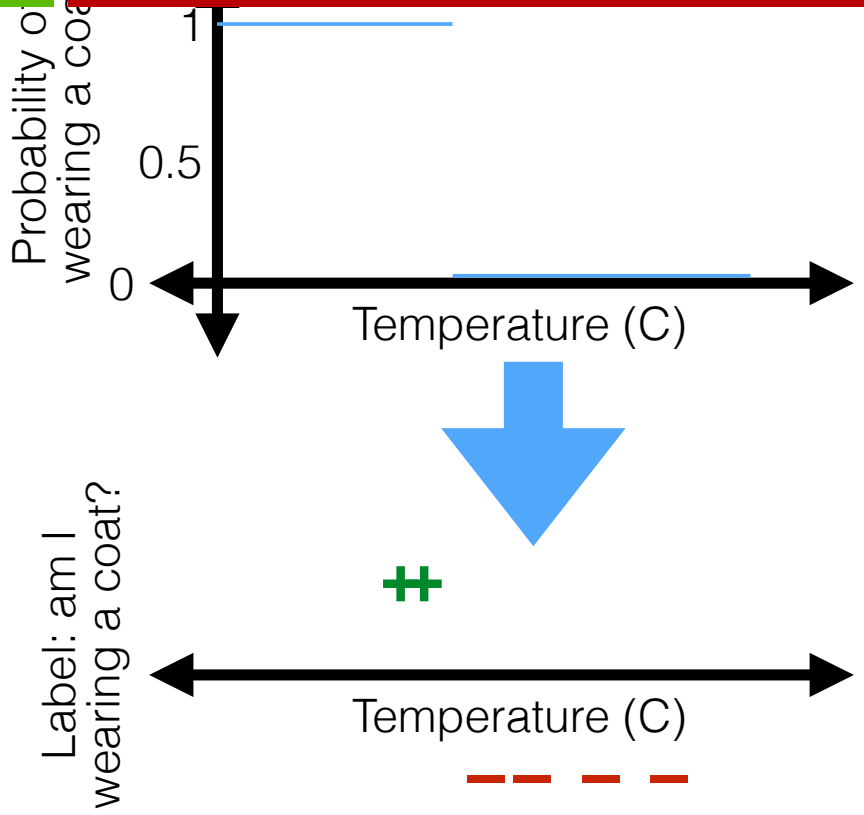

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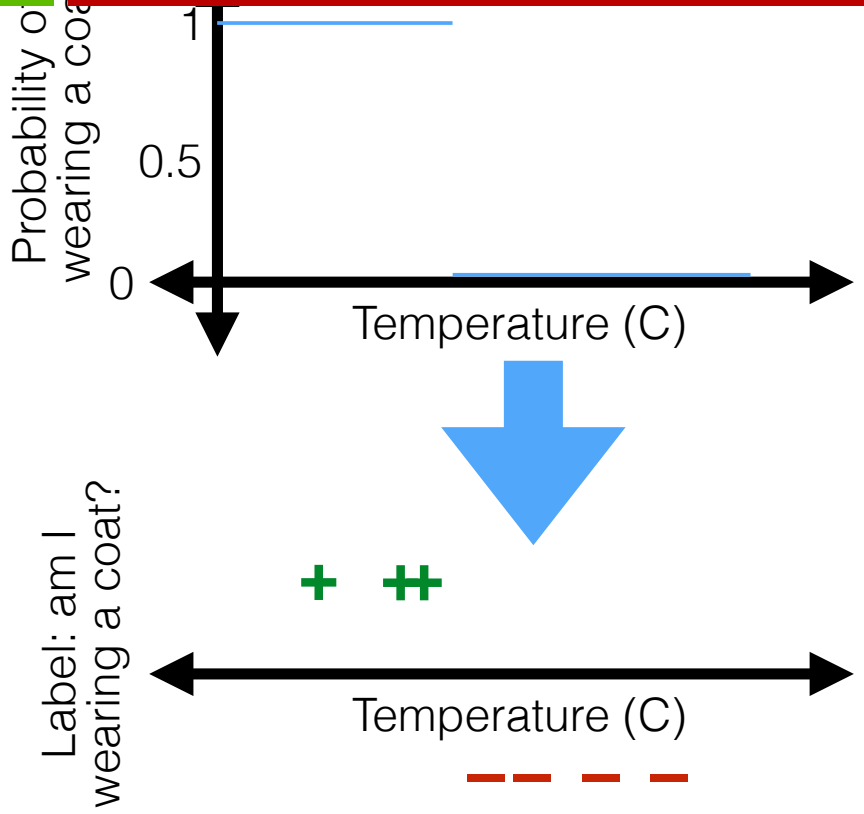

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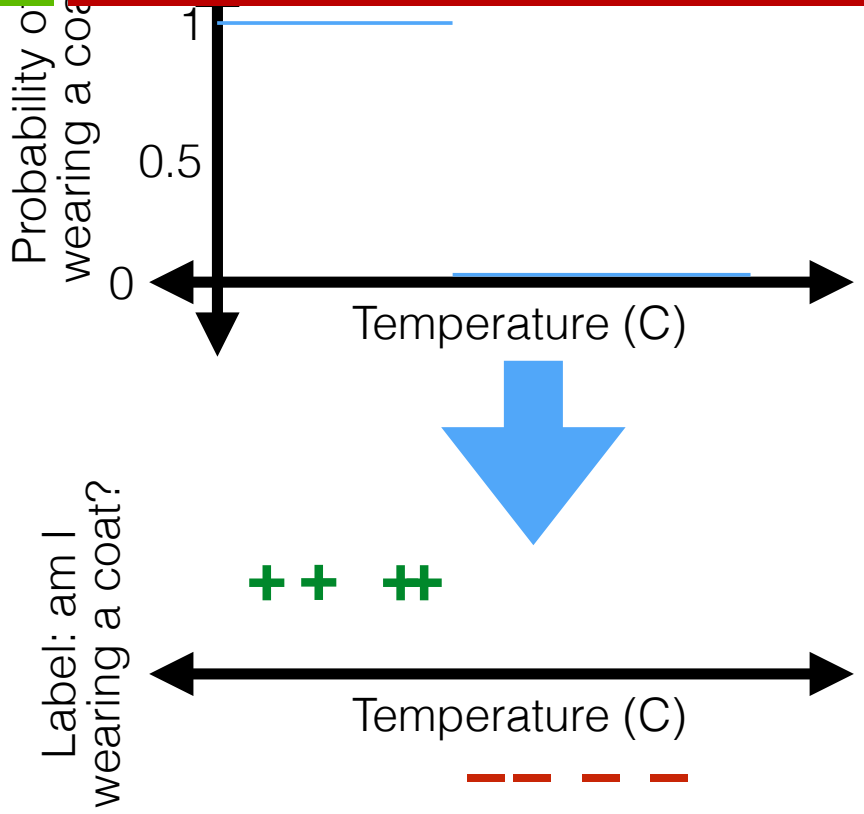

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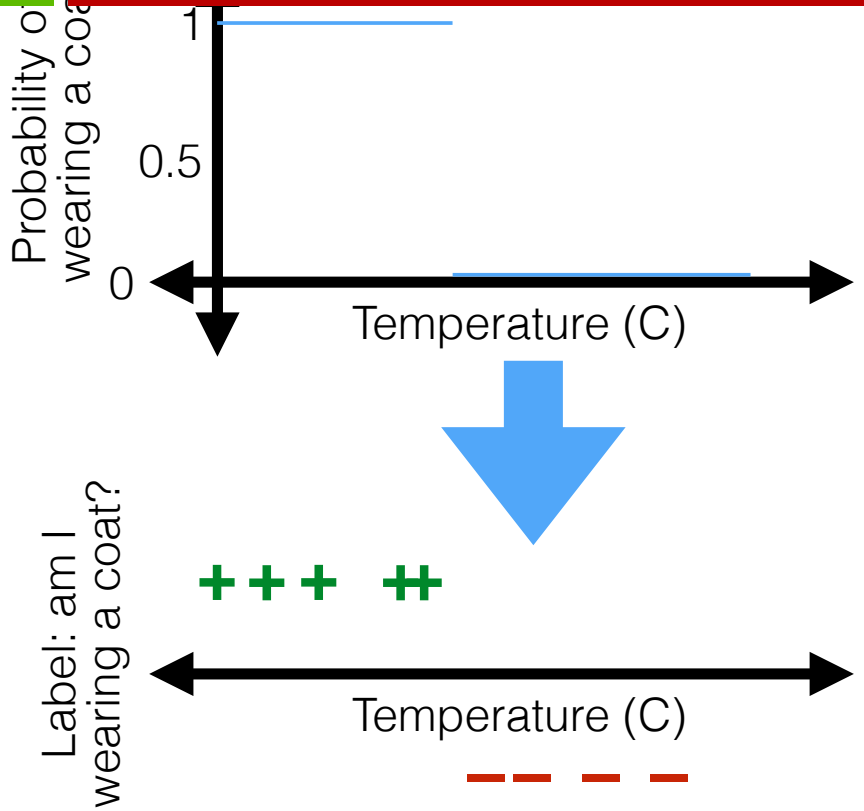

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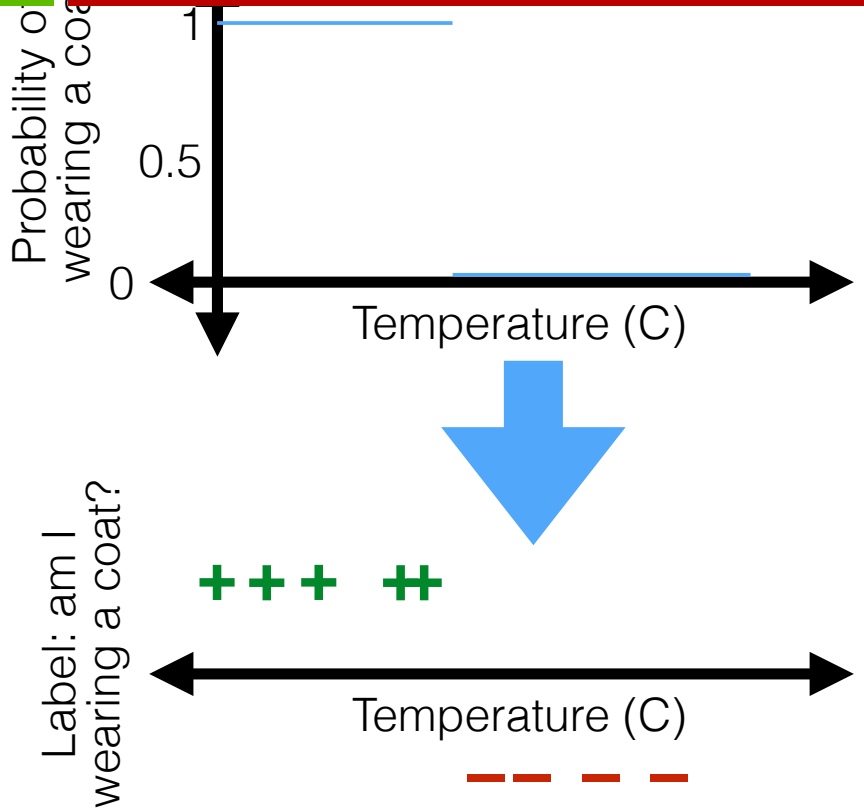

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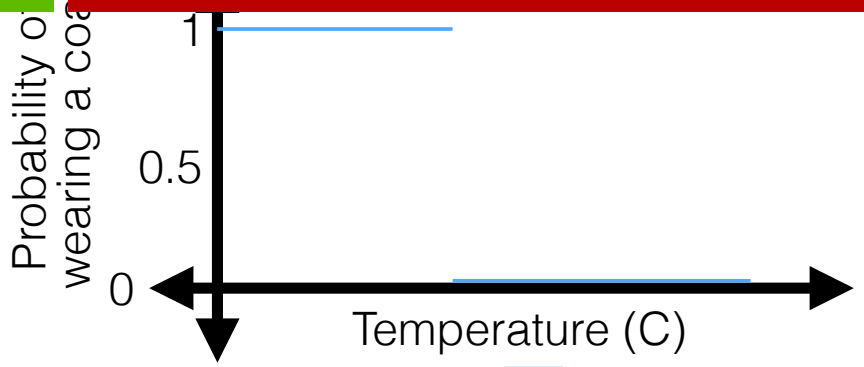

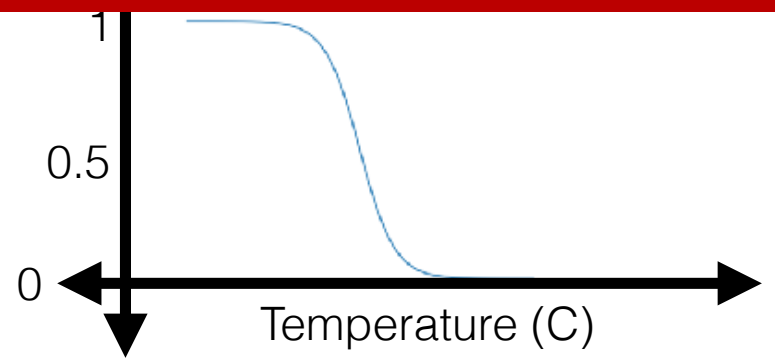

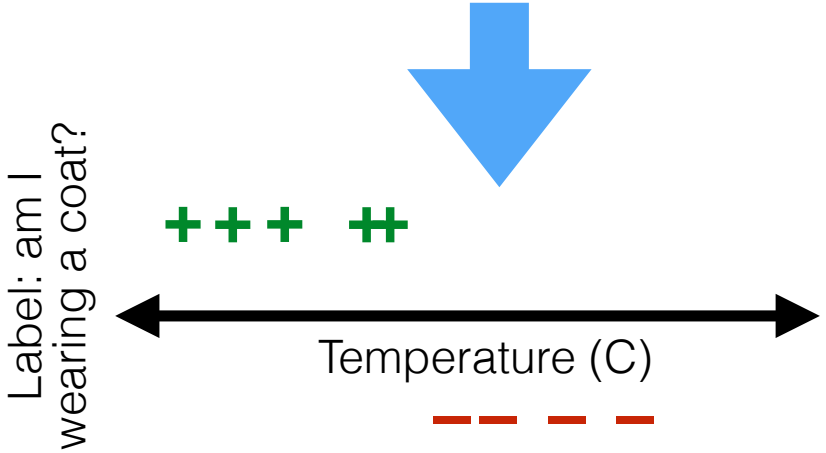

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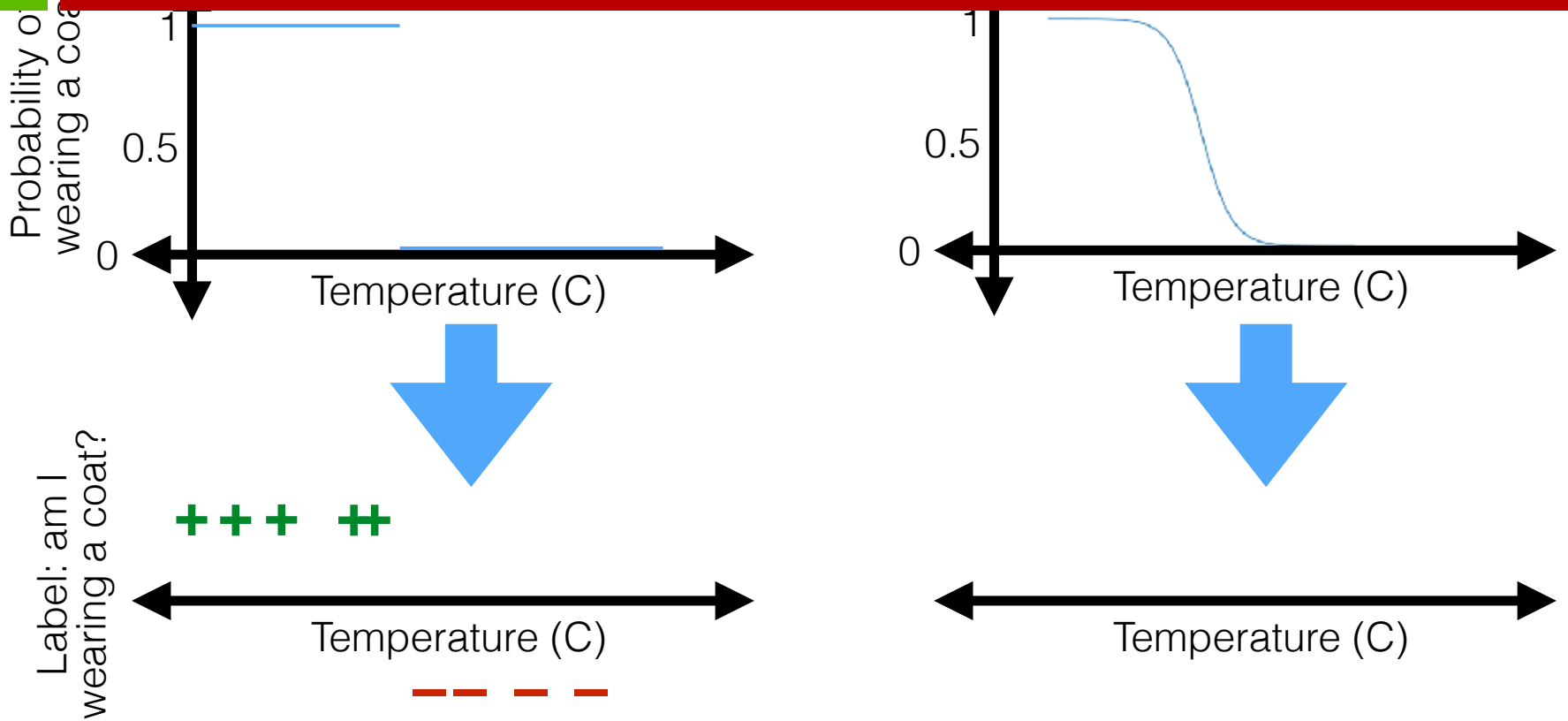

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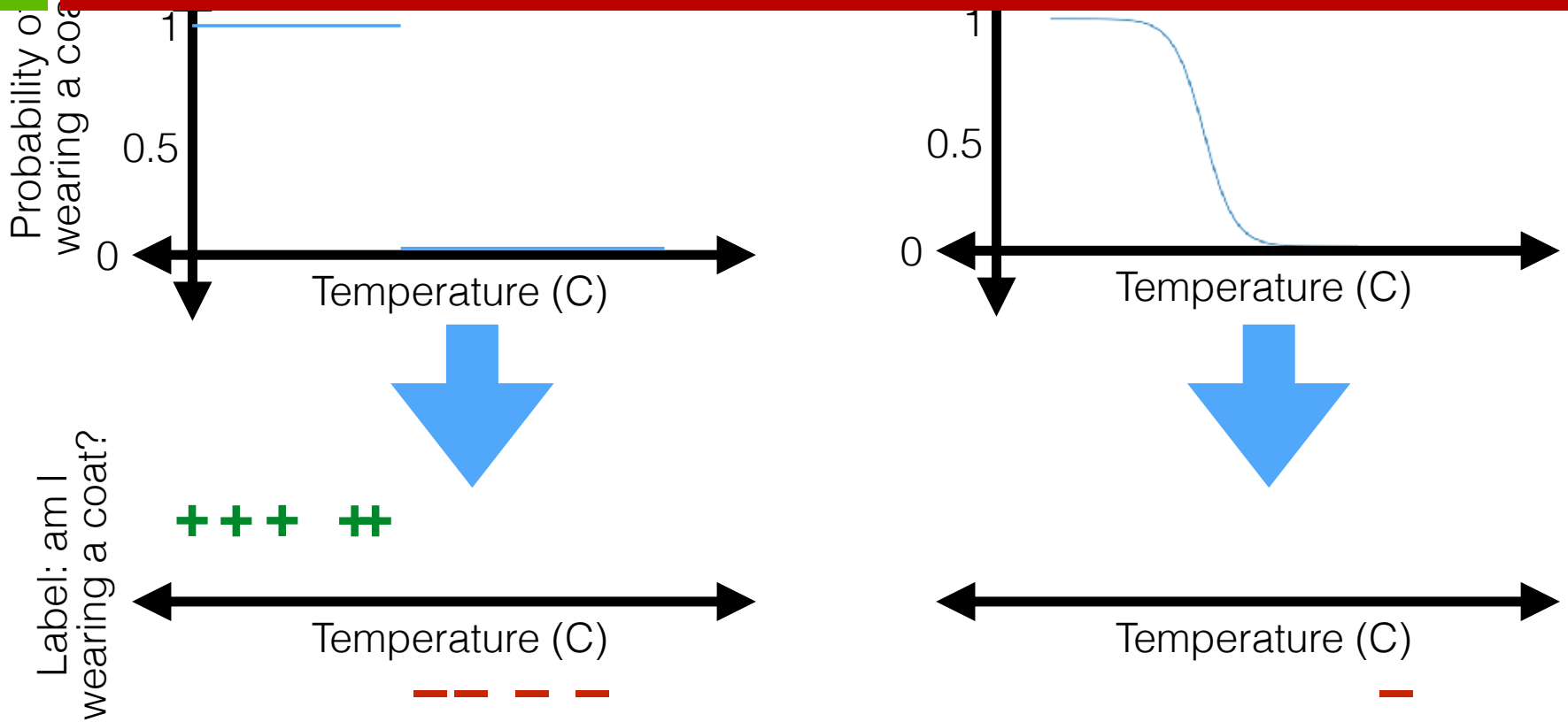

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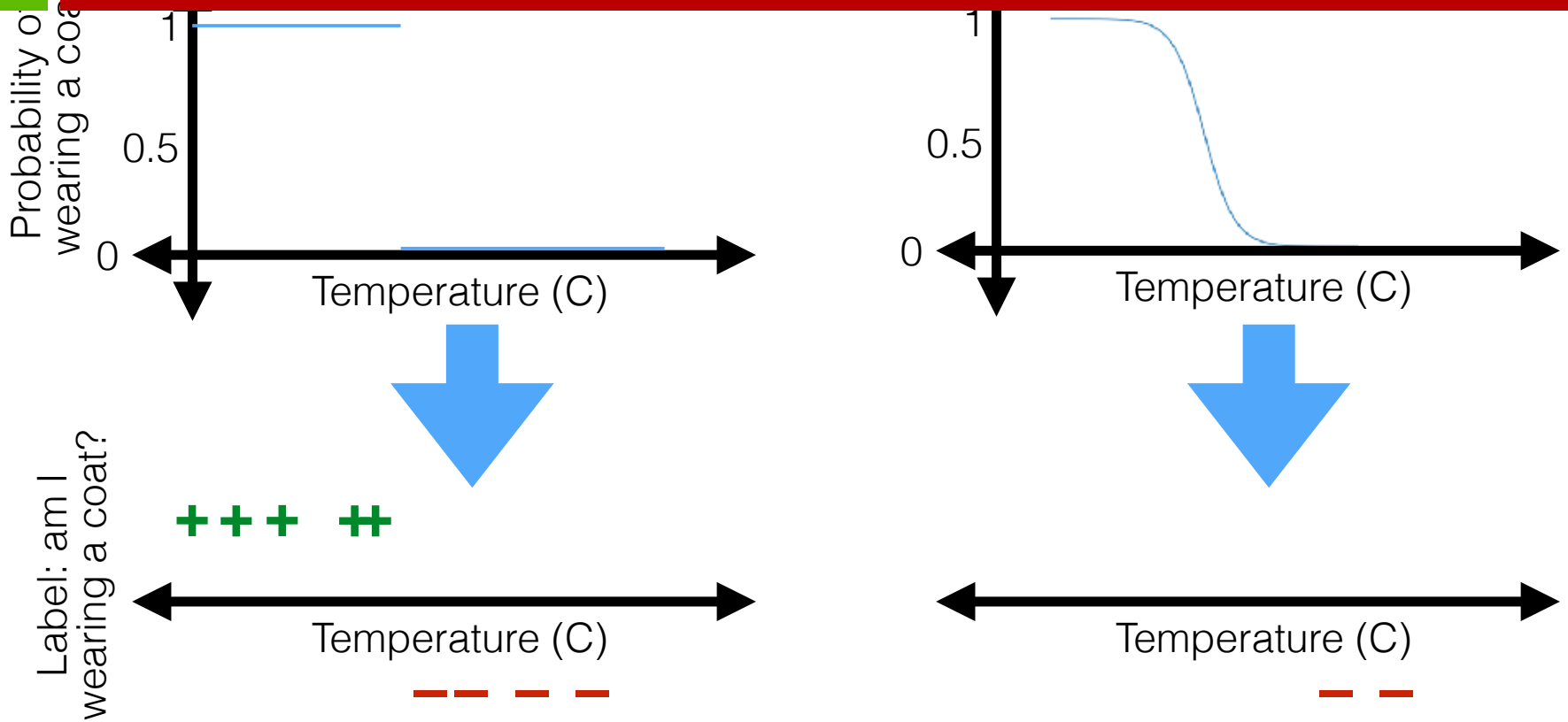

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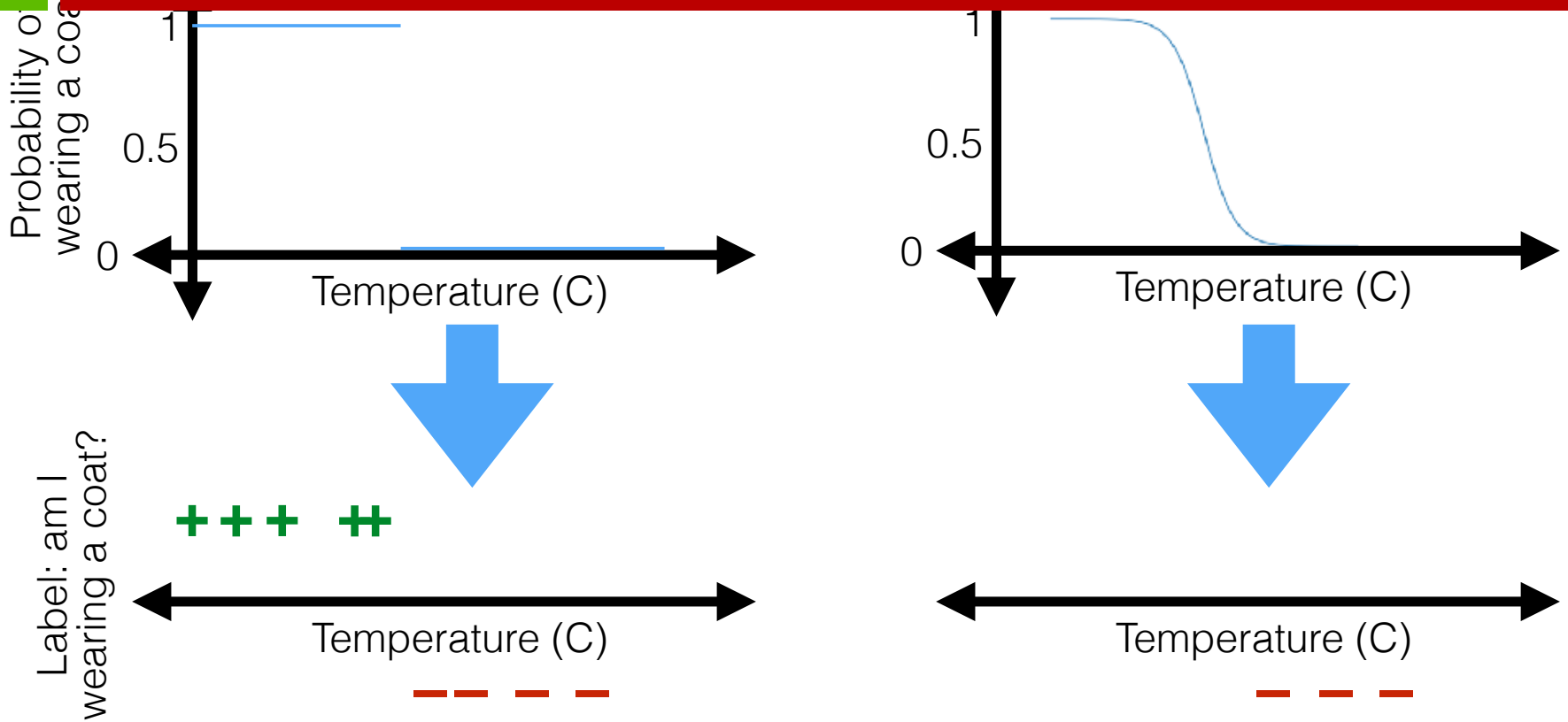

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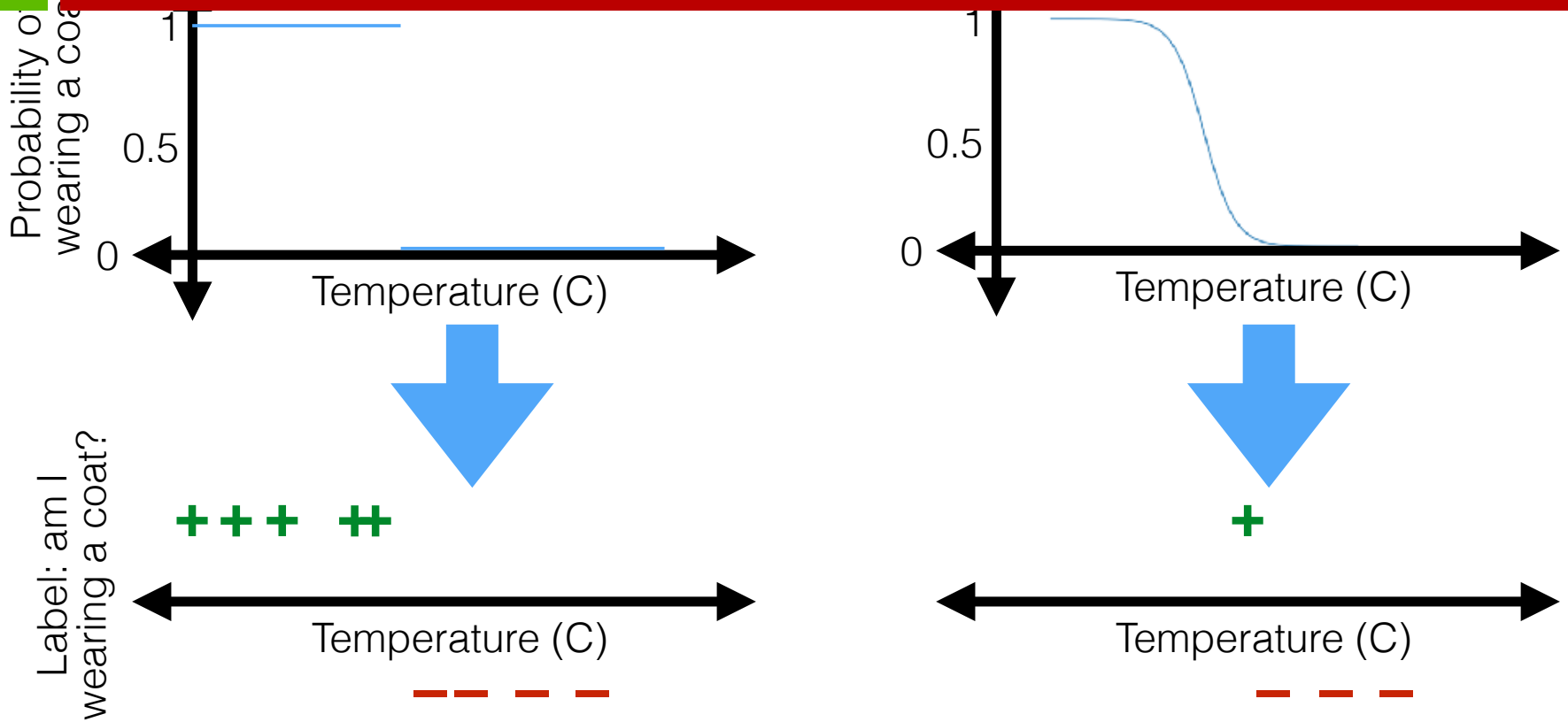

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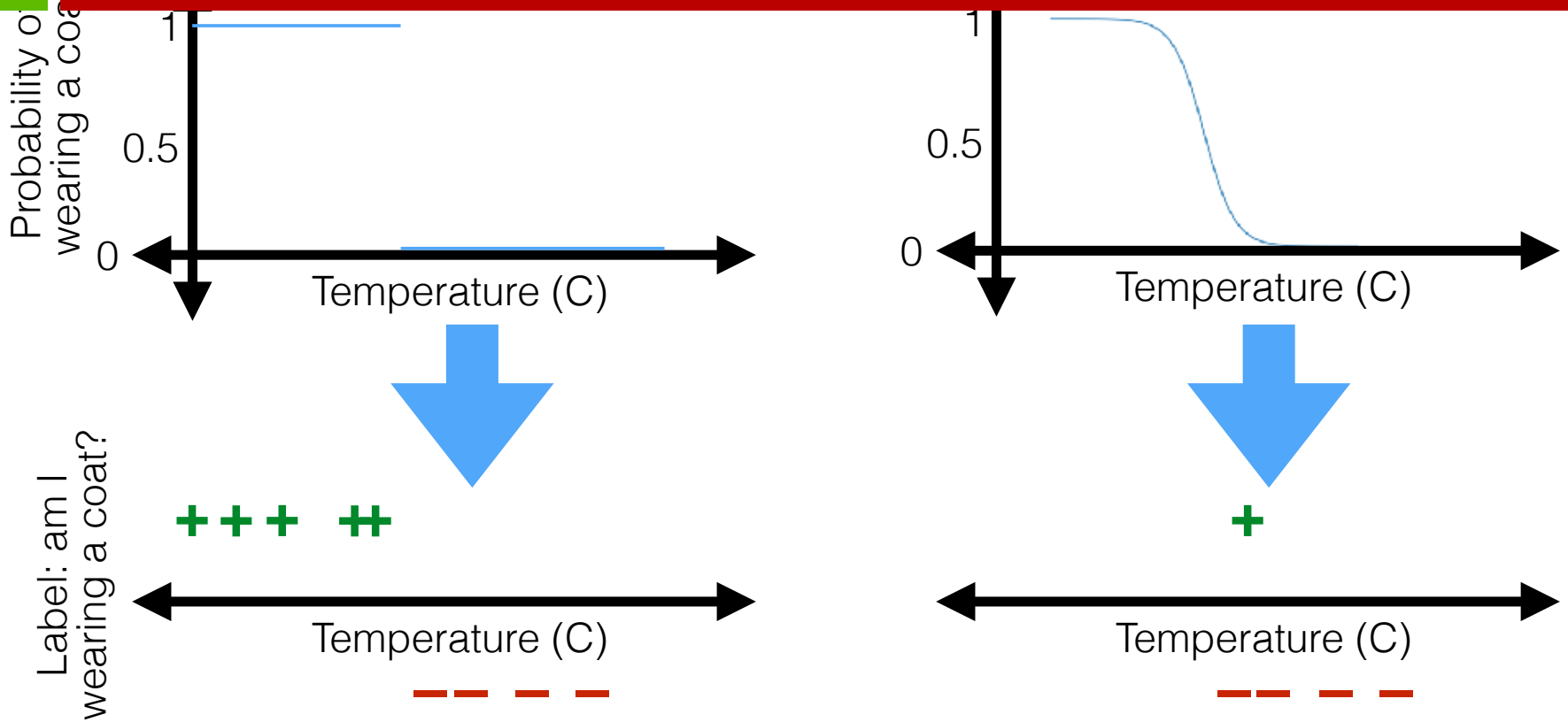

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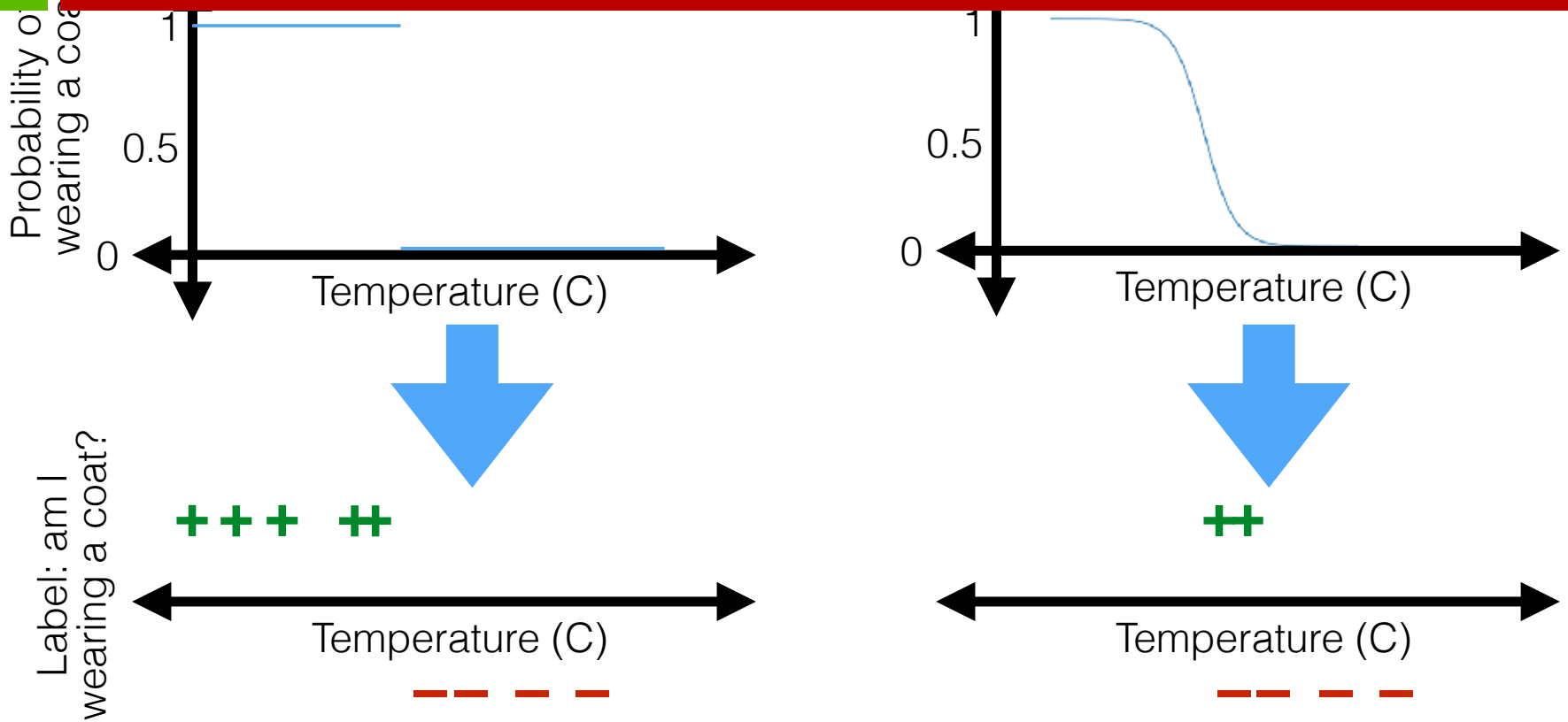

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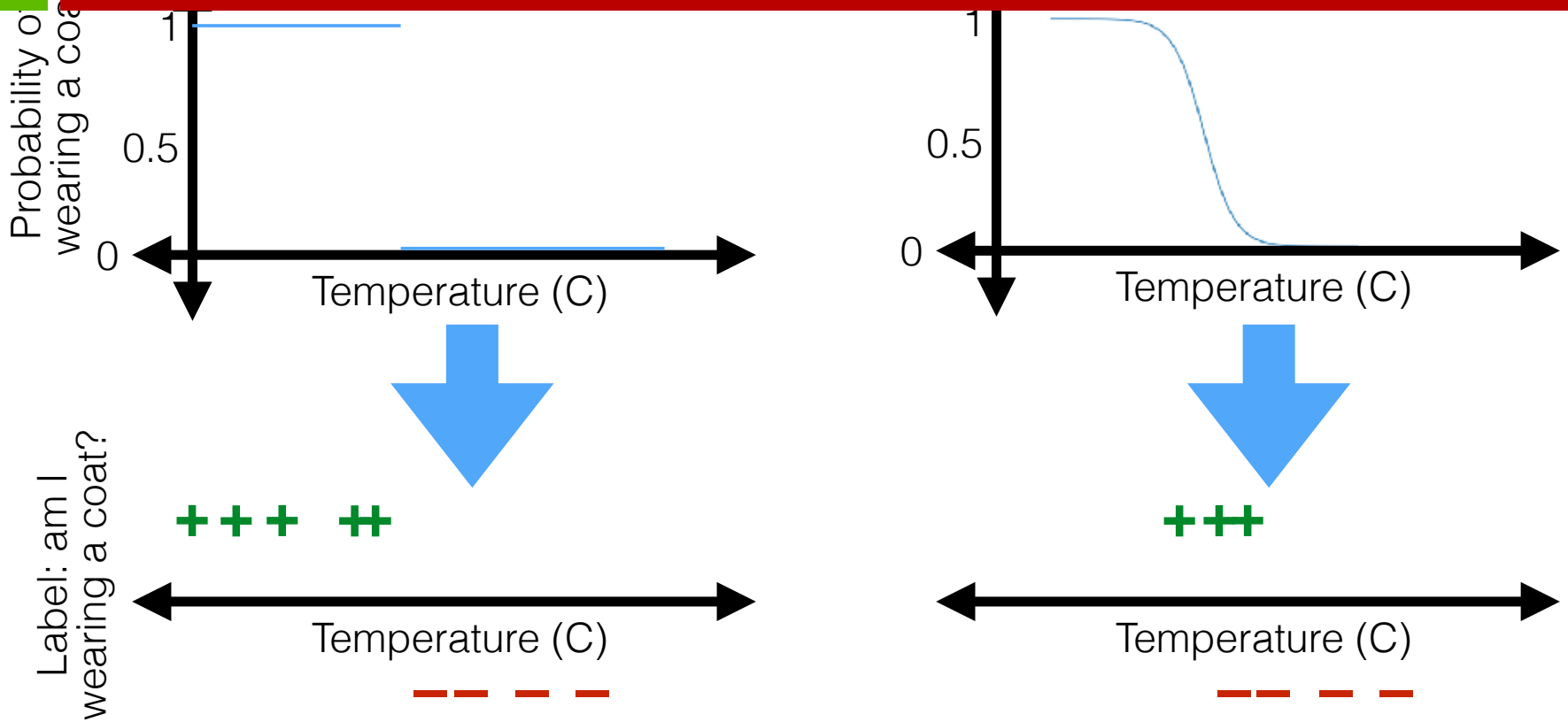

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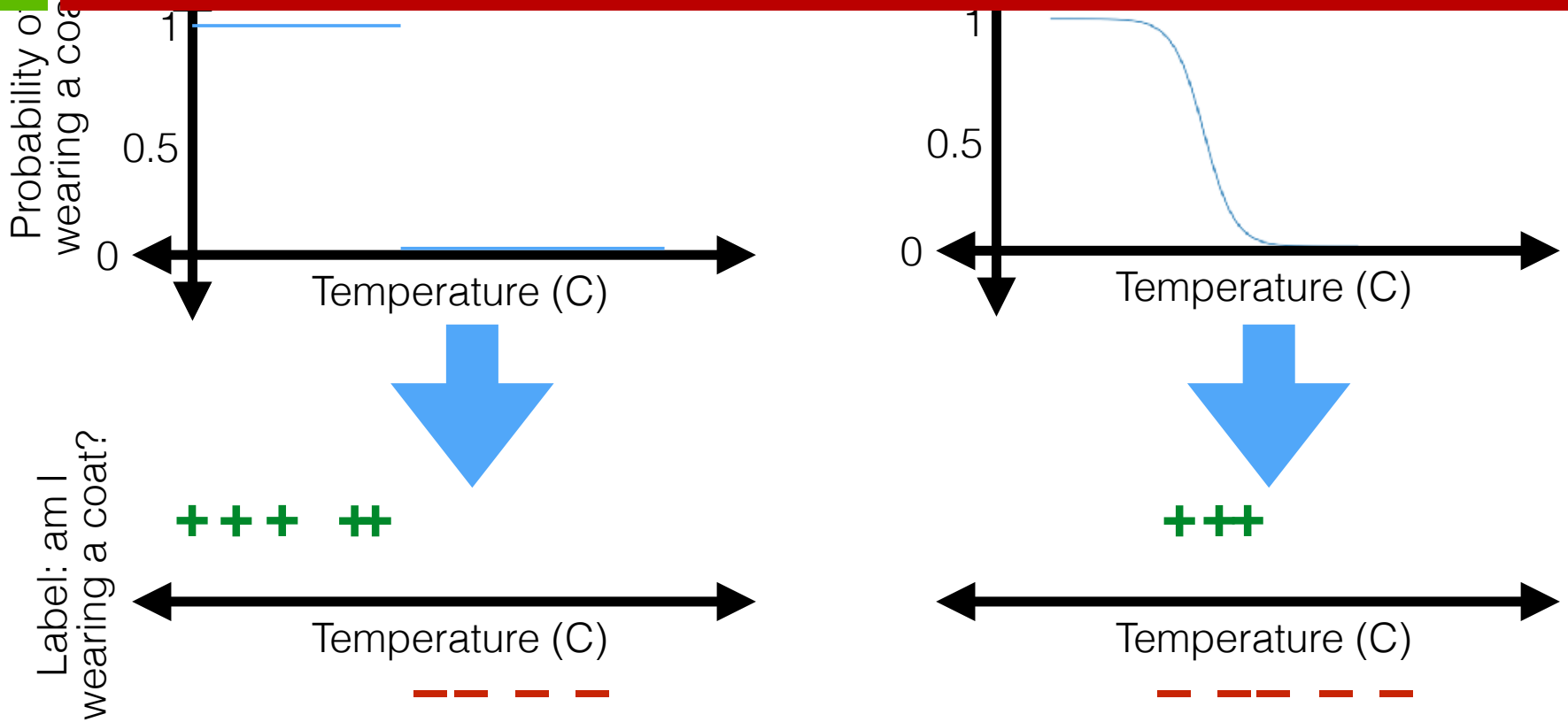

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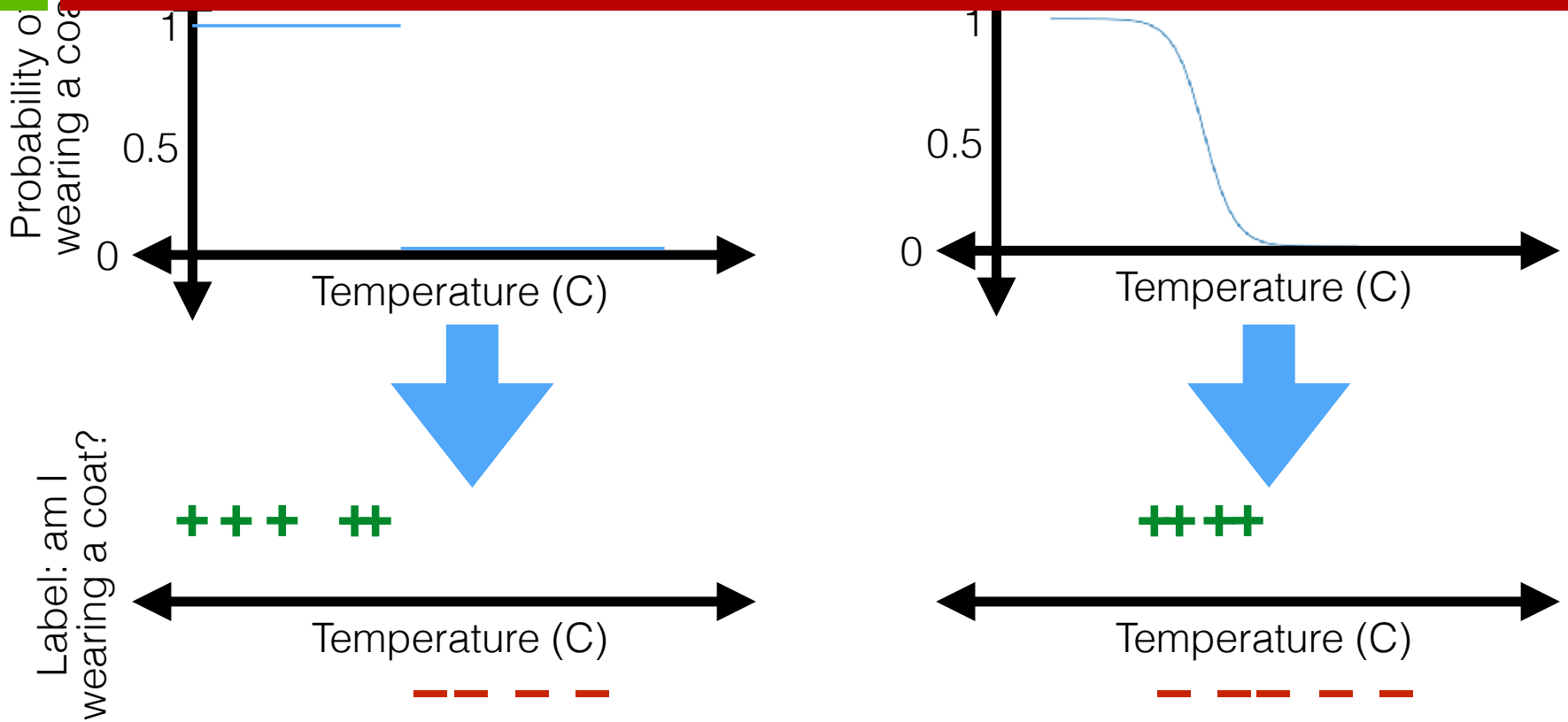

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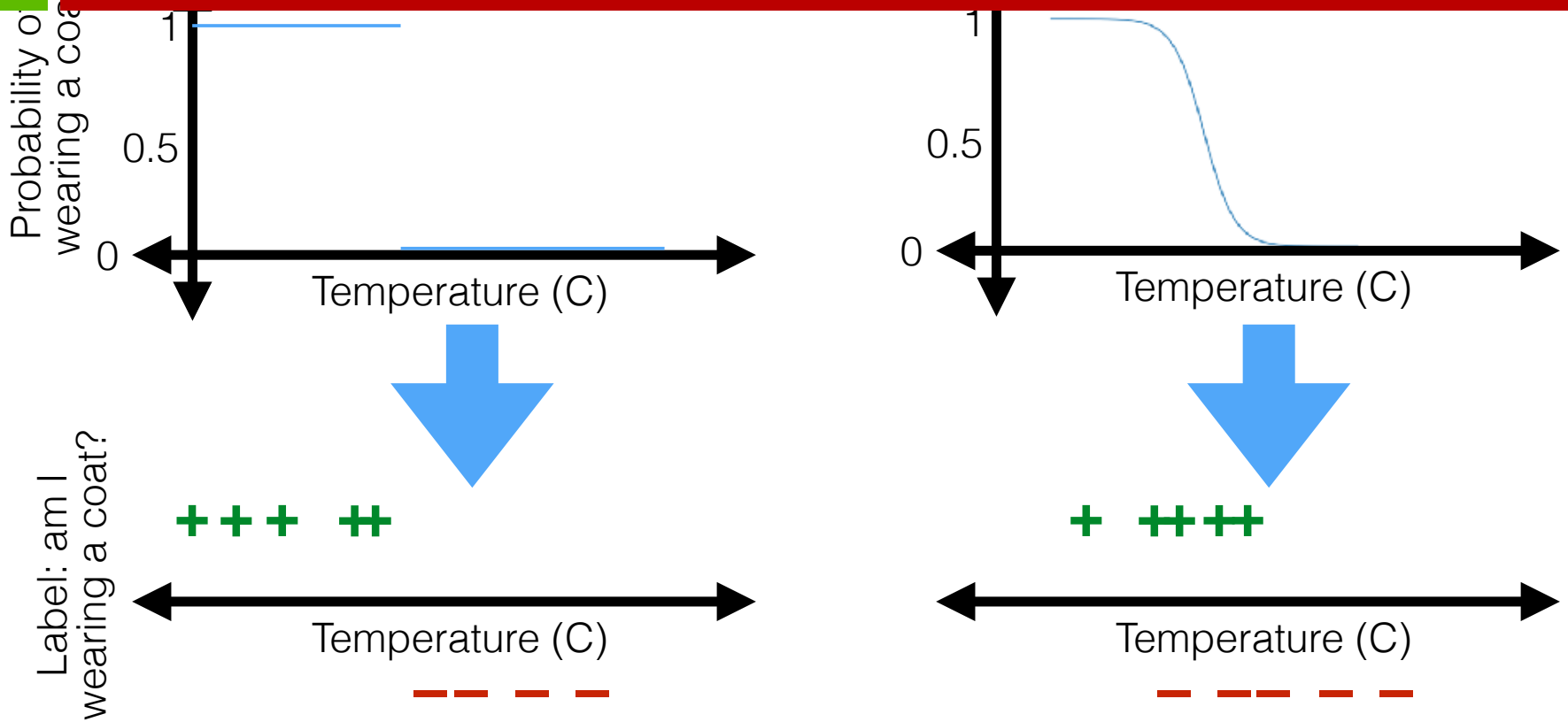

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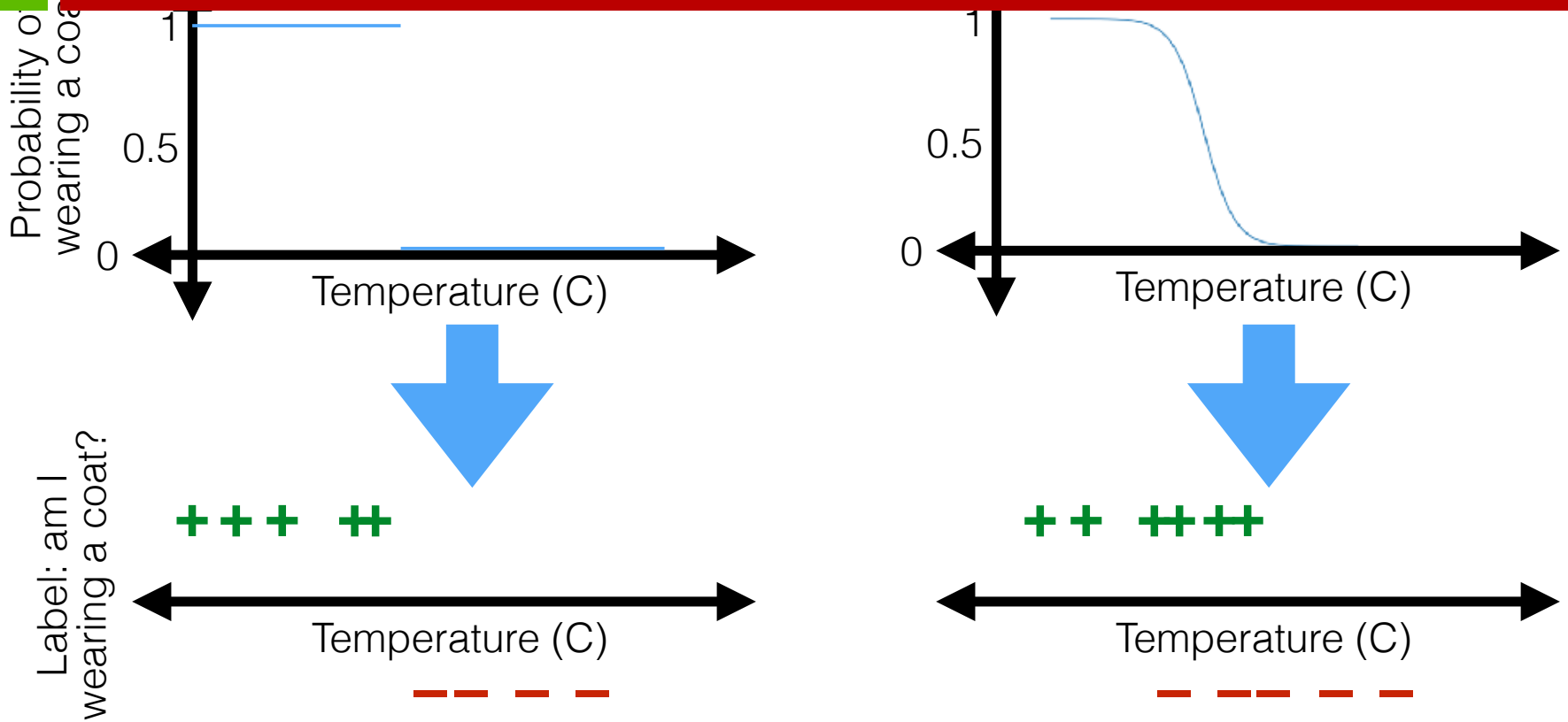

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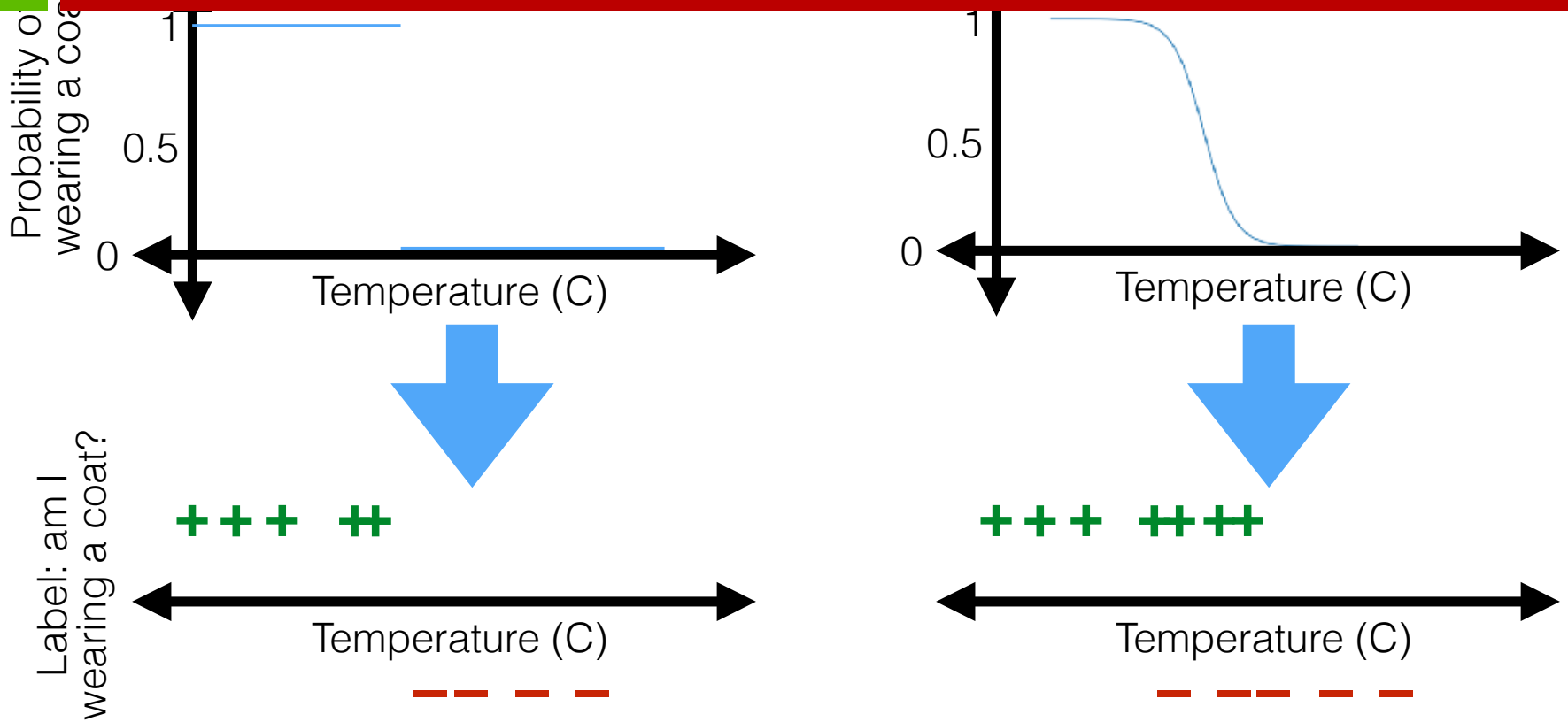

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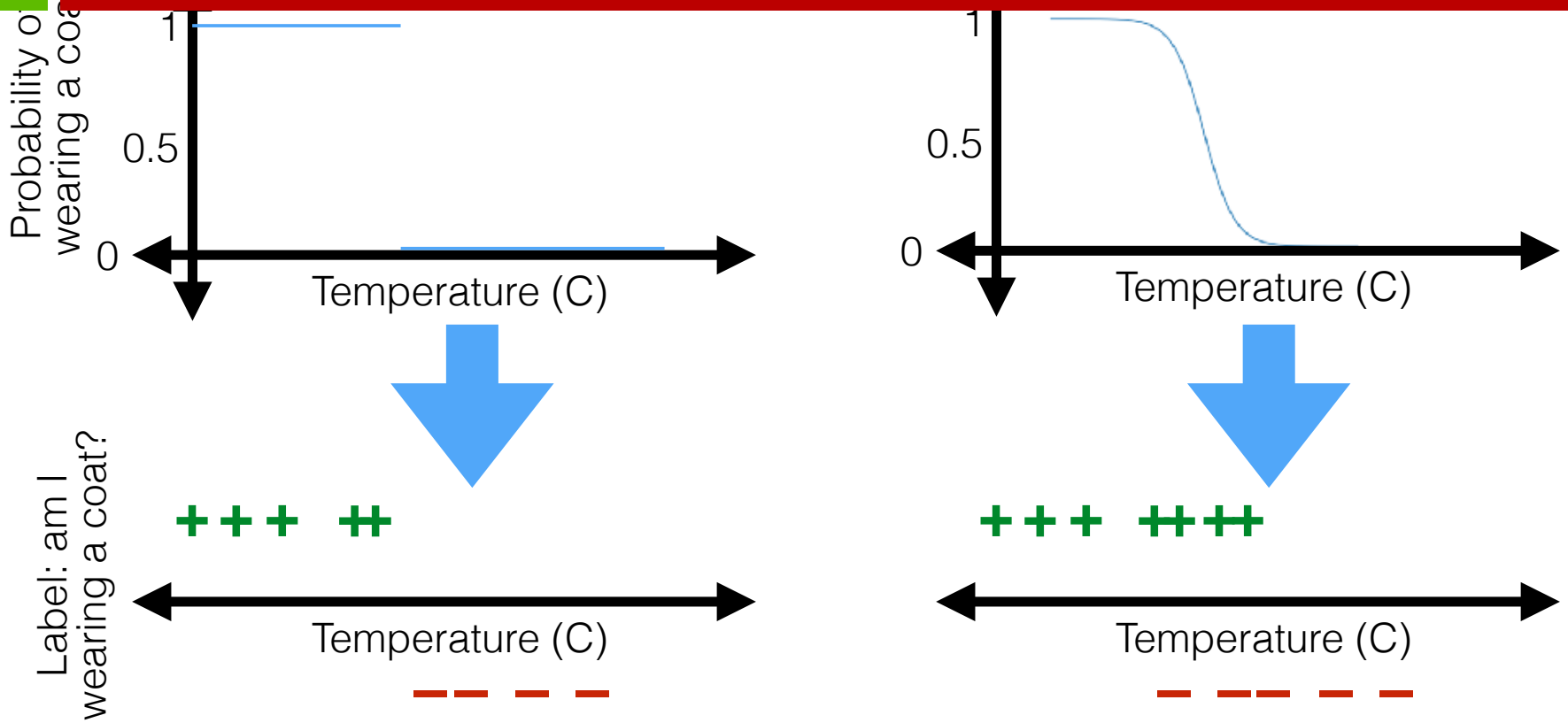
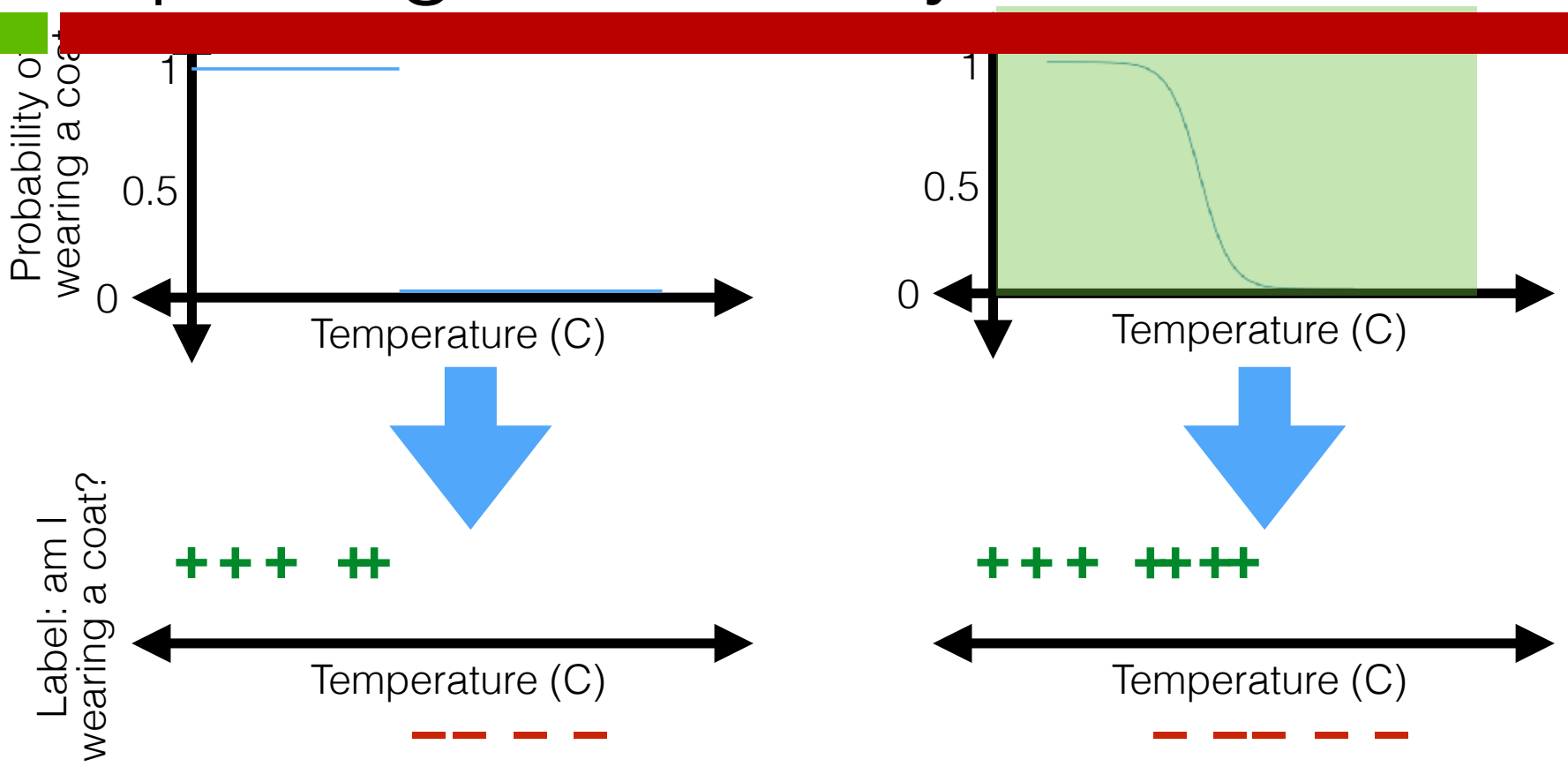


- How to make this shape?

Capturing uncertainty



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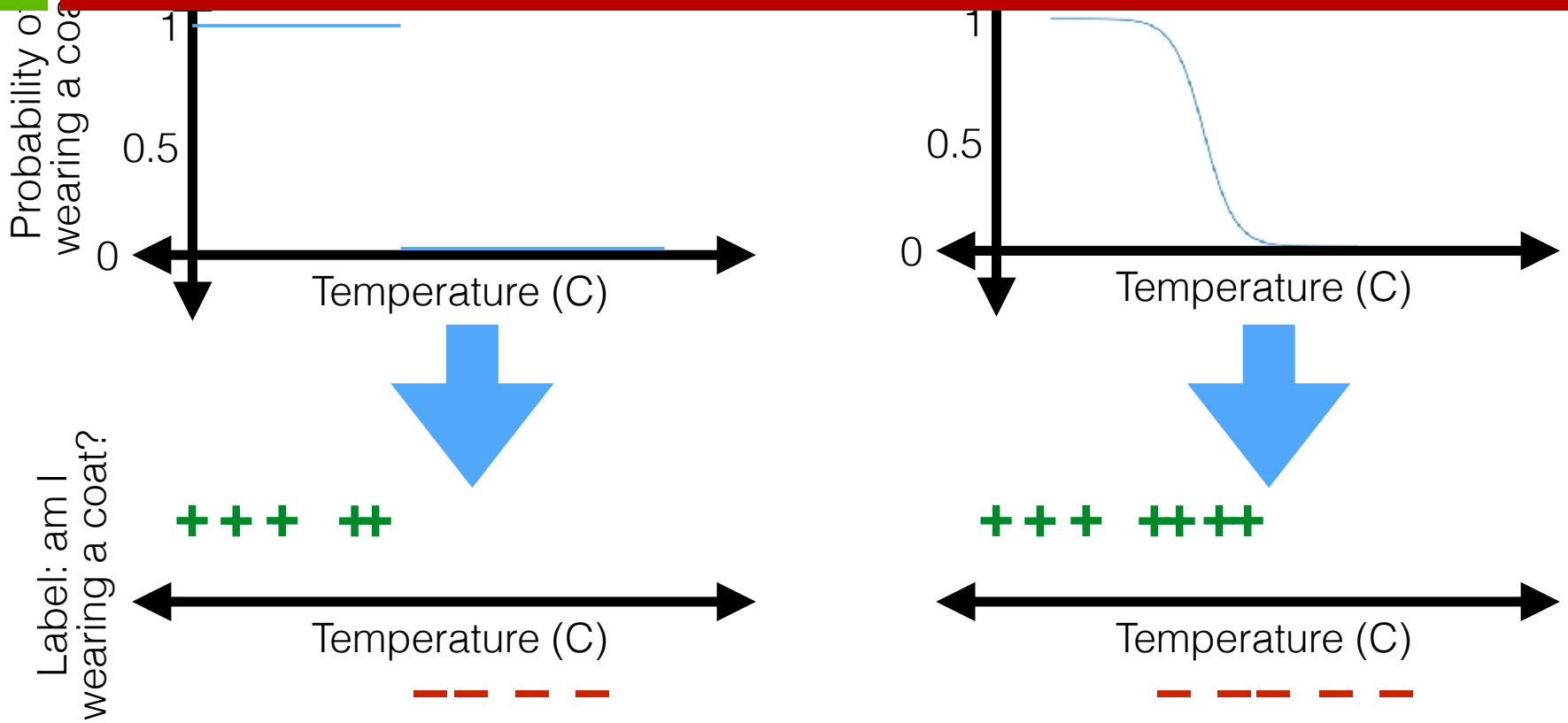

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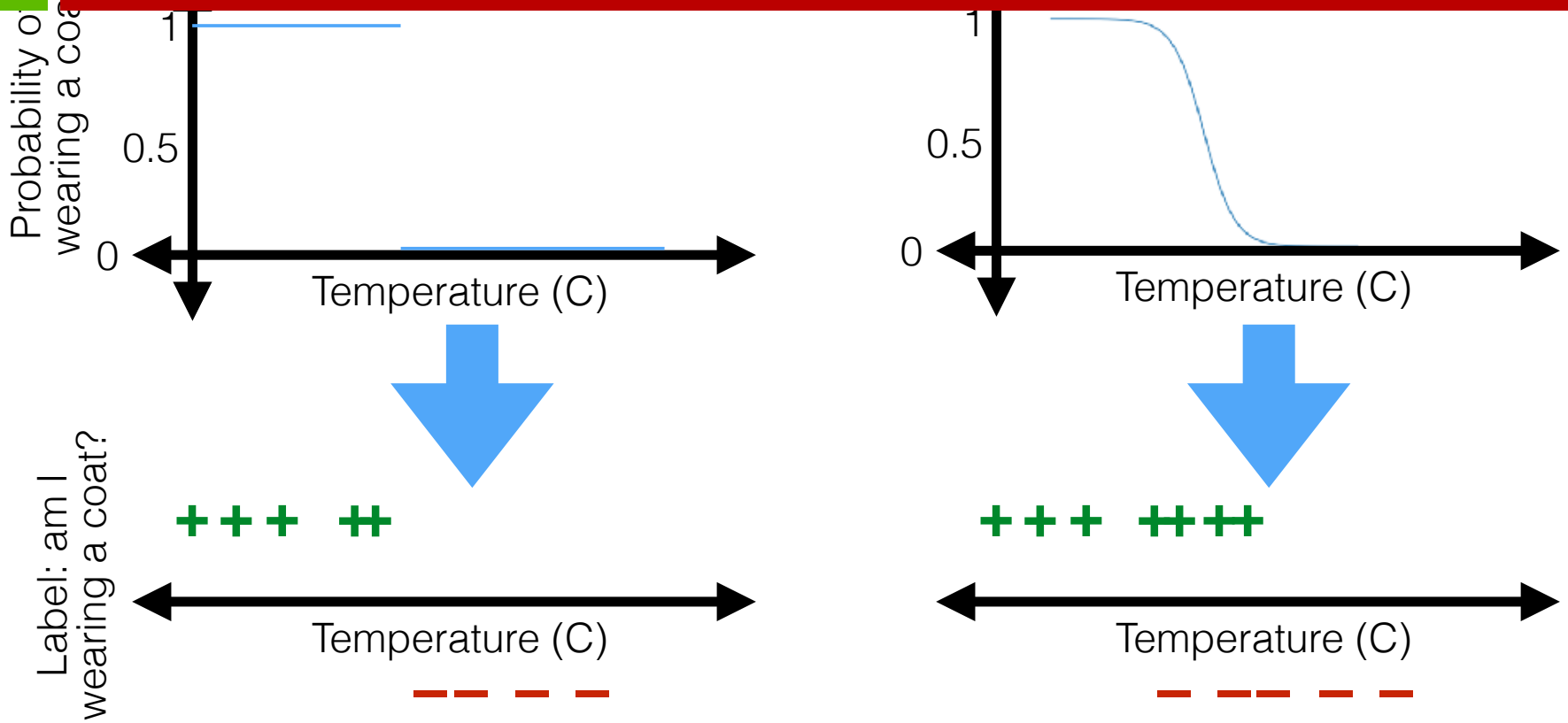
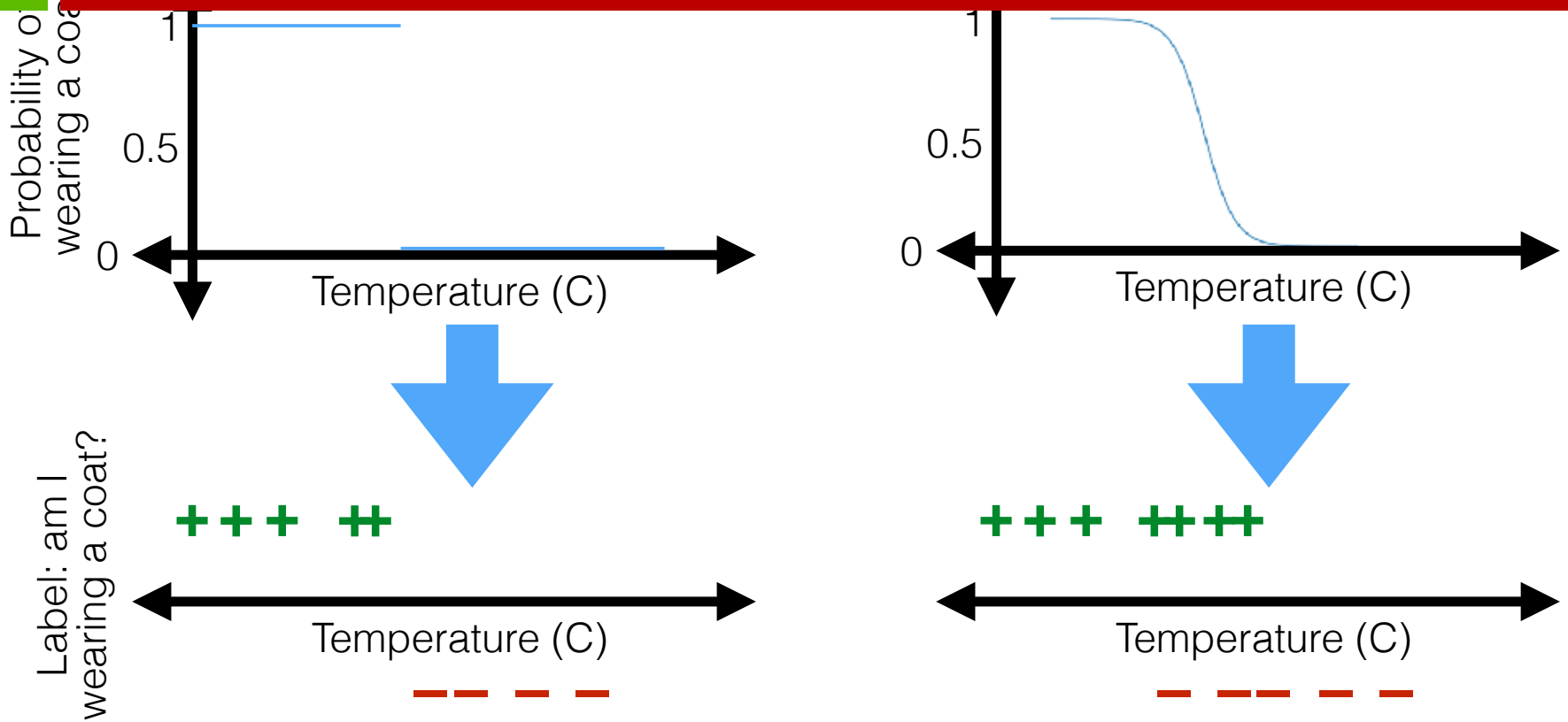


- How to make this shape?
  - Sigmoid/logistic function

Capturing uncertainty



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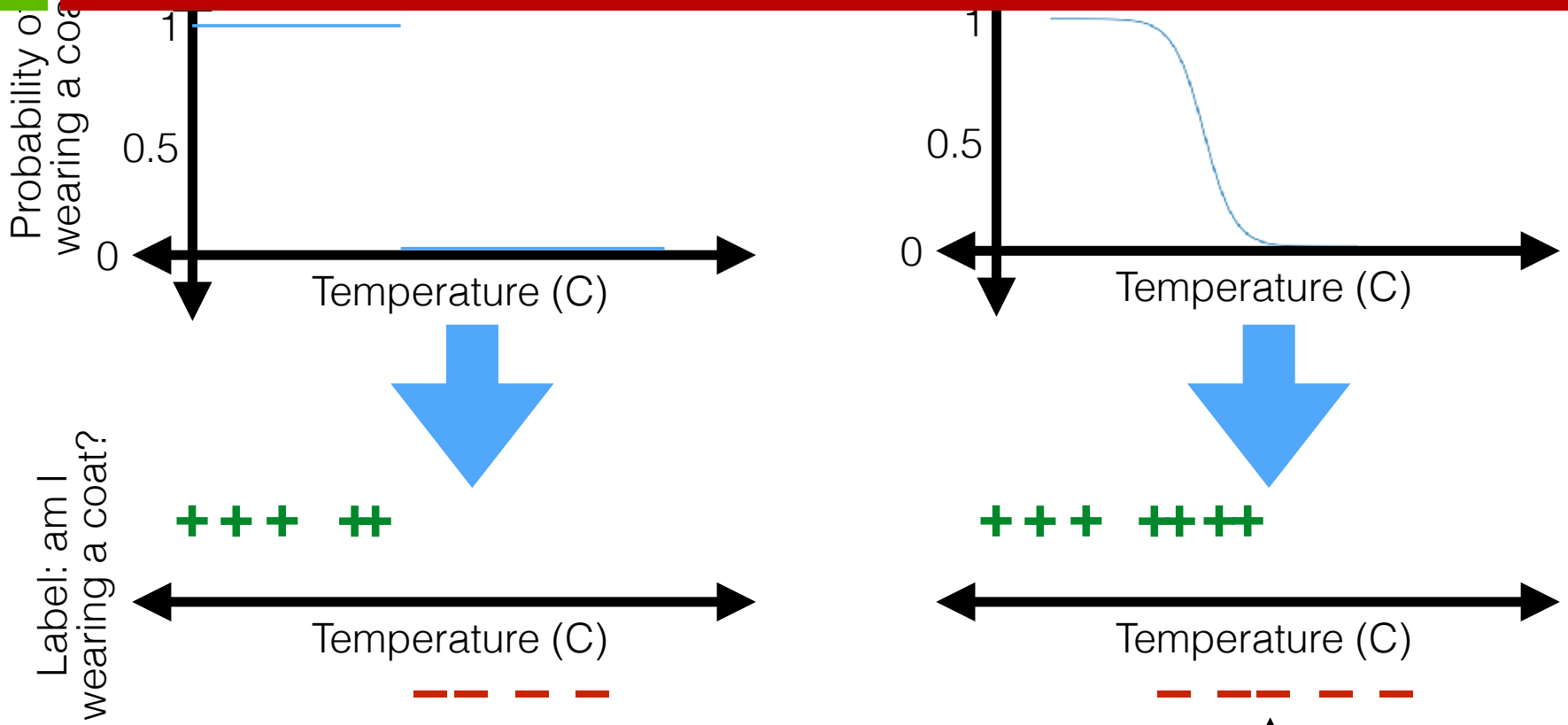
  - Sigmoid/logistic function

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$

Capturing uncertainty



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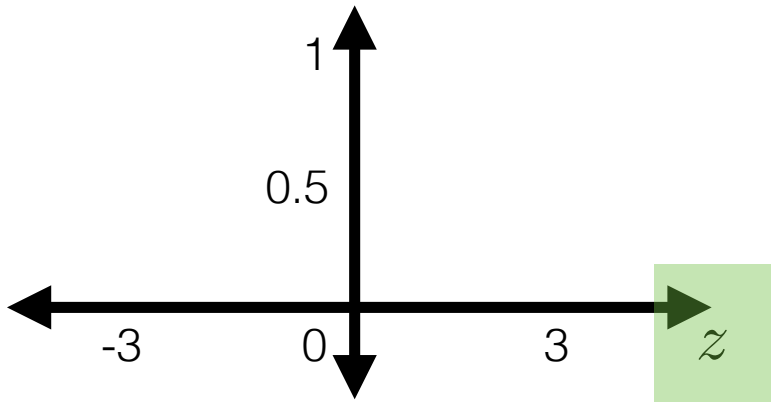

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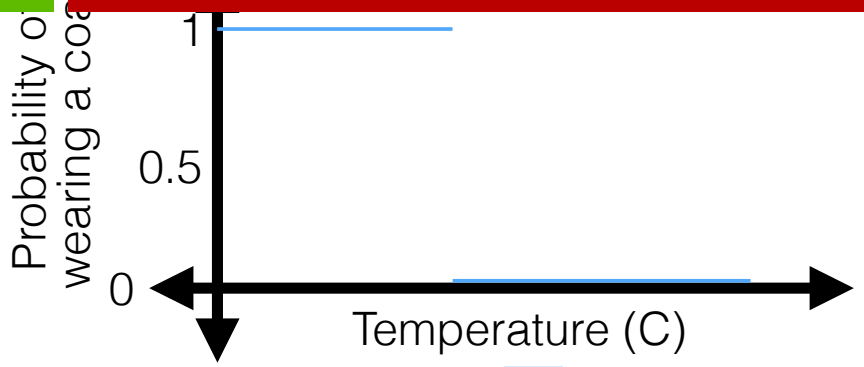

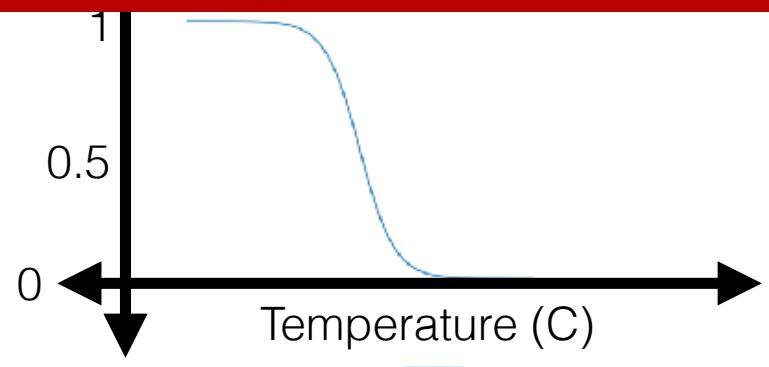

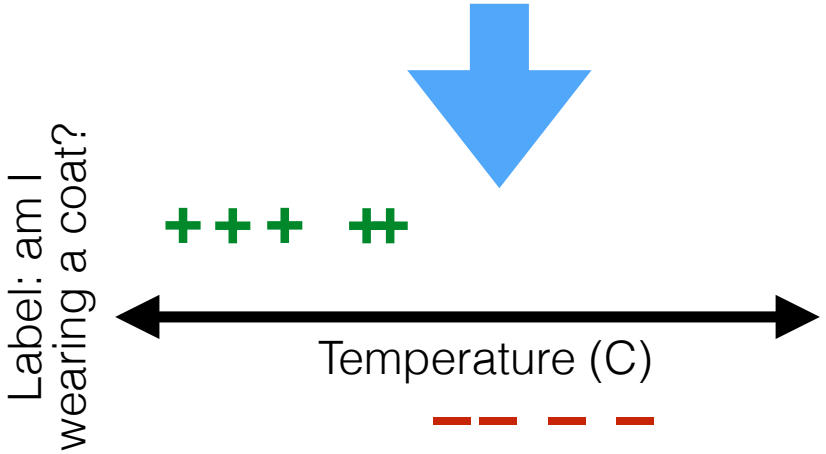

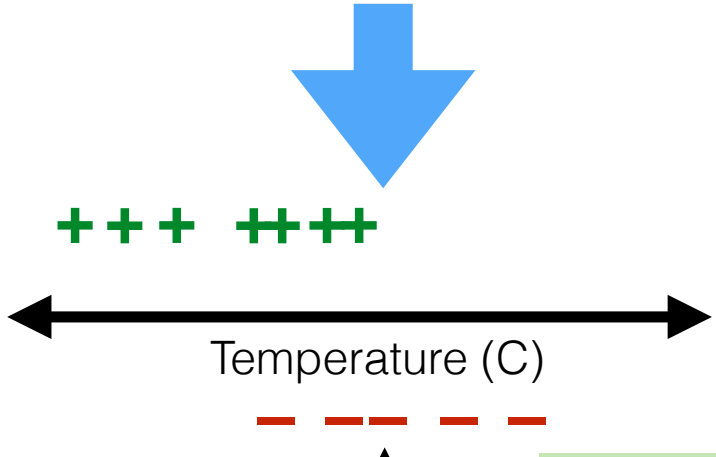

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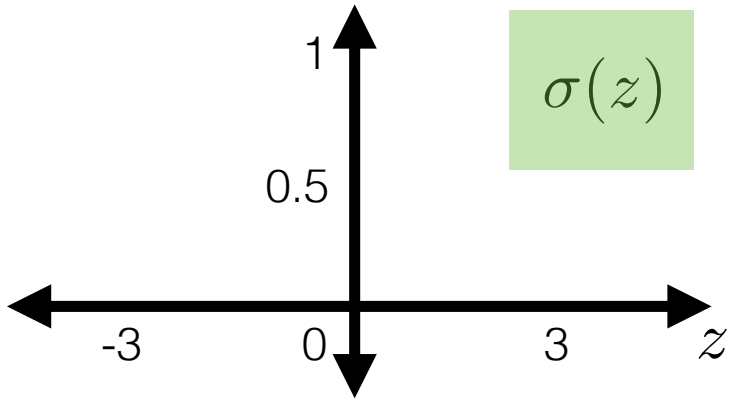
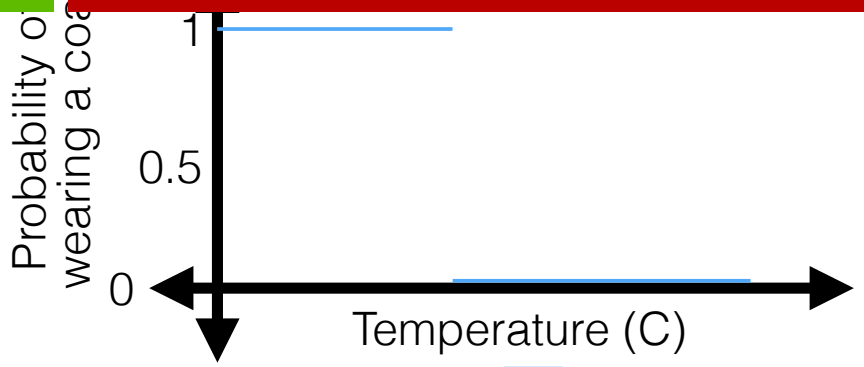
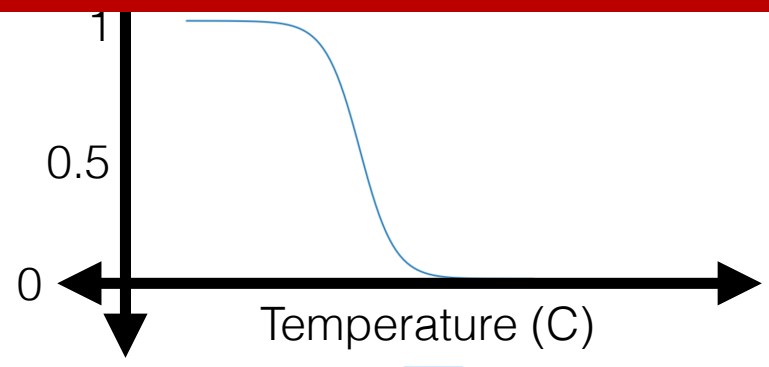
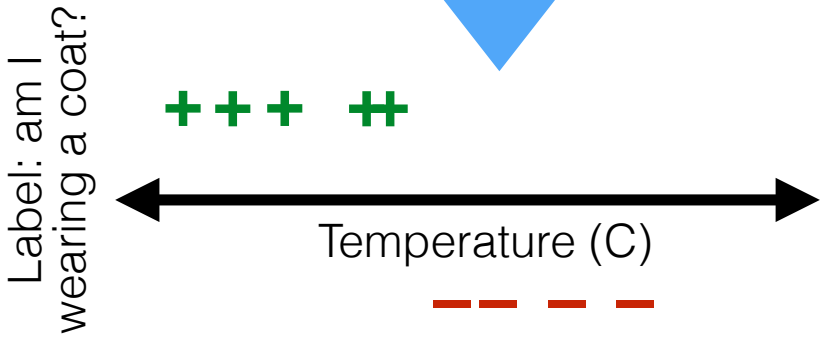


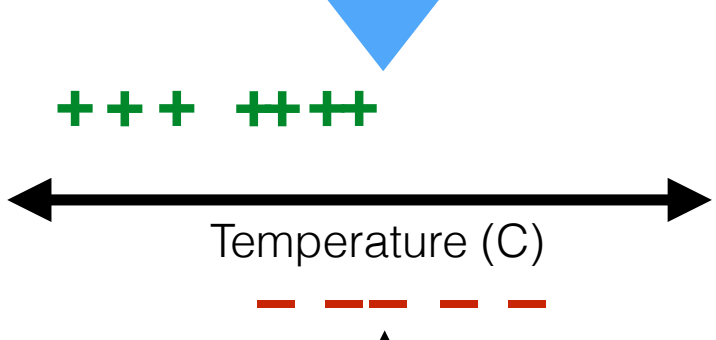
$\sigma(z)$

Capturing uncertainty









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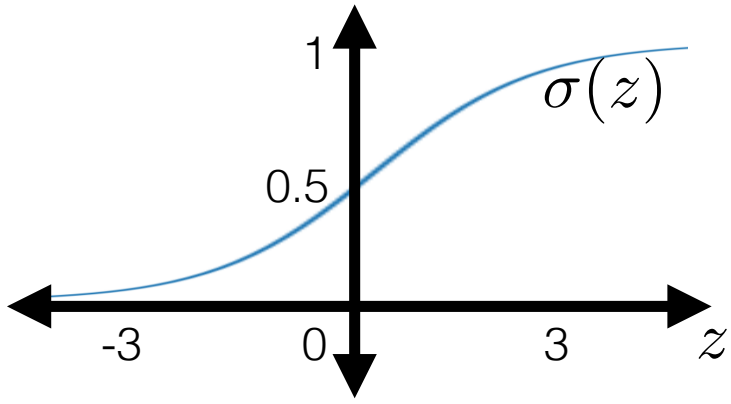

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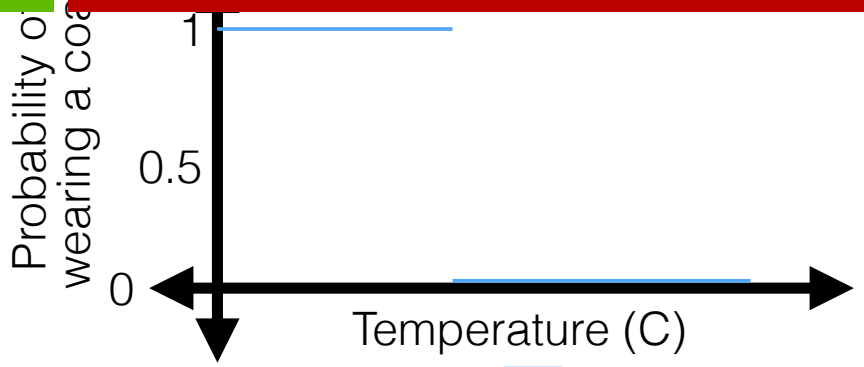

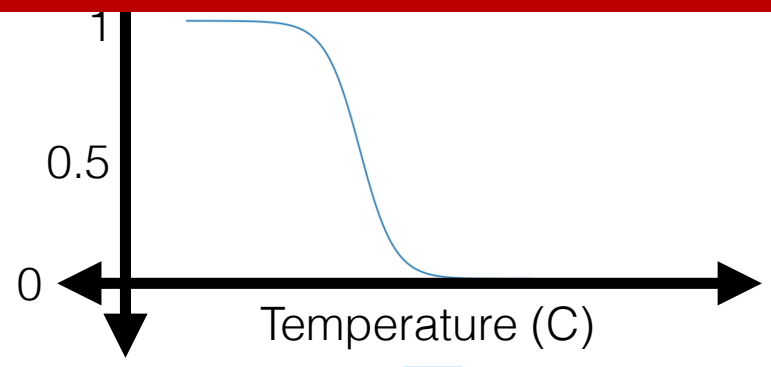

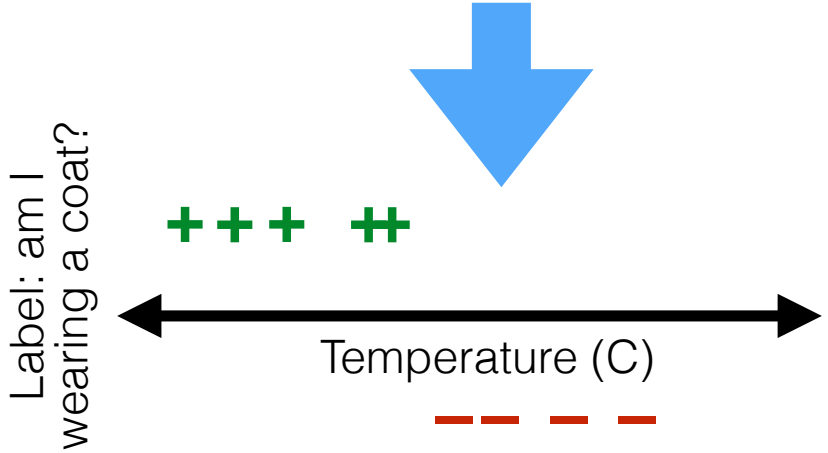

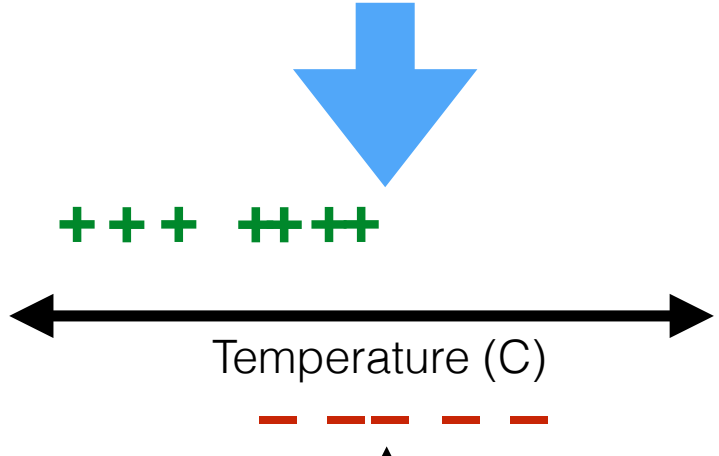

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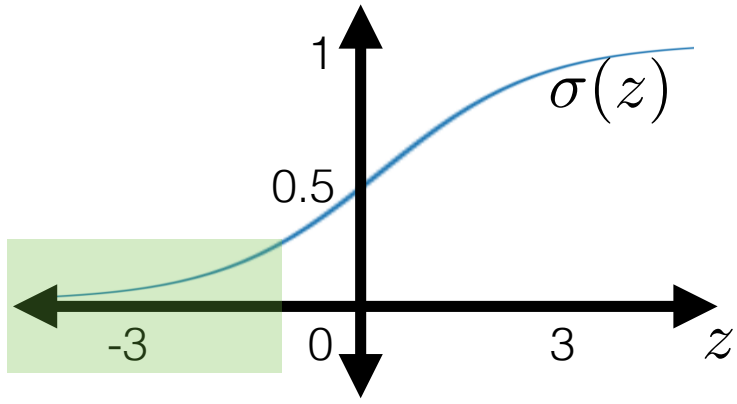

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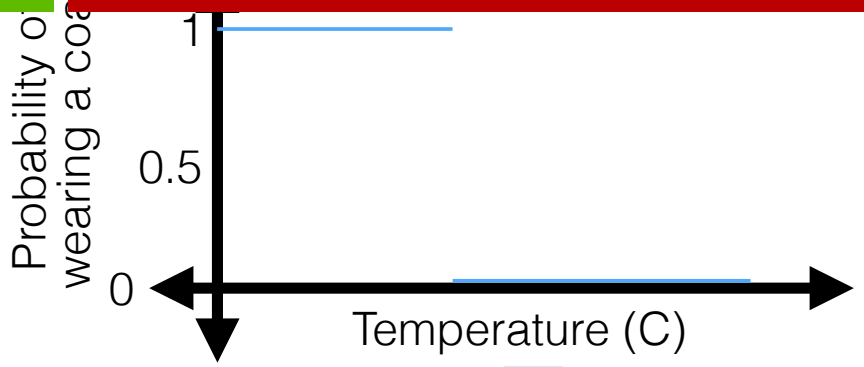

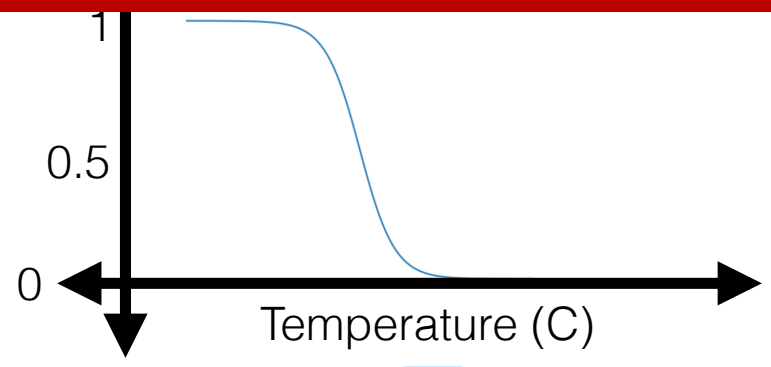

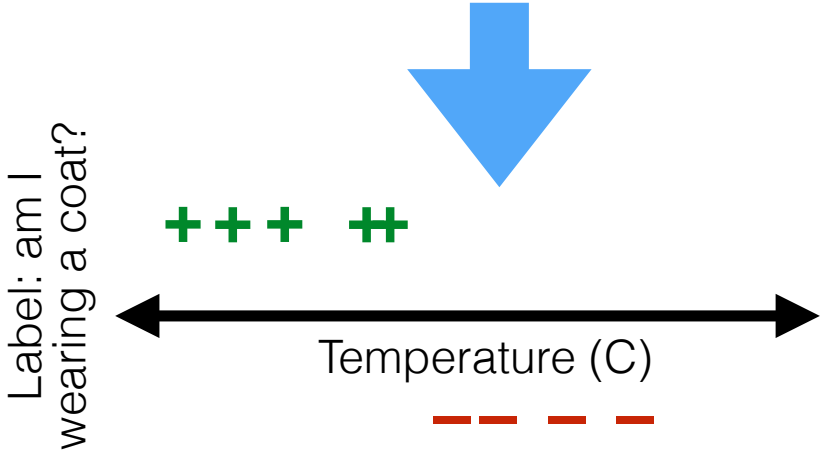

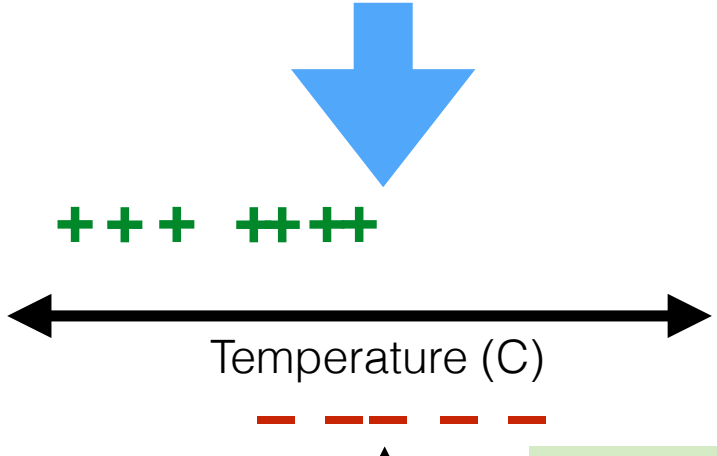

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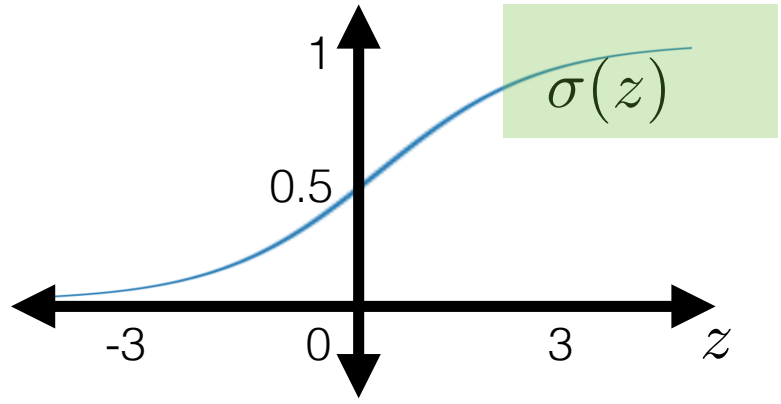
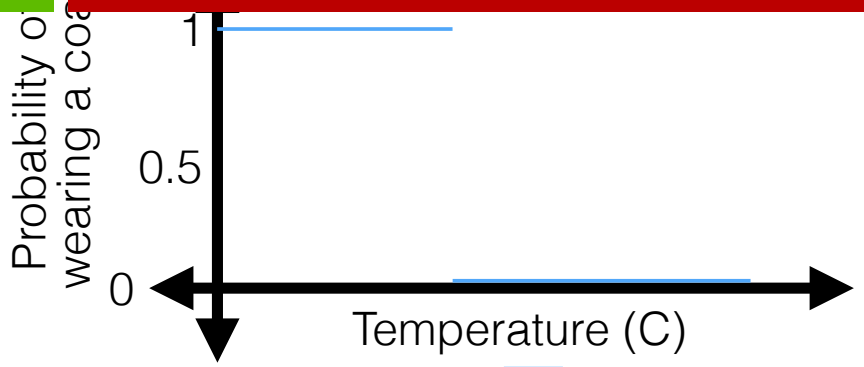
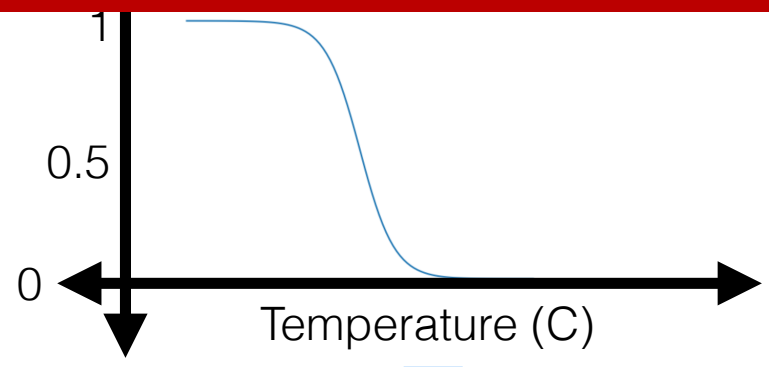
  - Sigmoid/logistic function

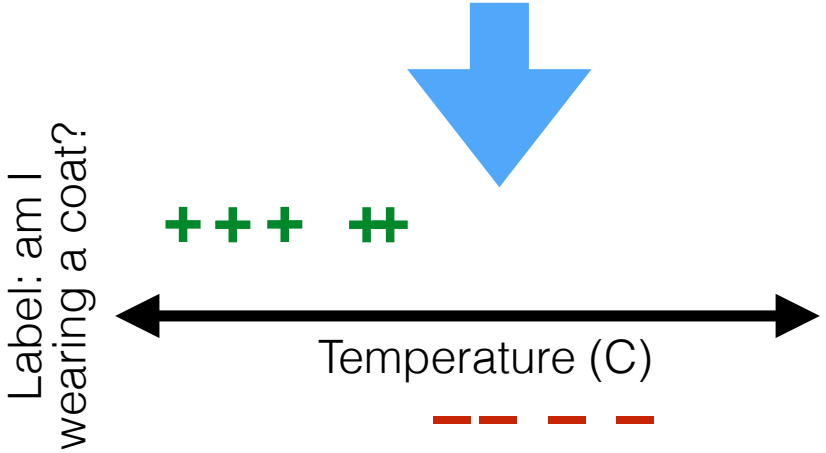
$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$

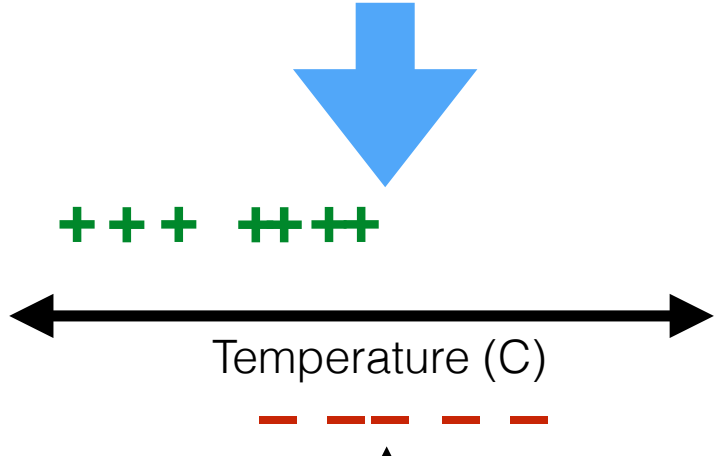


Capturing uncertainty









- How to make this shape?
  - Sigmoid/logistic function

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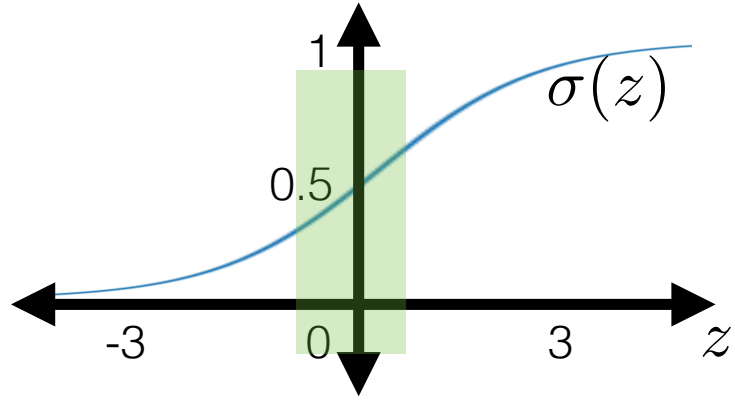

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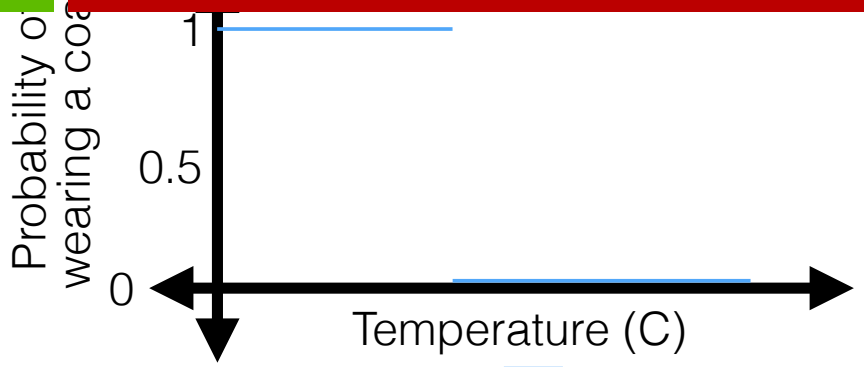

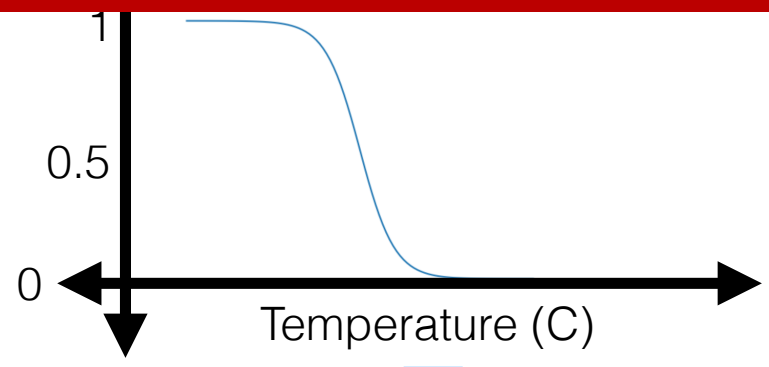

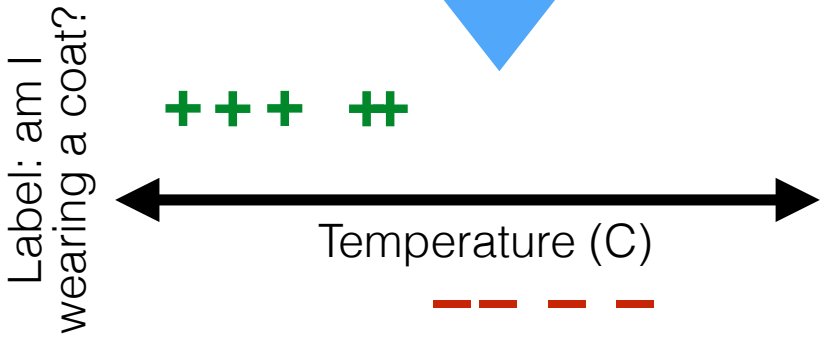

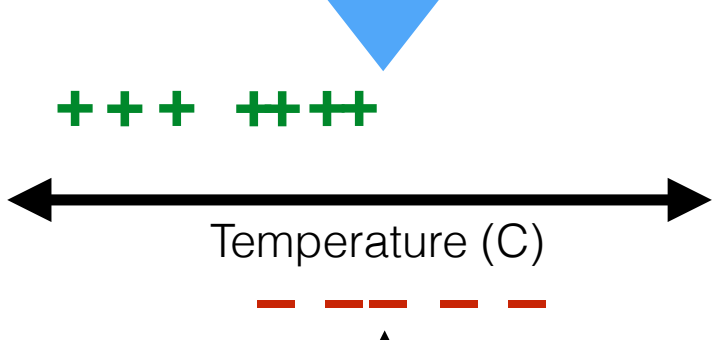

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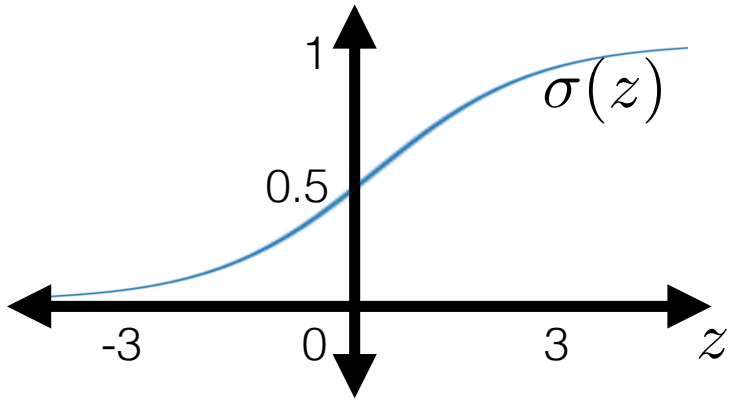

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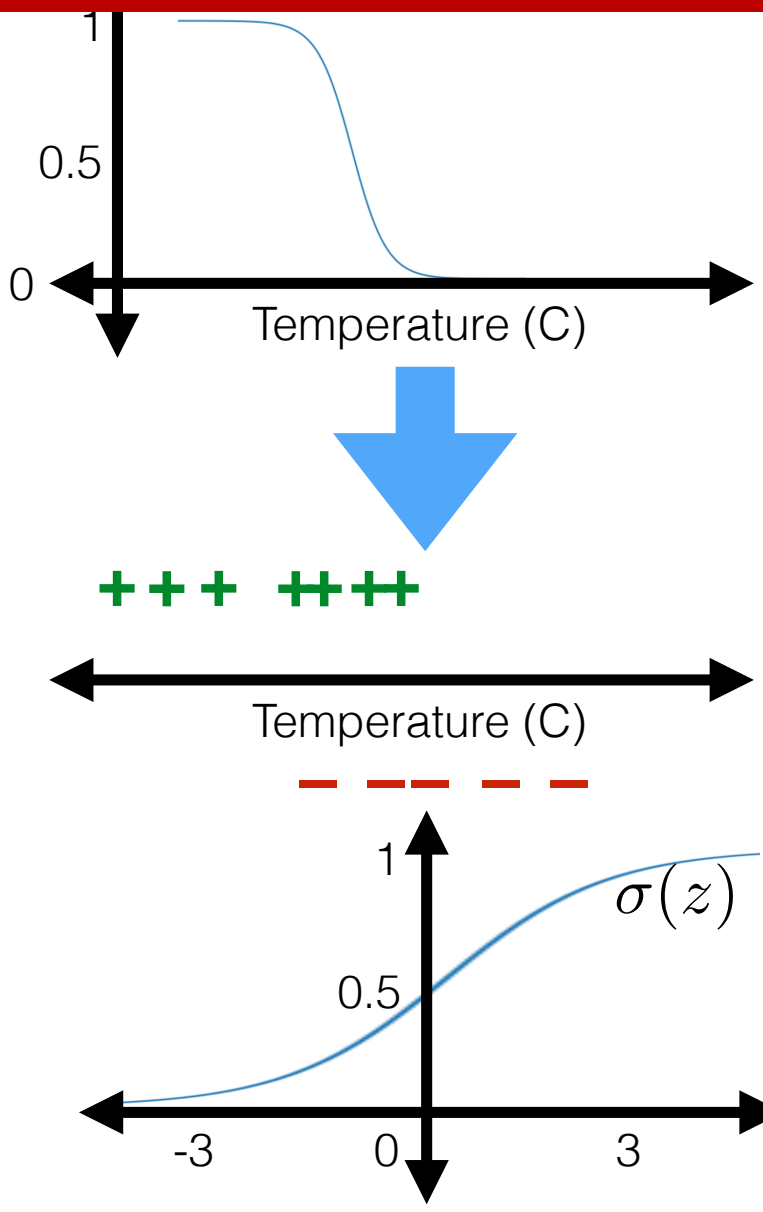

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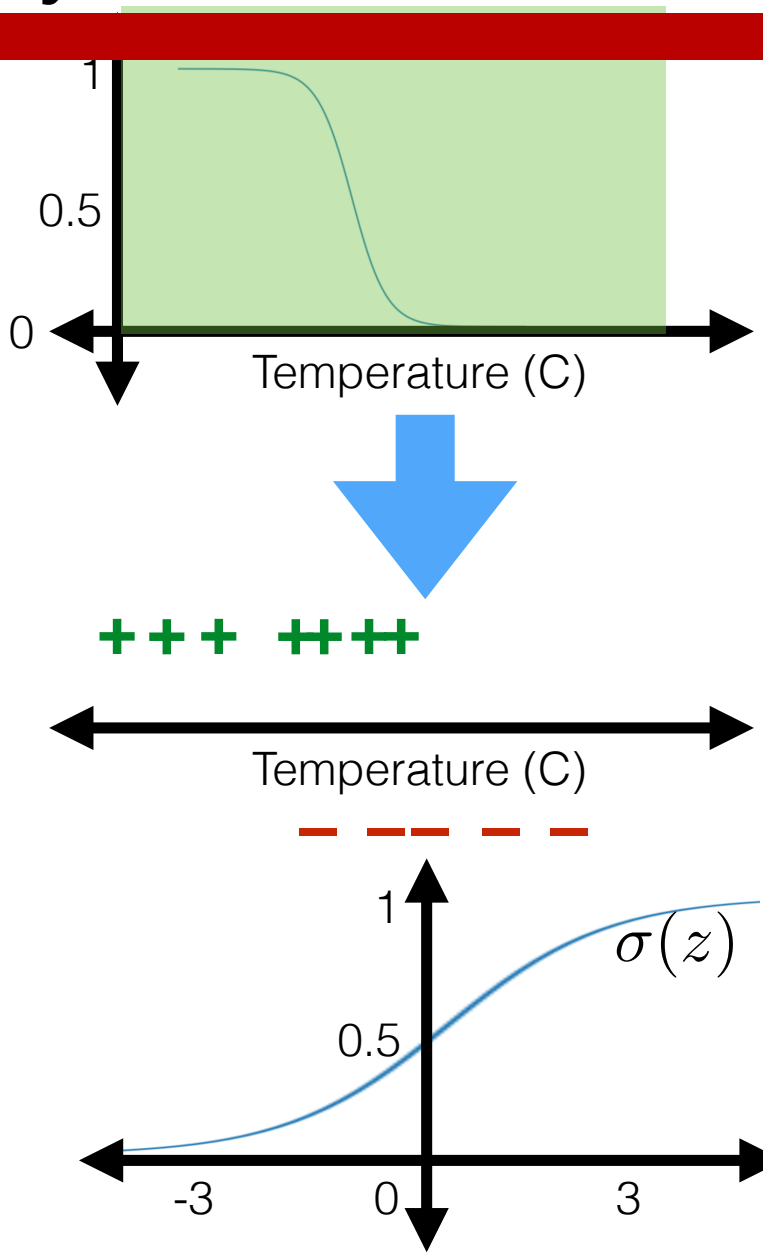

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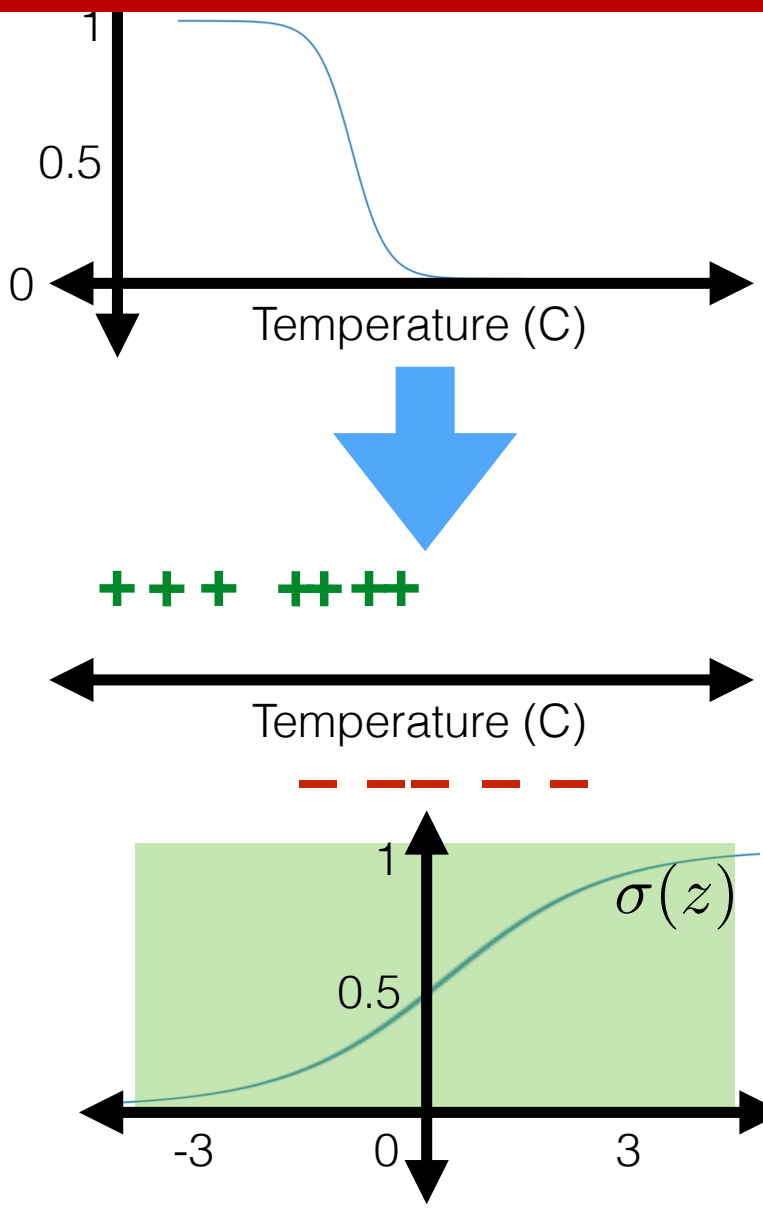

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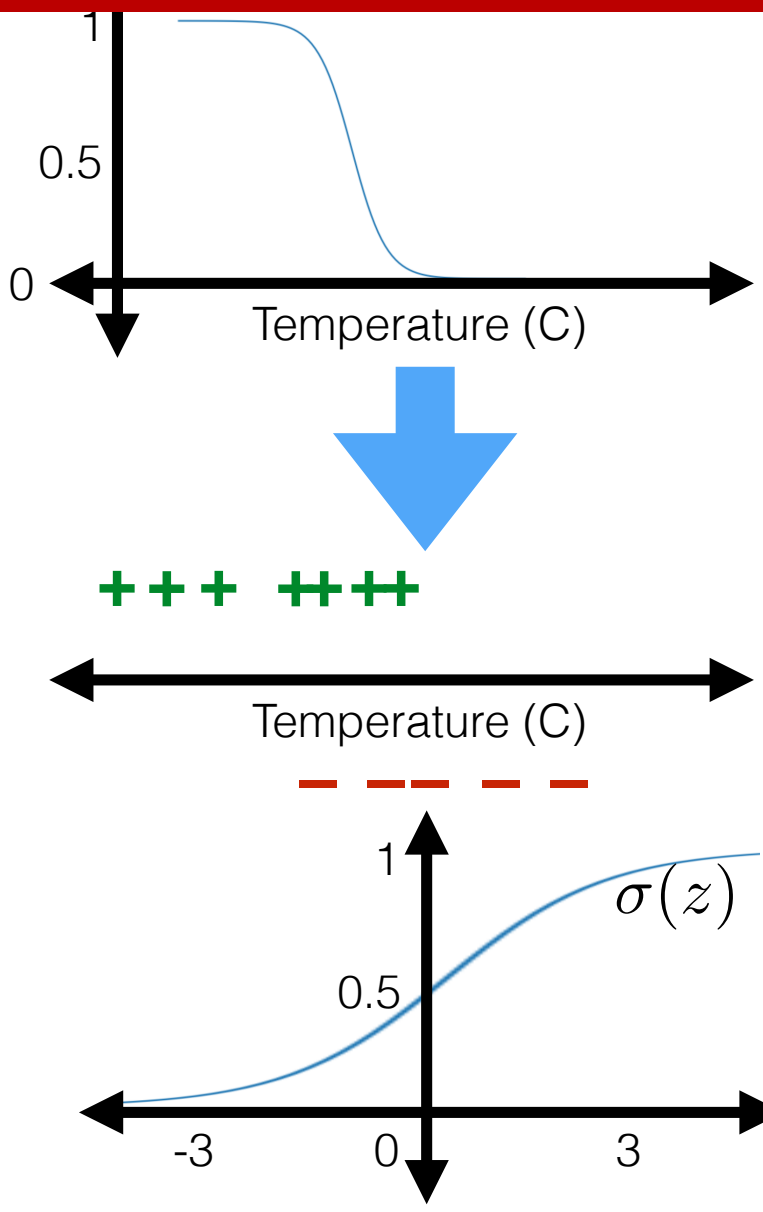

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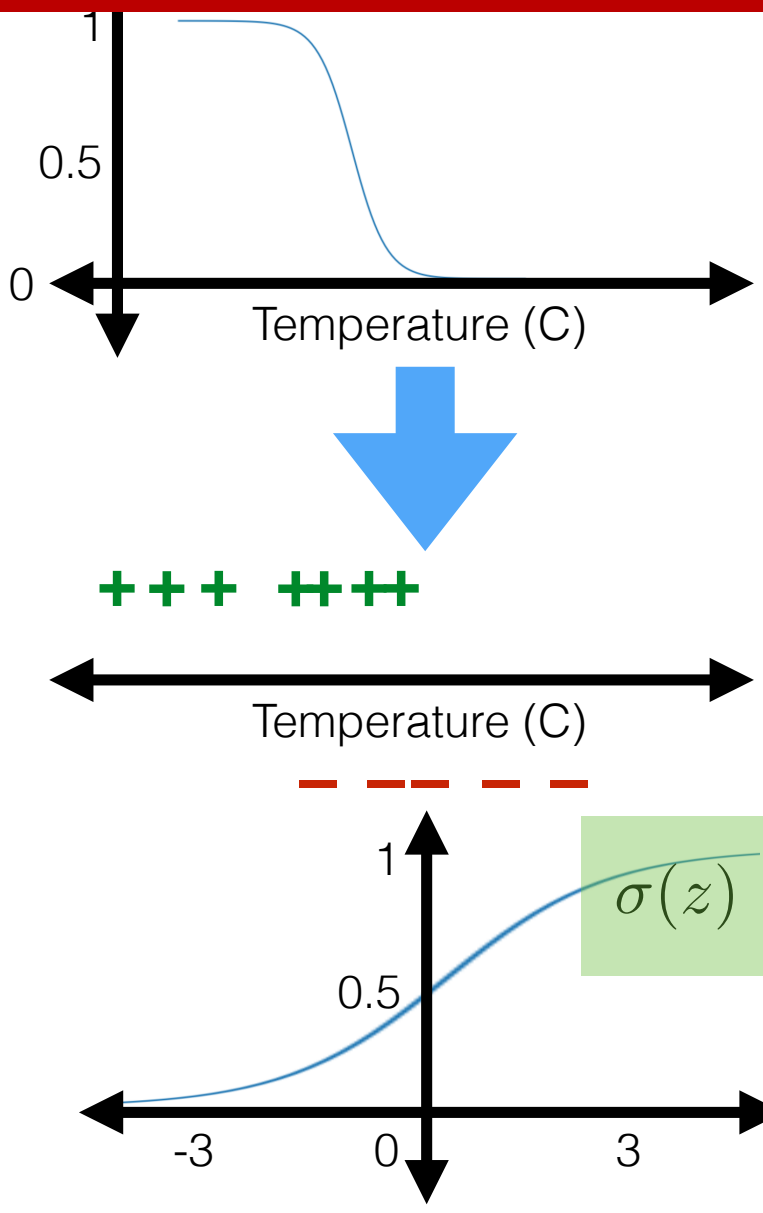

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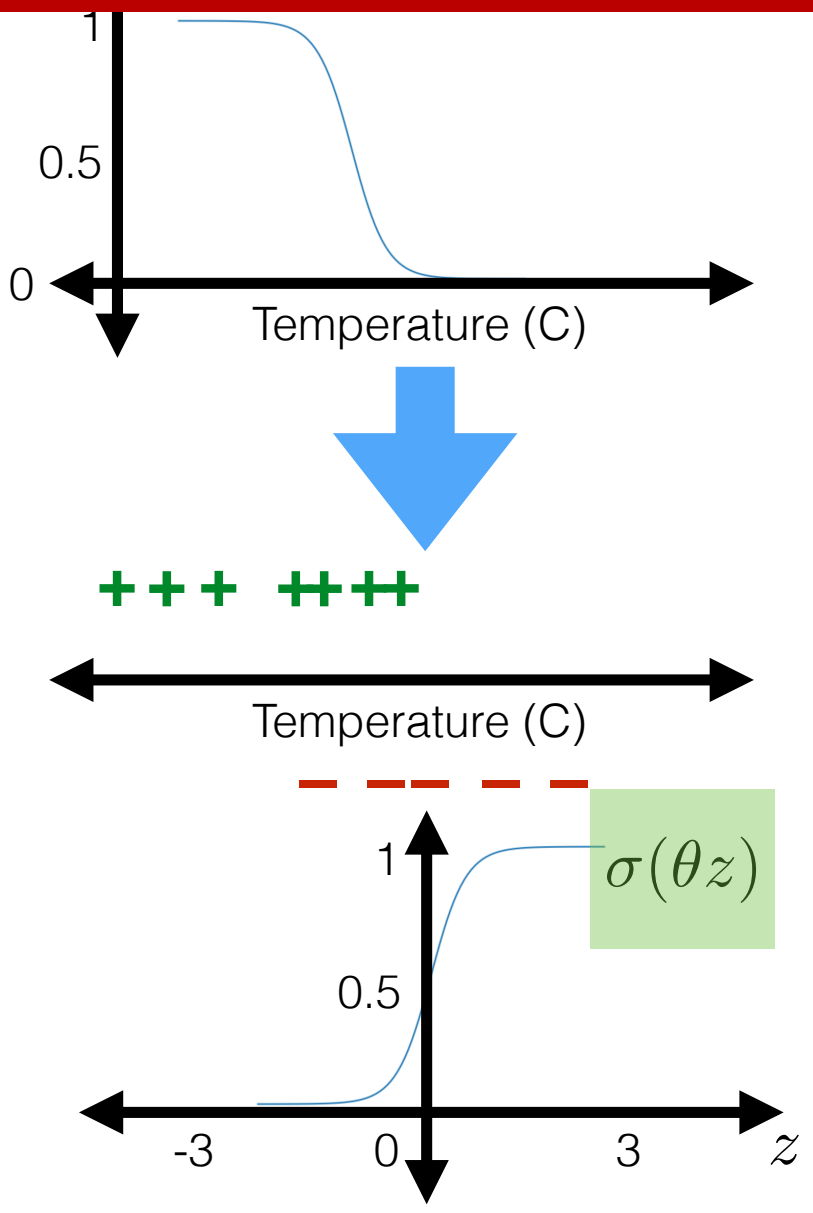

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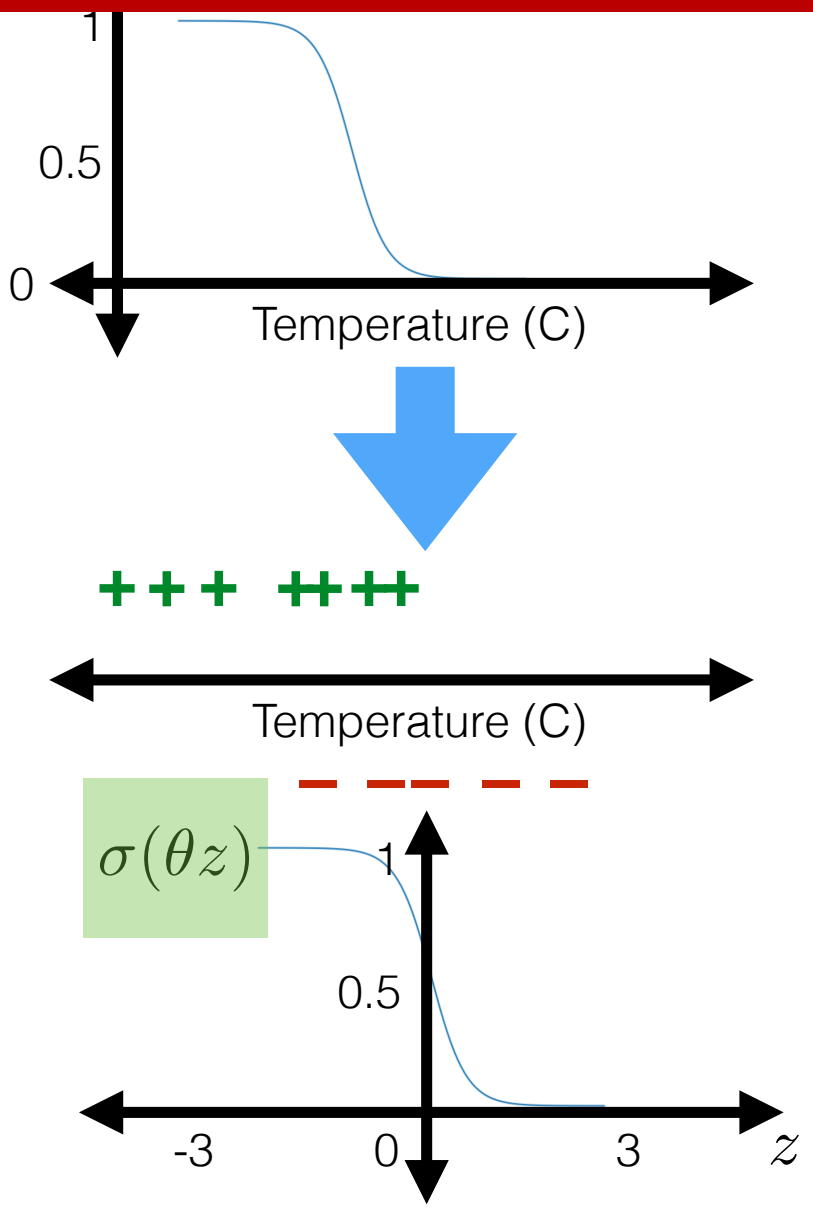

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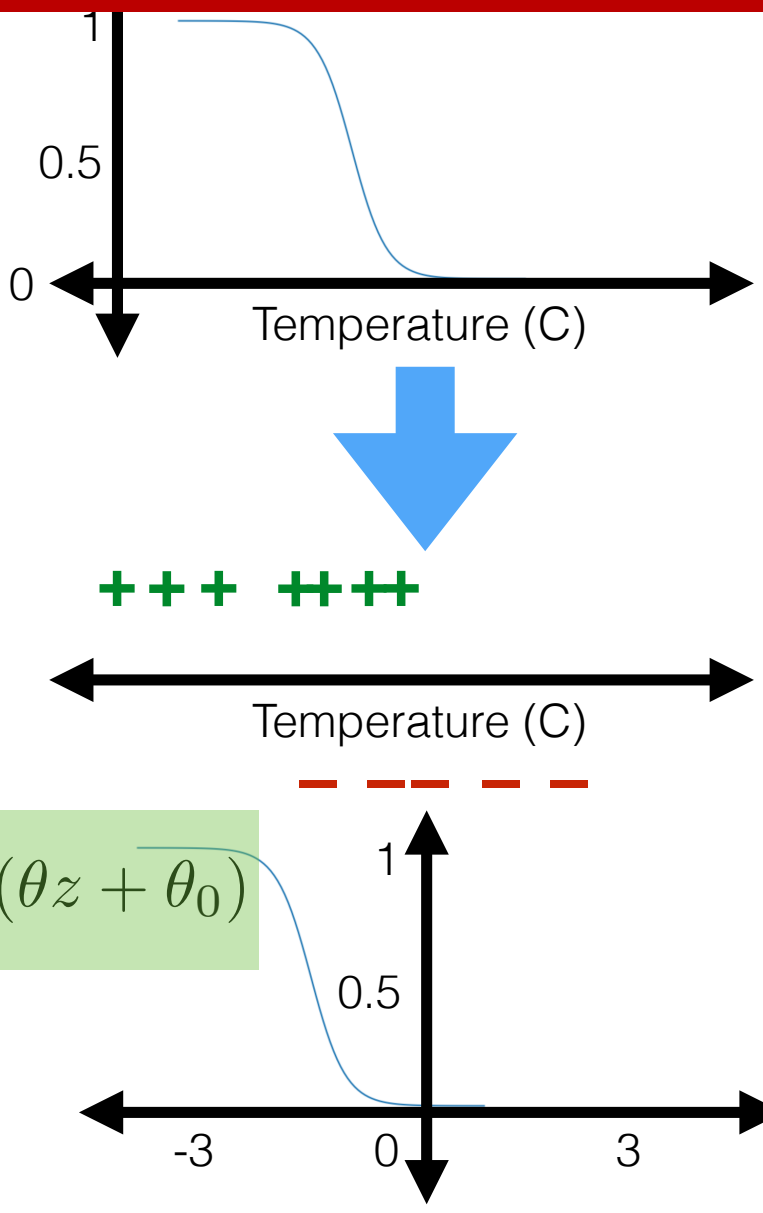

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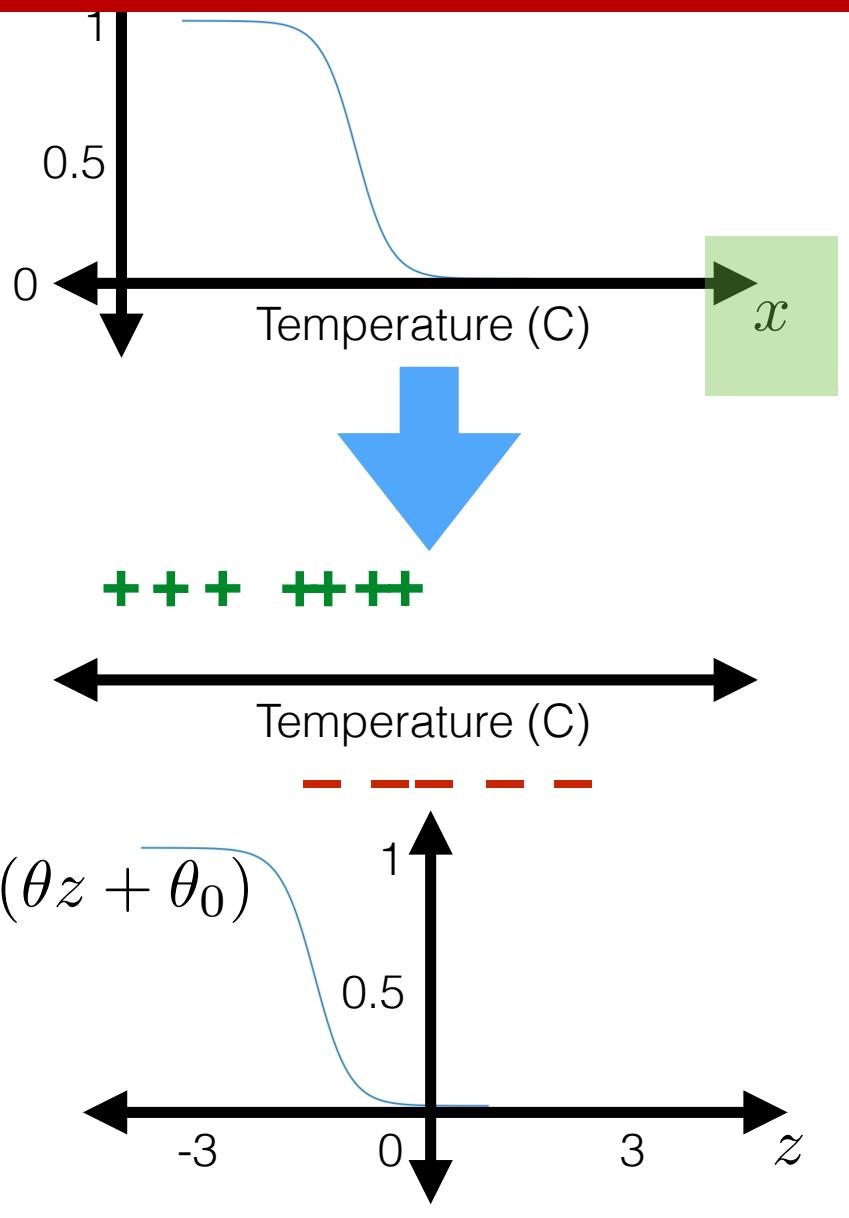

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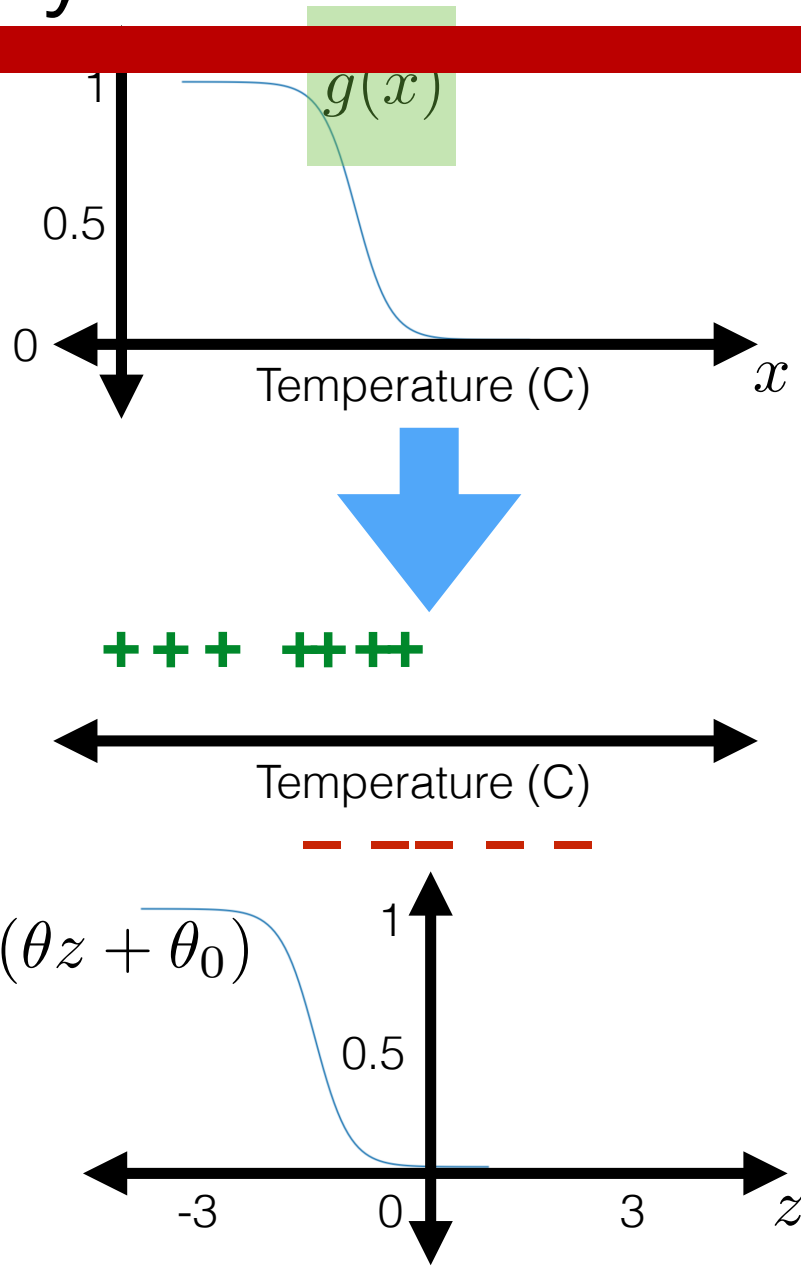

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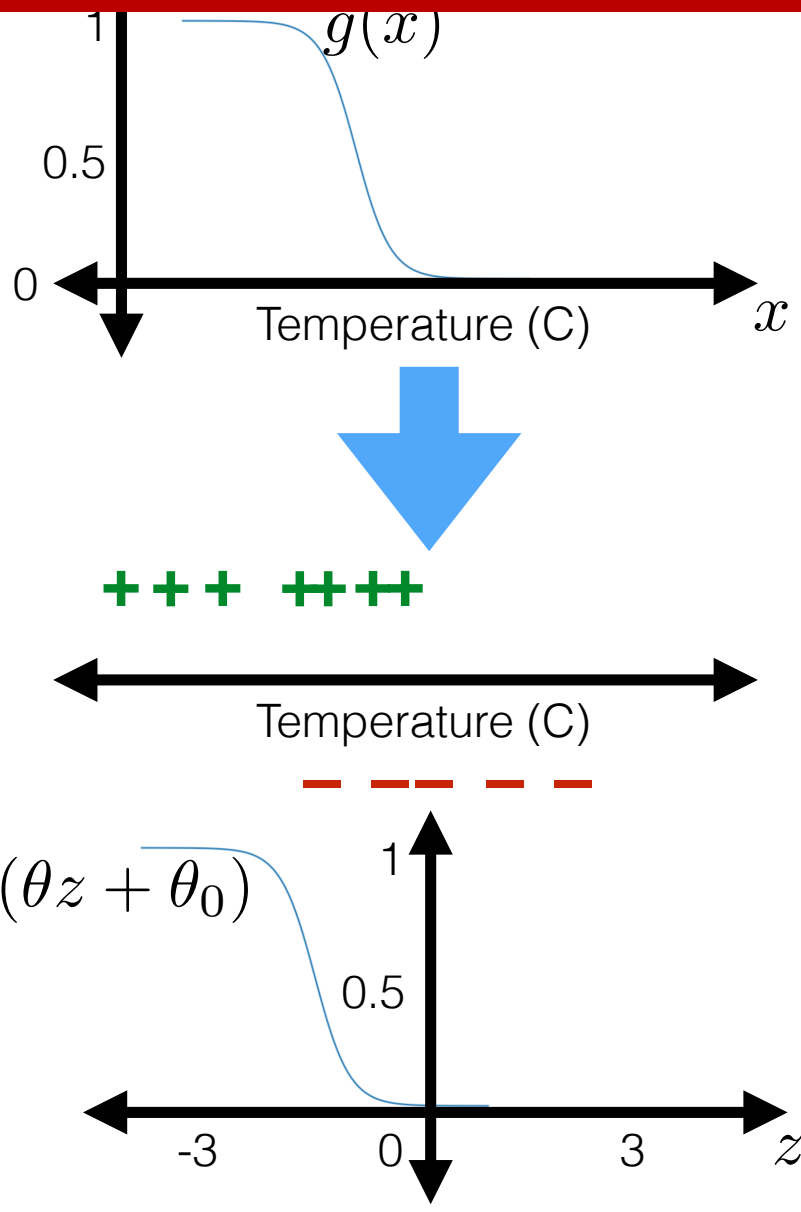
Capturing uncertainty

$$g(x) = \sigma(\theta x + \theta_0)$$



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Capturing uncertainty

$$g(x) = \sigma(\theta x + \theta_0)$$
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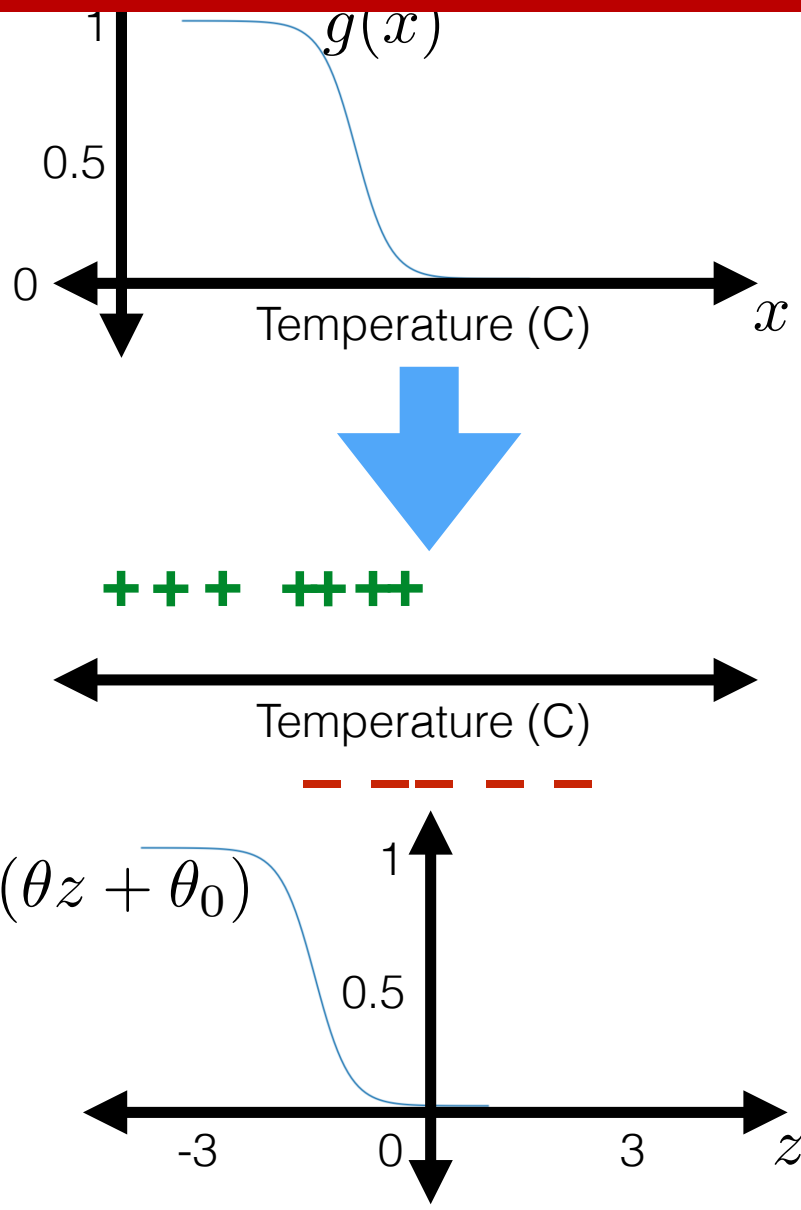

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Capturing uncertainty

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1 feature:

$$g(x) = \sigma(\theta x + \theta_0)$$

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Capturing uncertainty

Feature:

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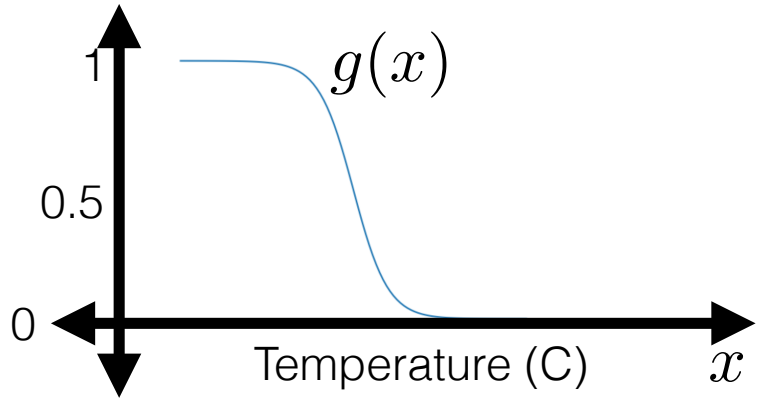

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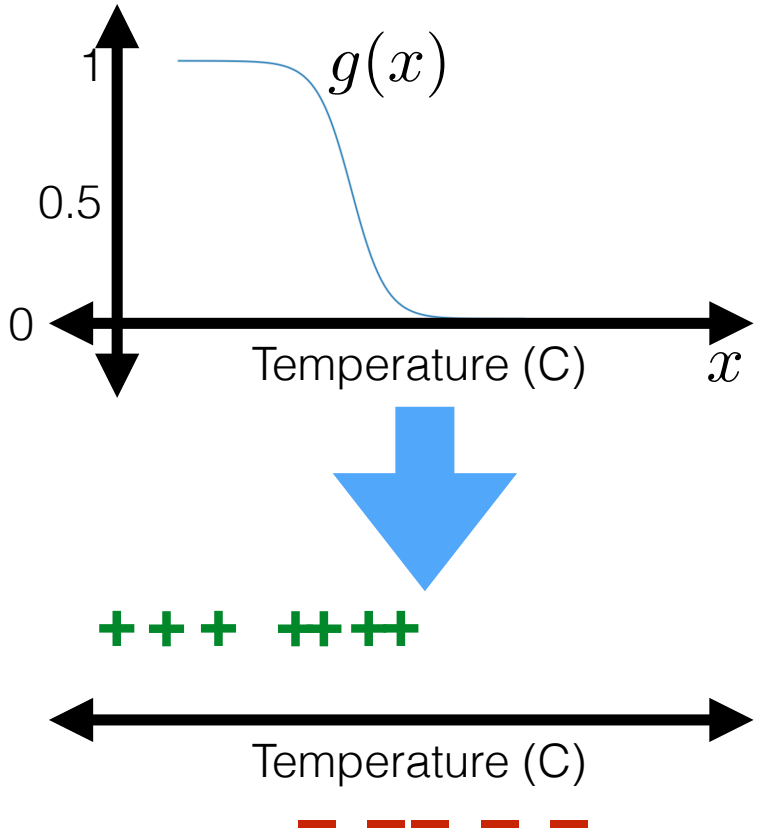

Capturing uncertainty

2 features:

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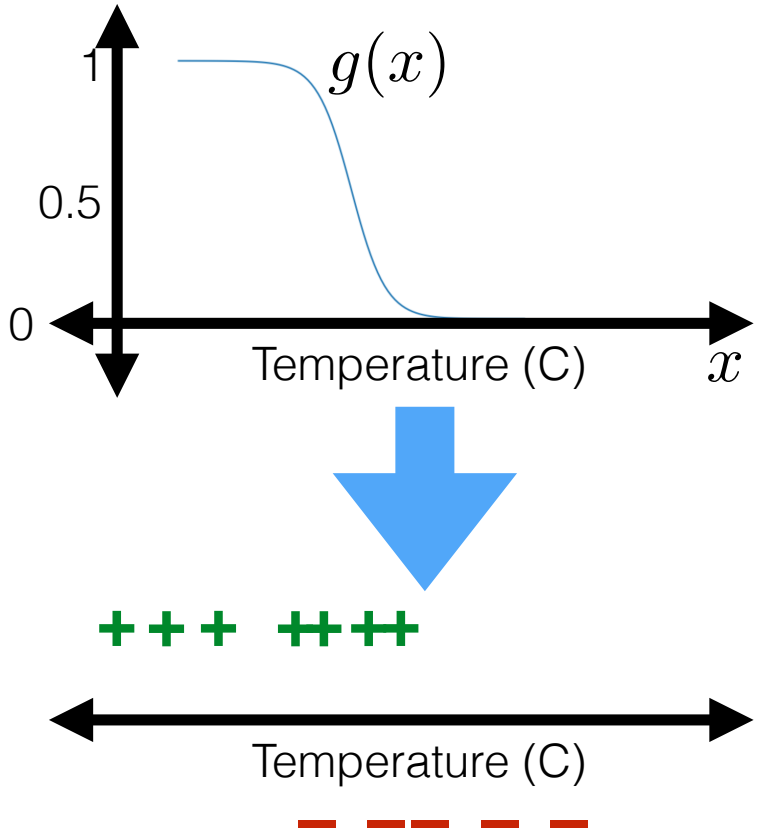

Capturing uncertainty

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2 features:

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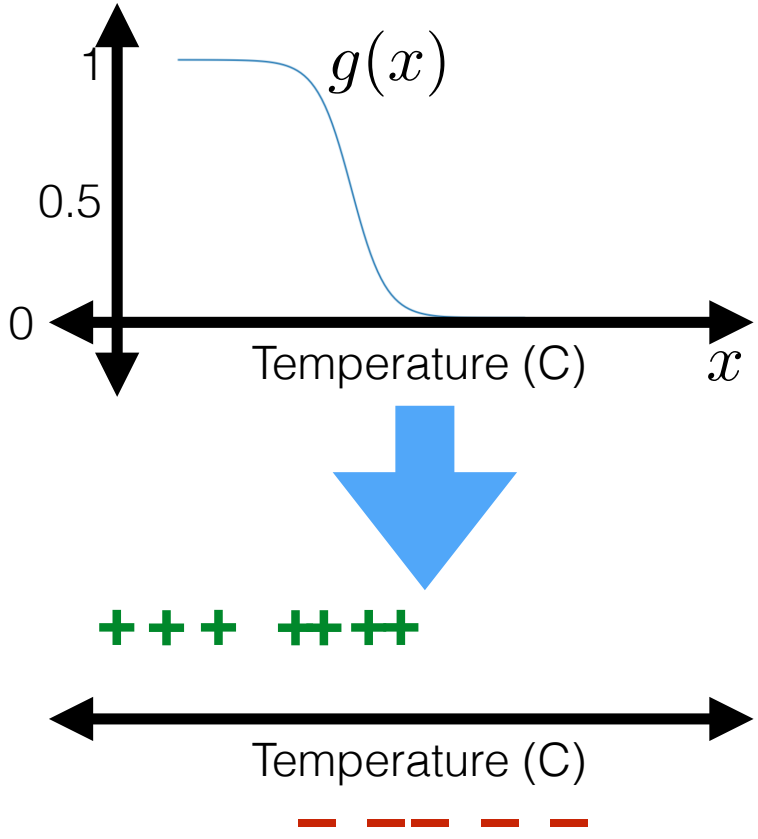

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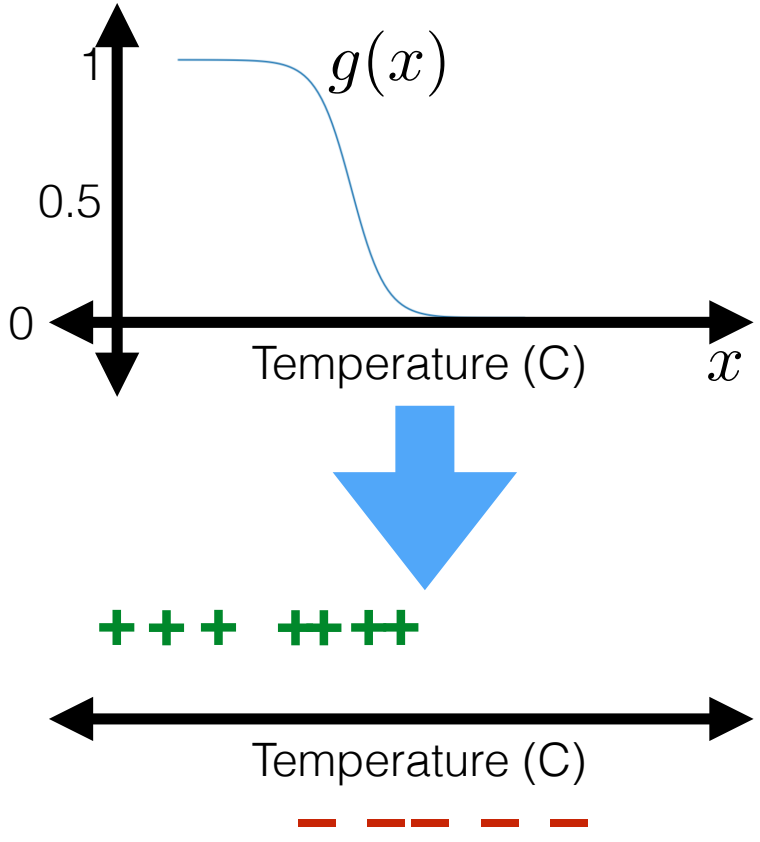

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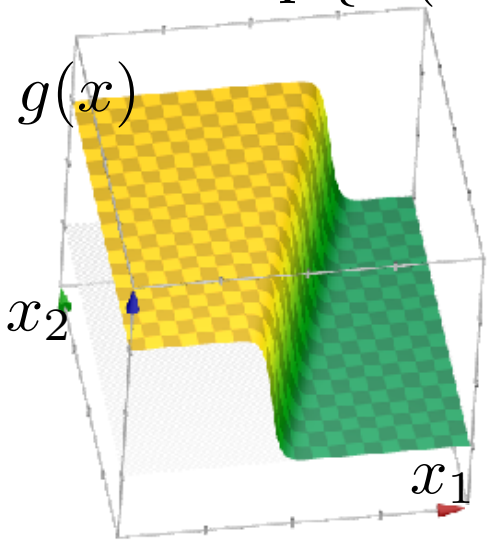

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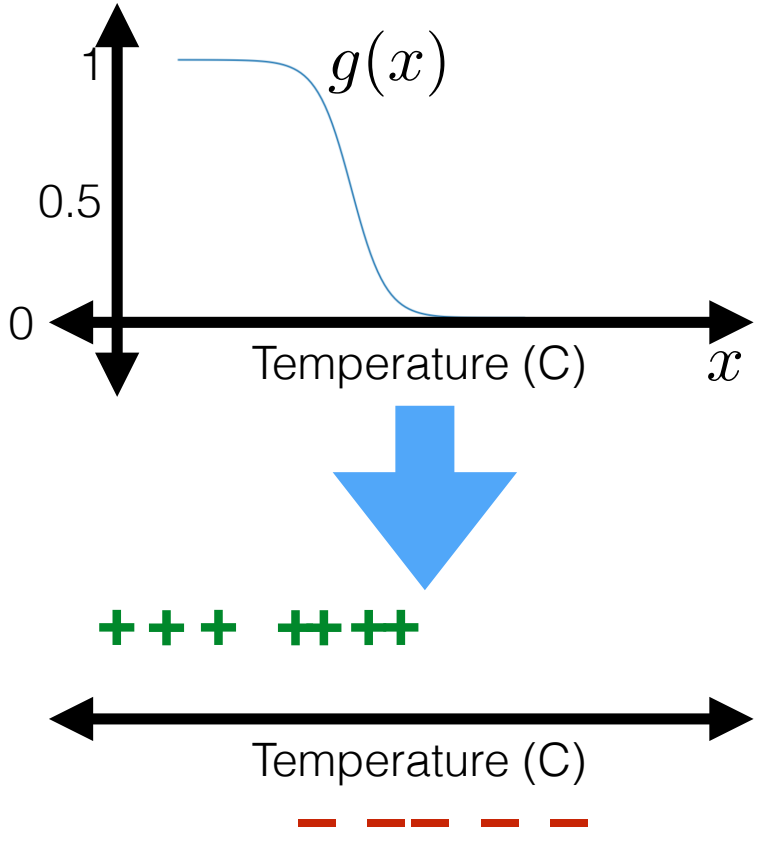

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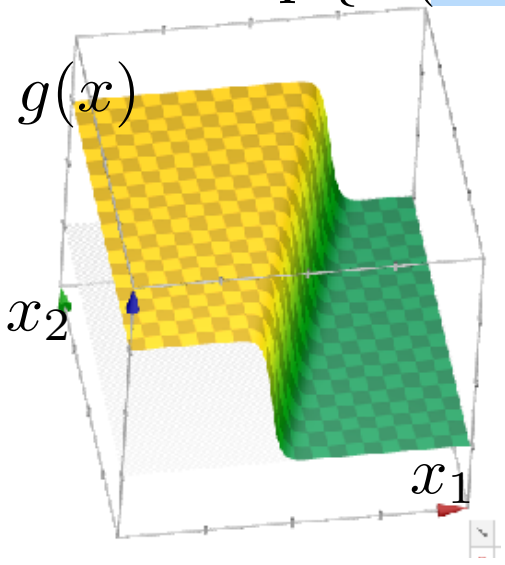

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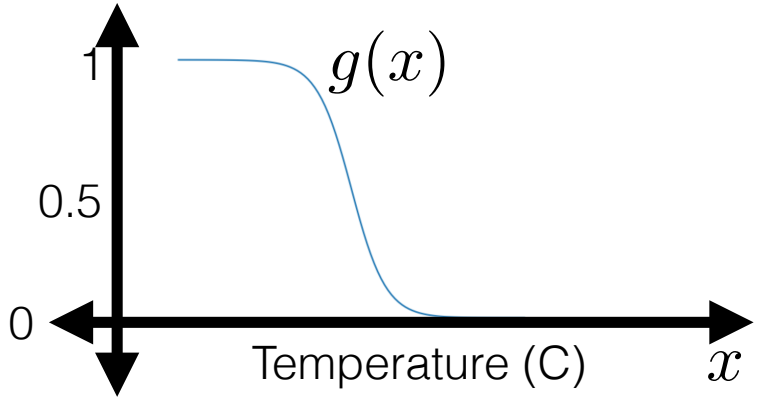

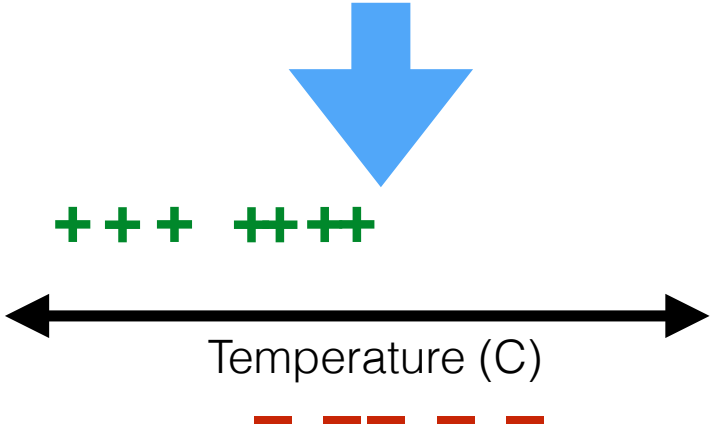

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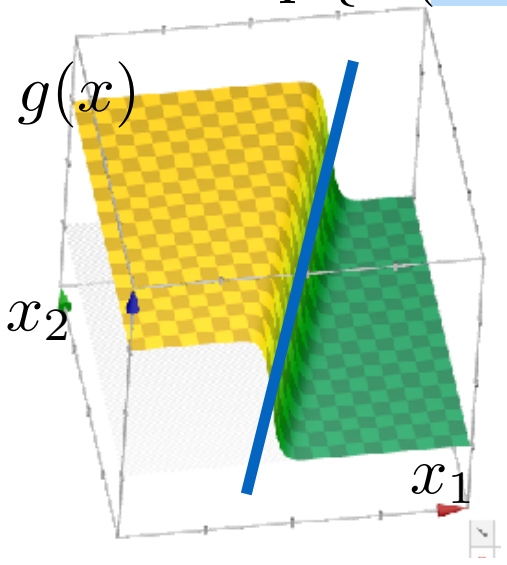

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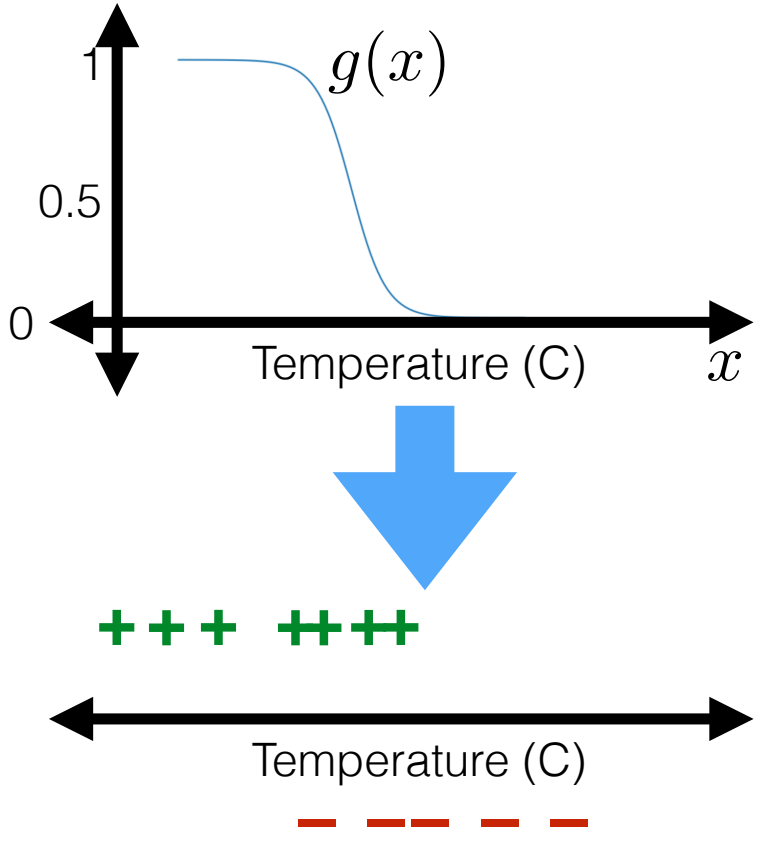

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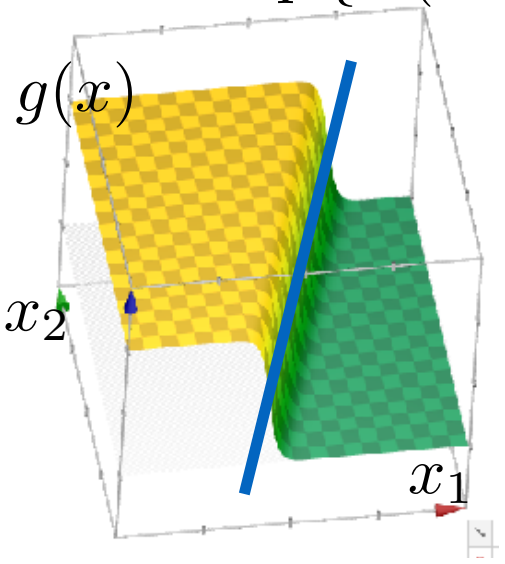

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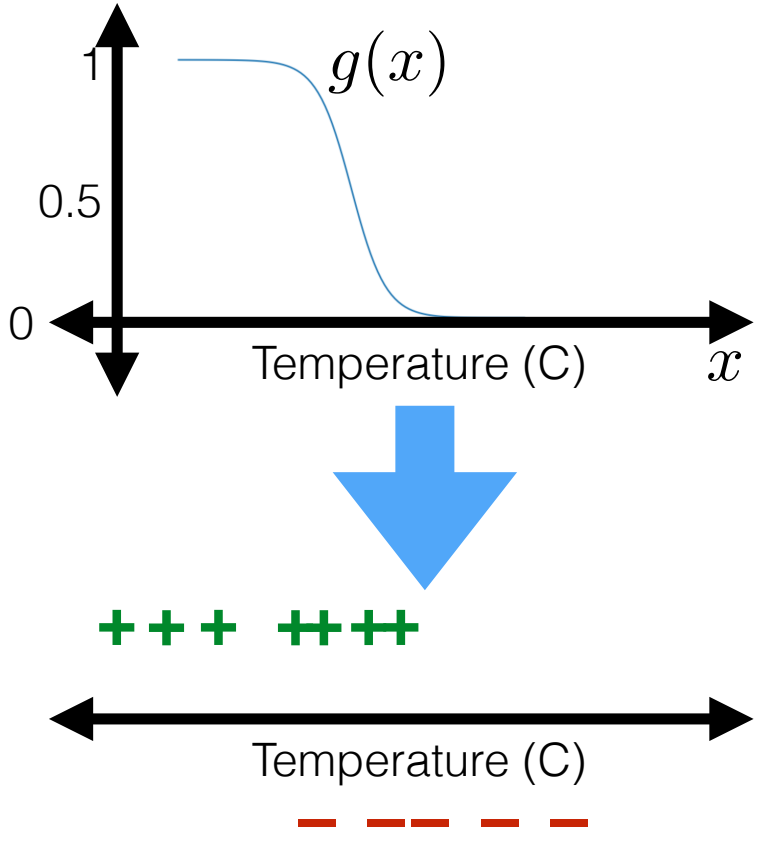

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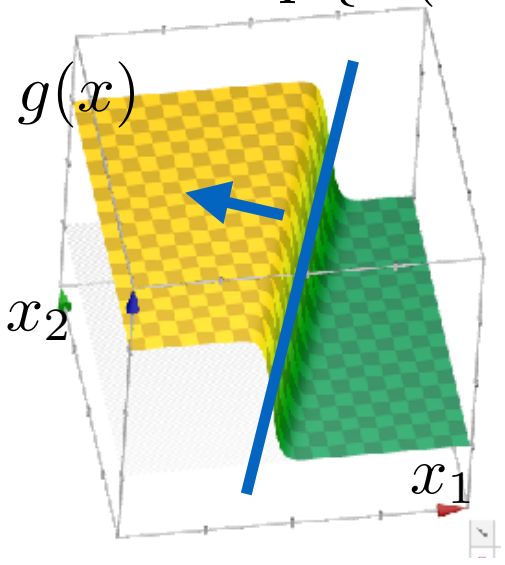

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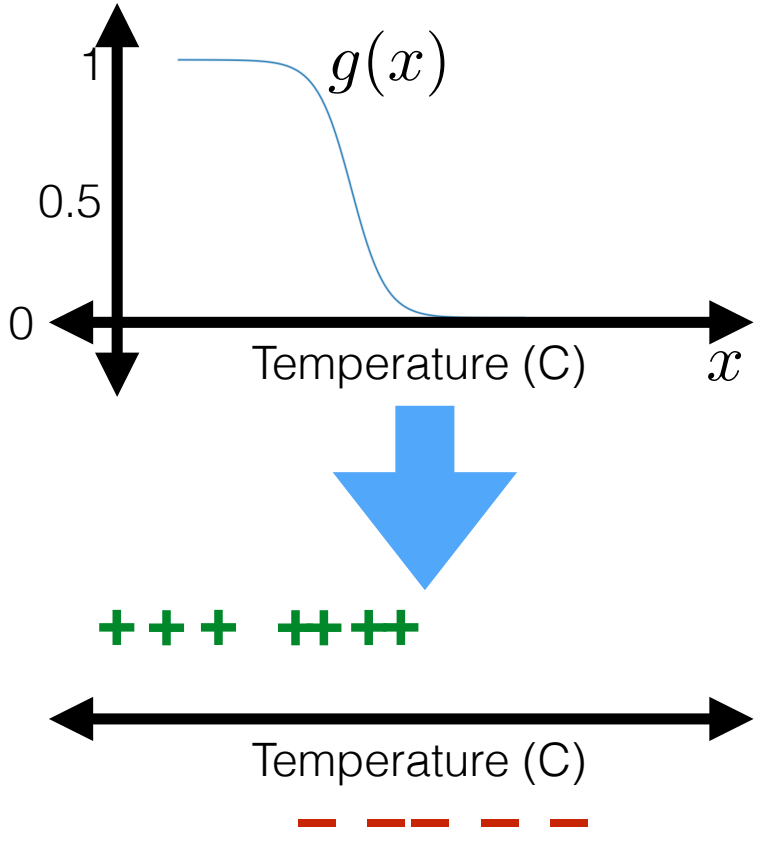

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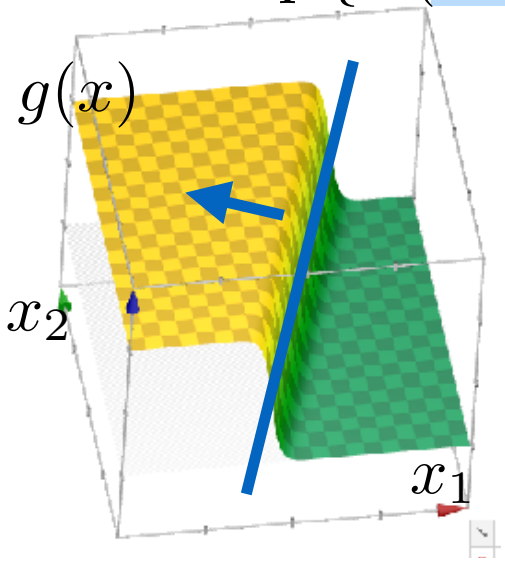

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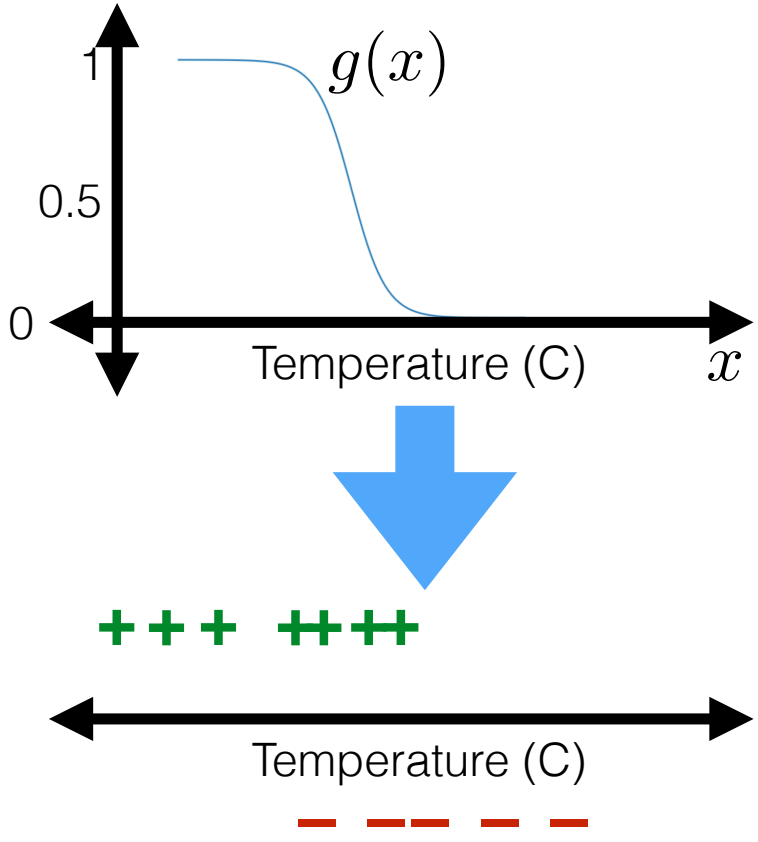

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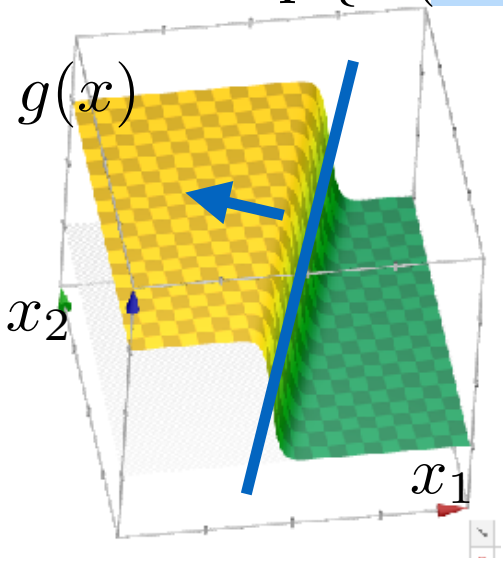

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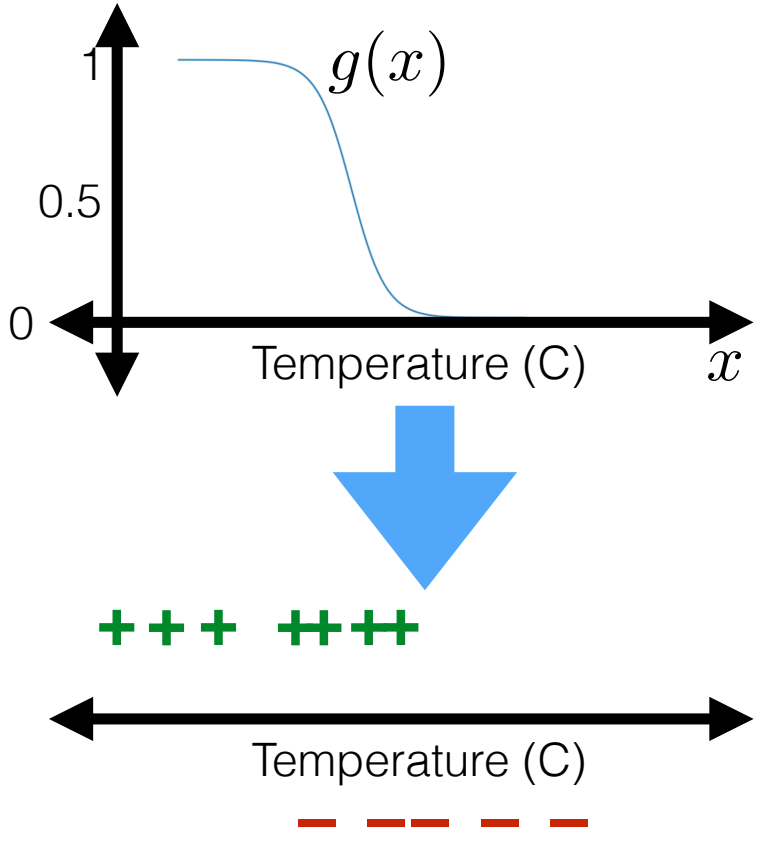

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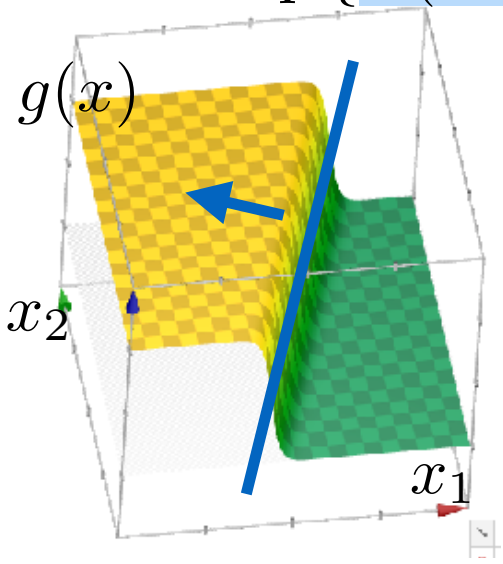

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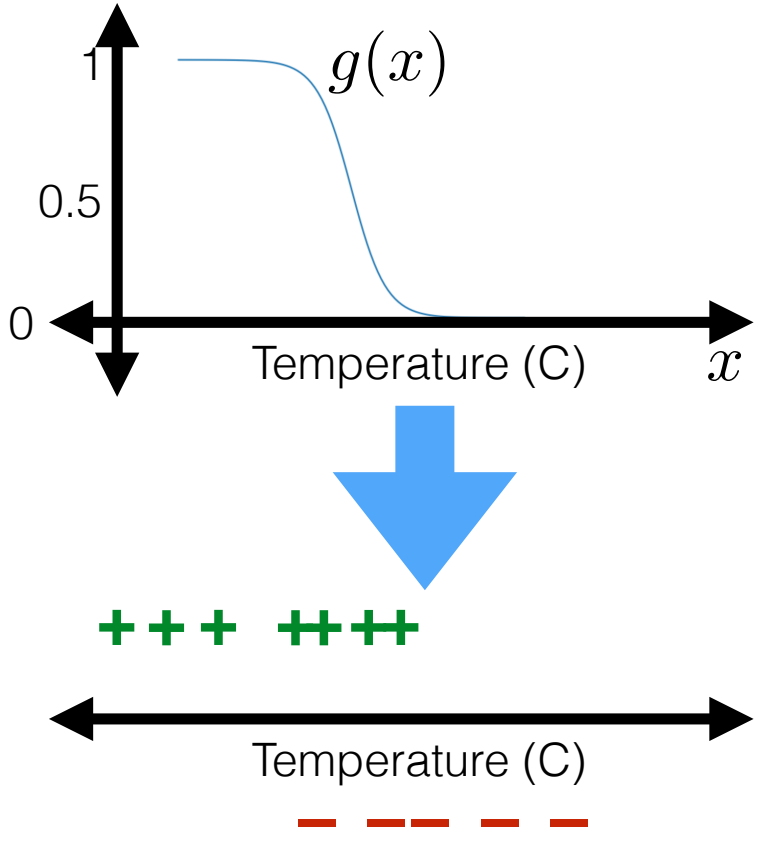

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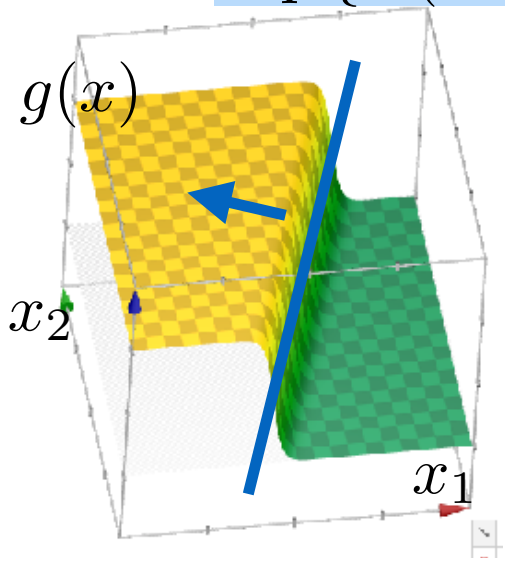

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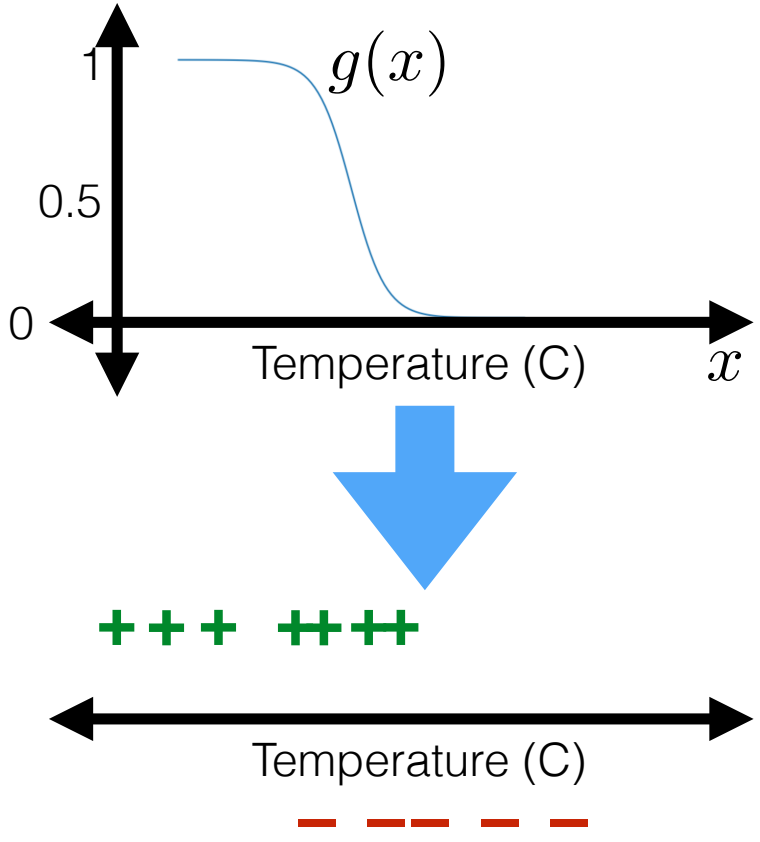

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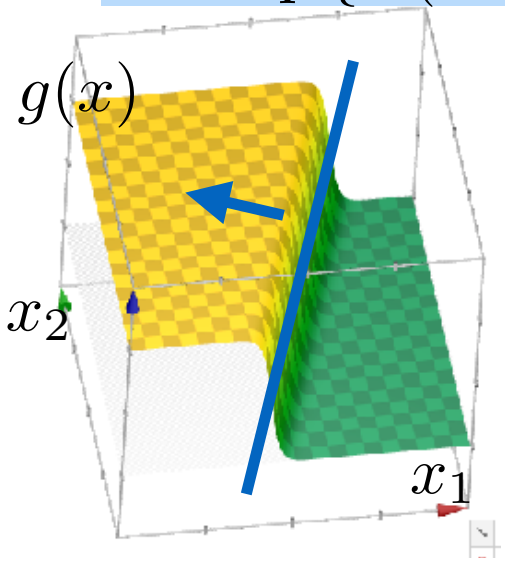

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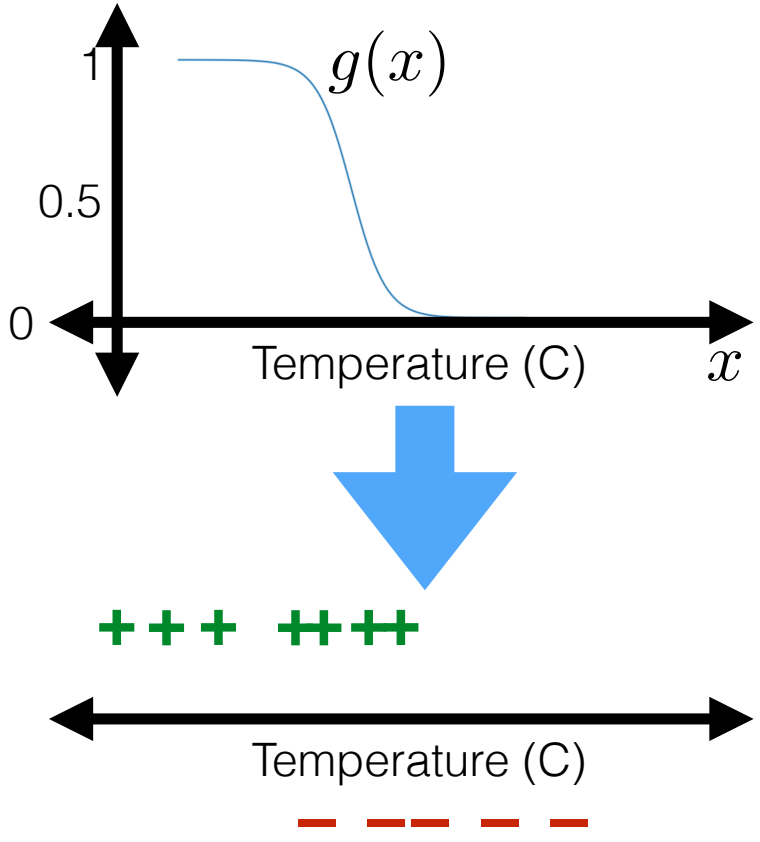

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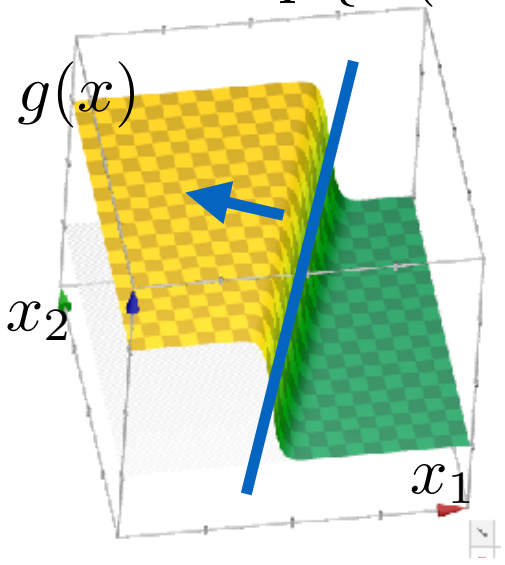

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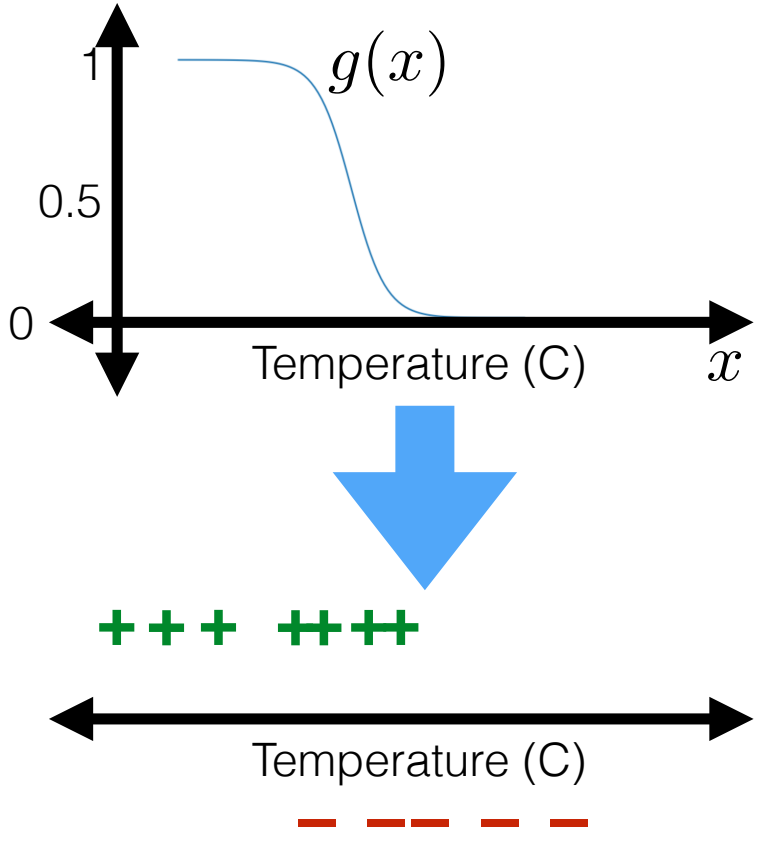

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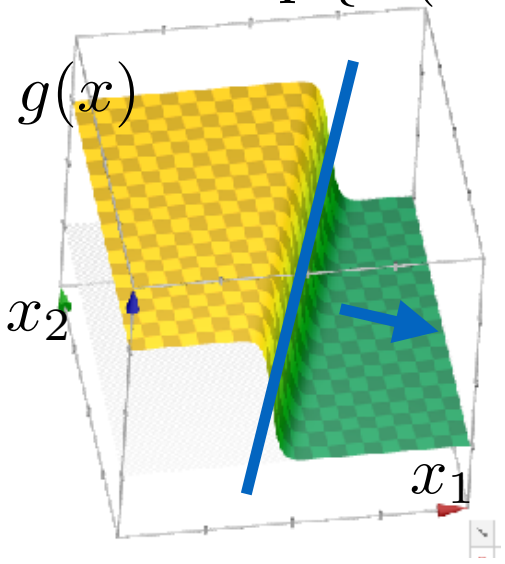

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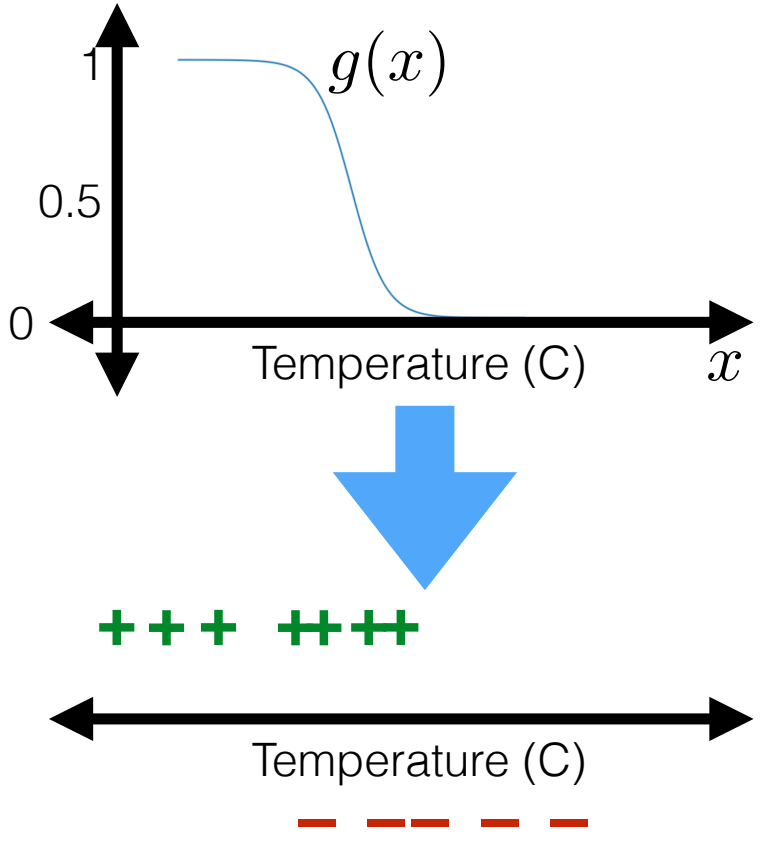

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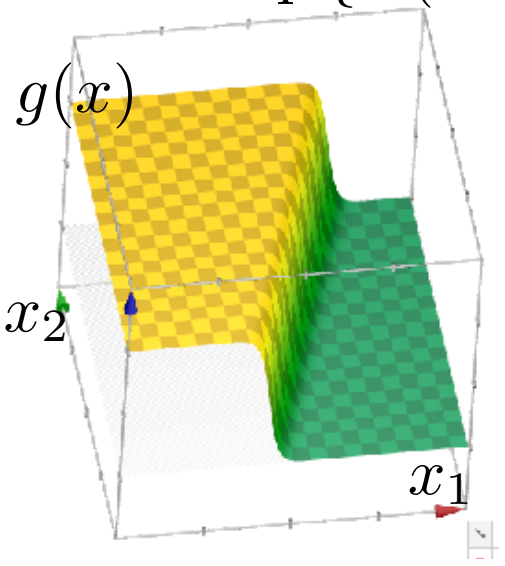

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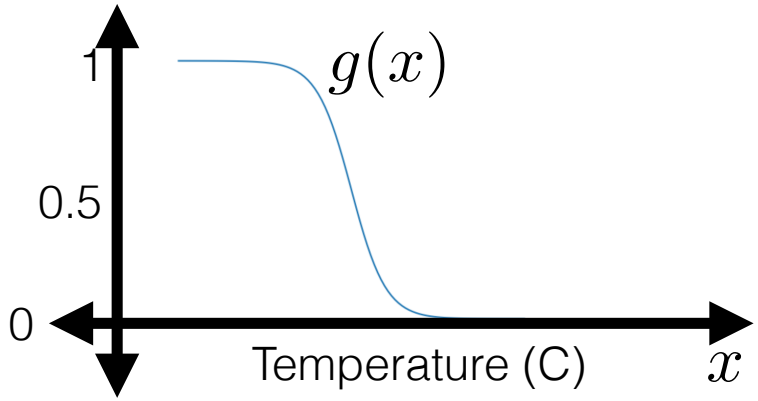

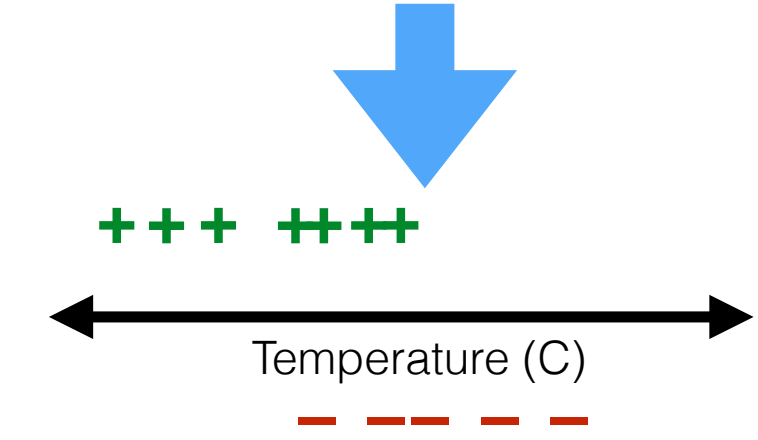

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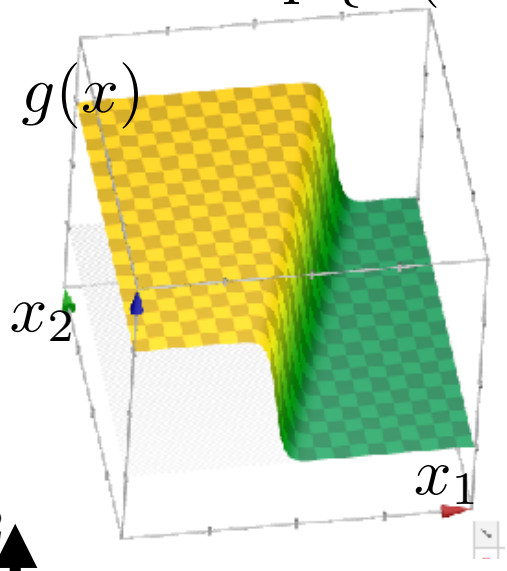

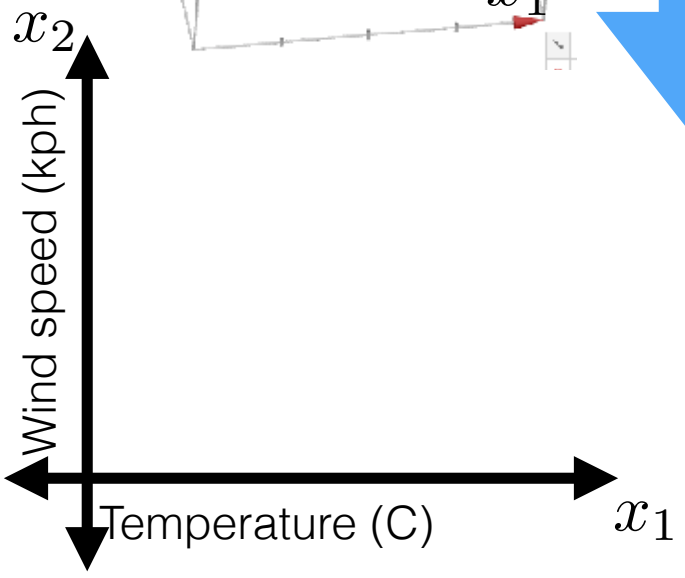

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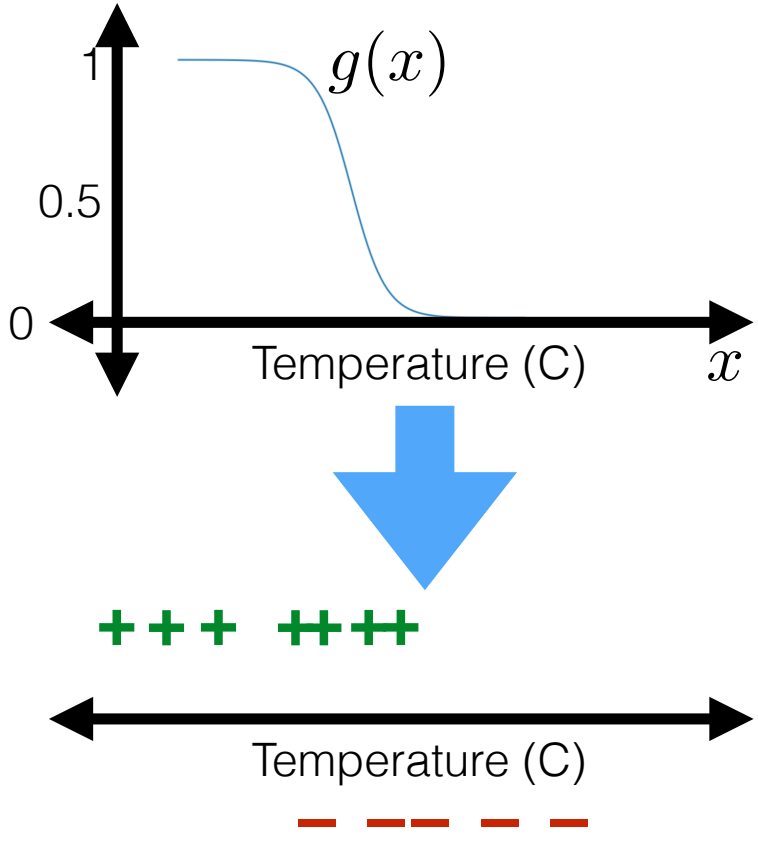

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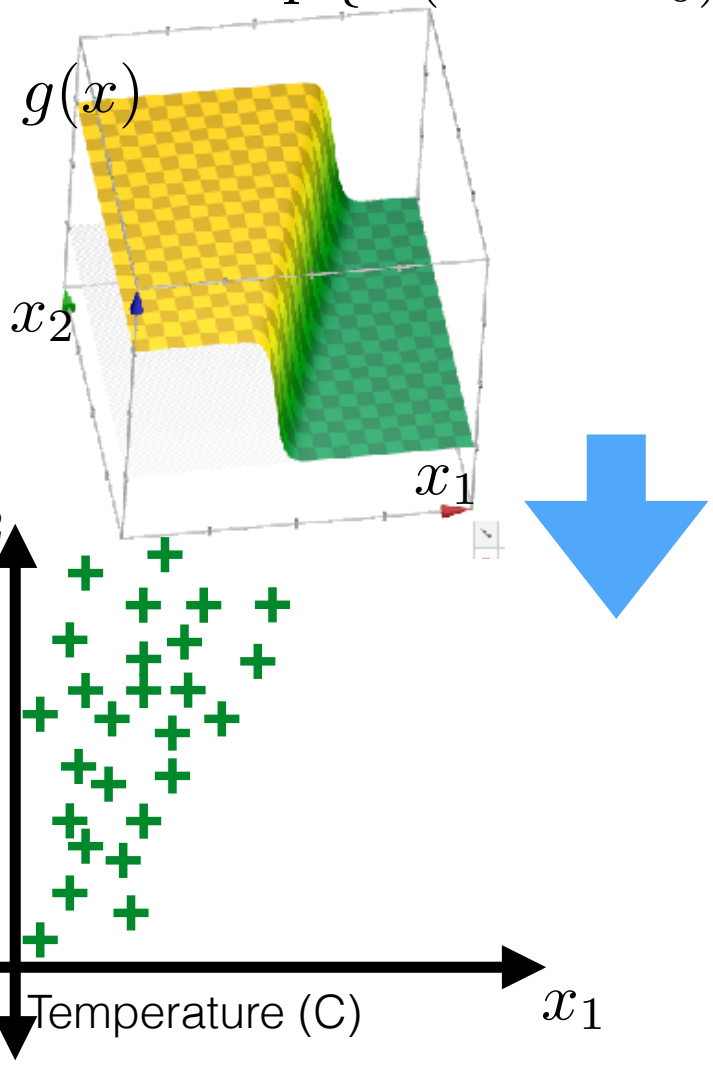

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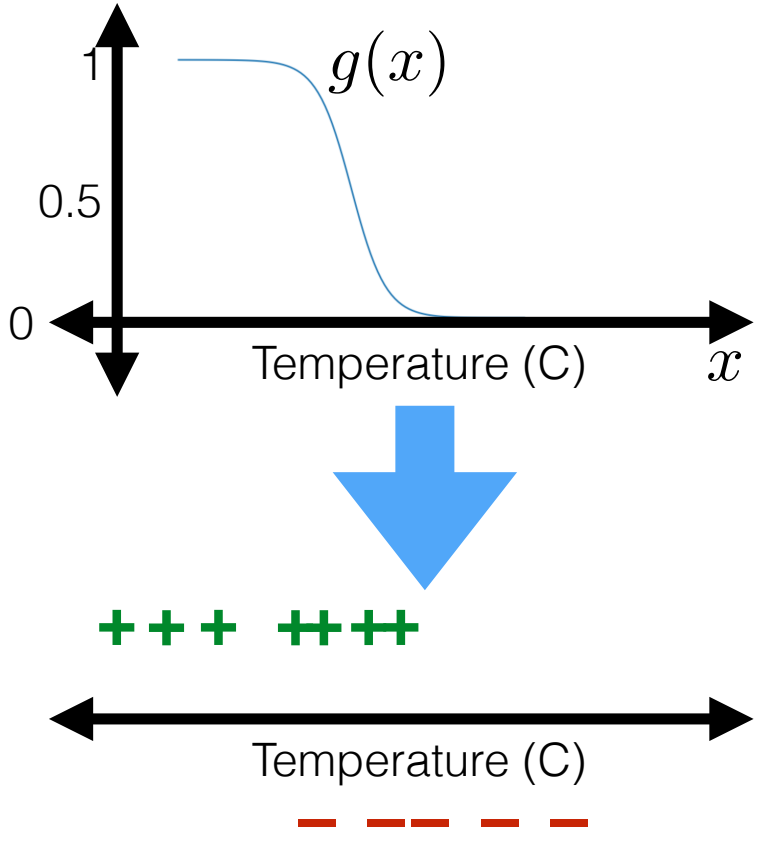

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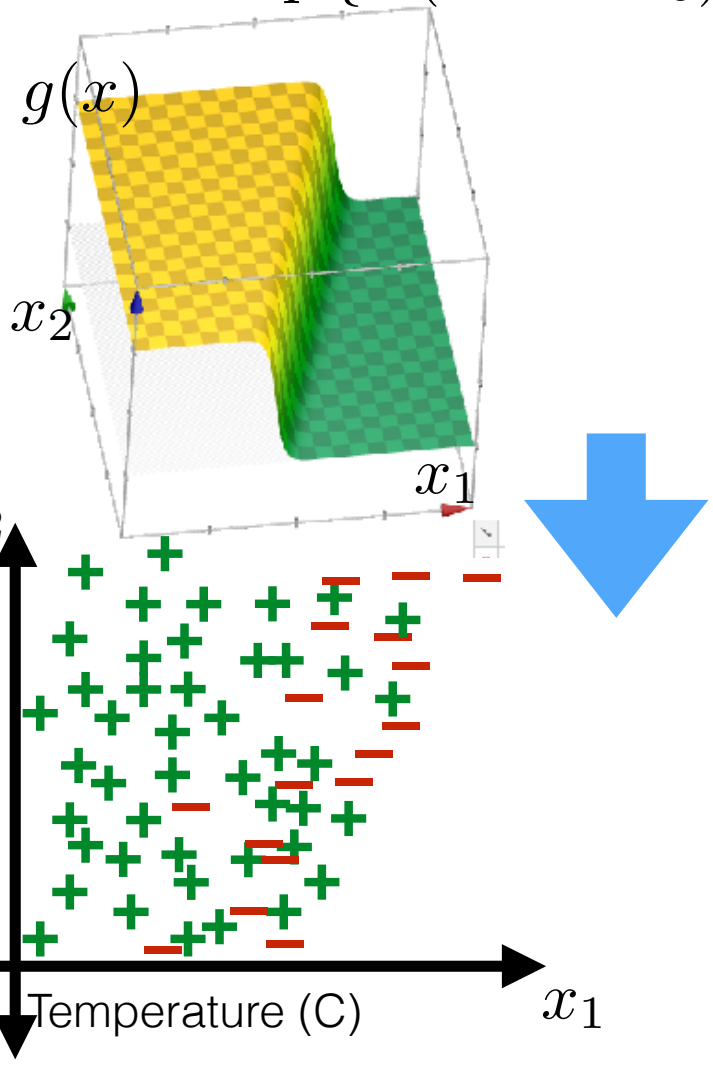

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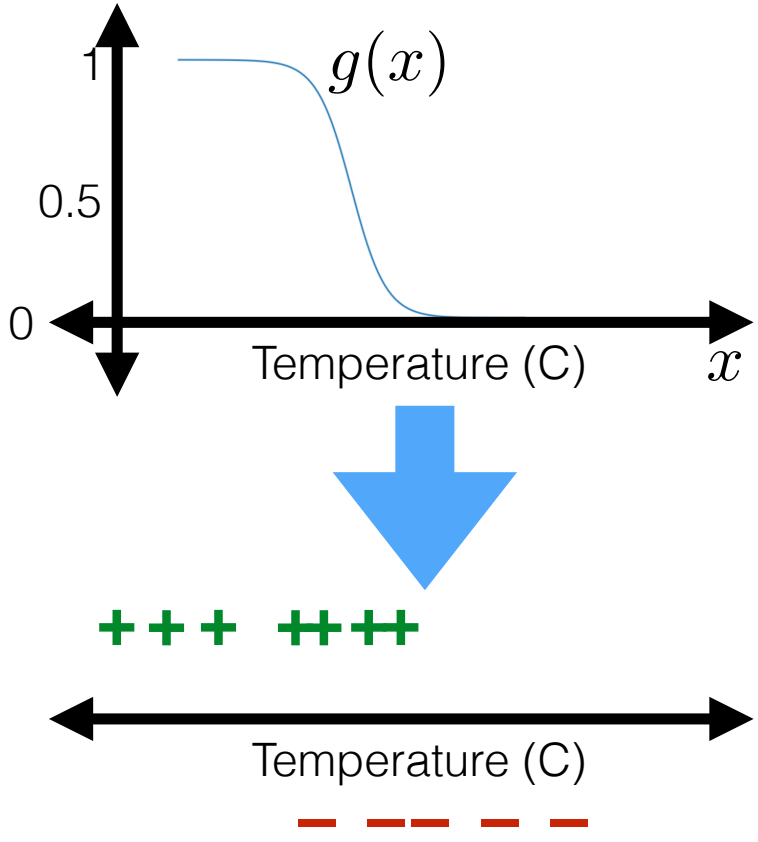

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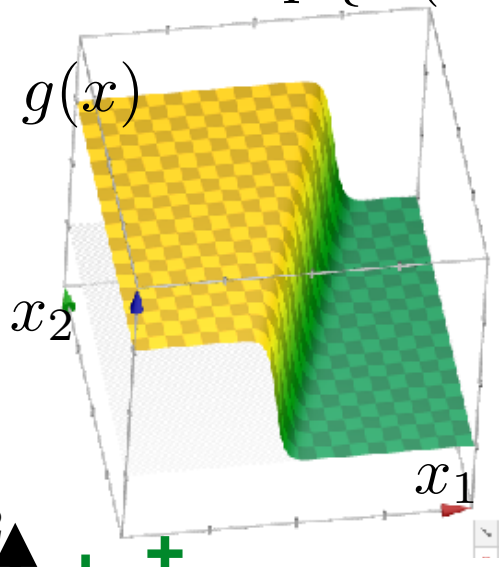

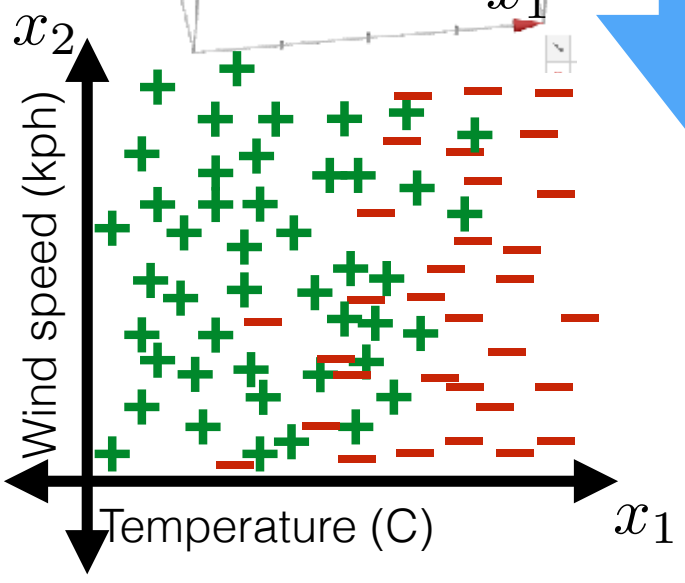

Linear logistic classification

aka logistic regression

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- How do we learn a classifier (i.e. learn θ, θ_0)?

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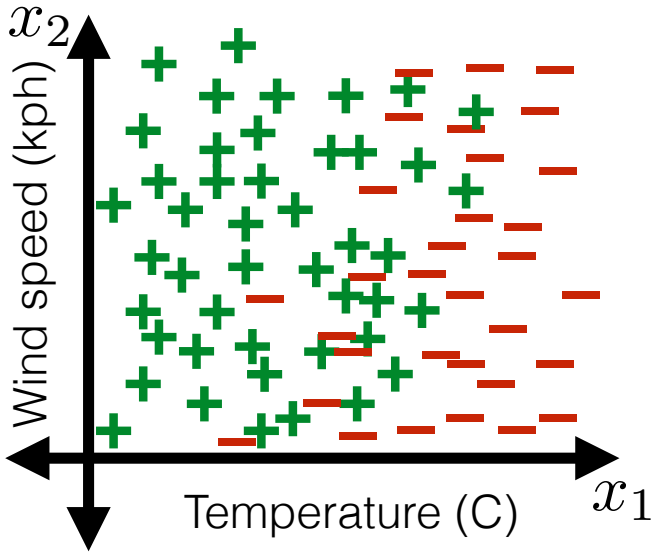

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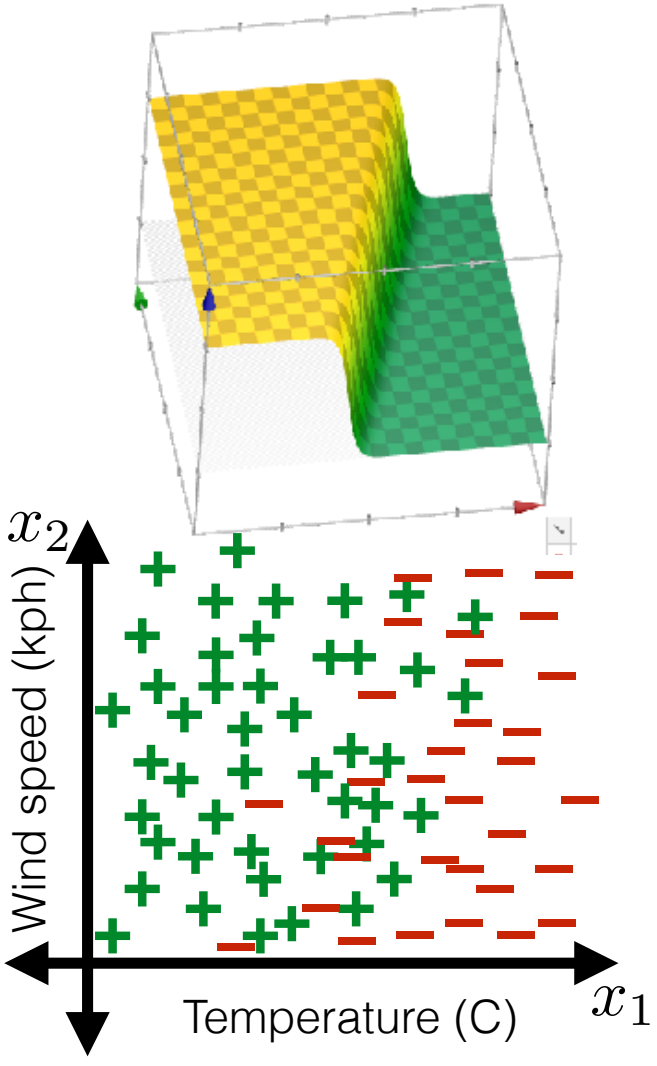

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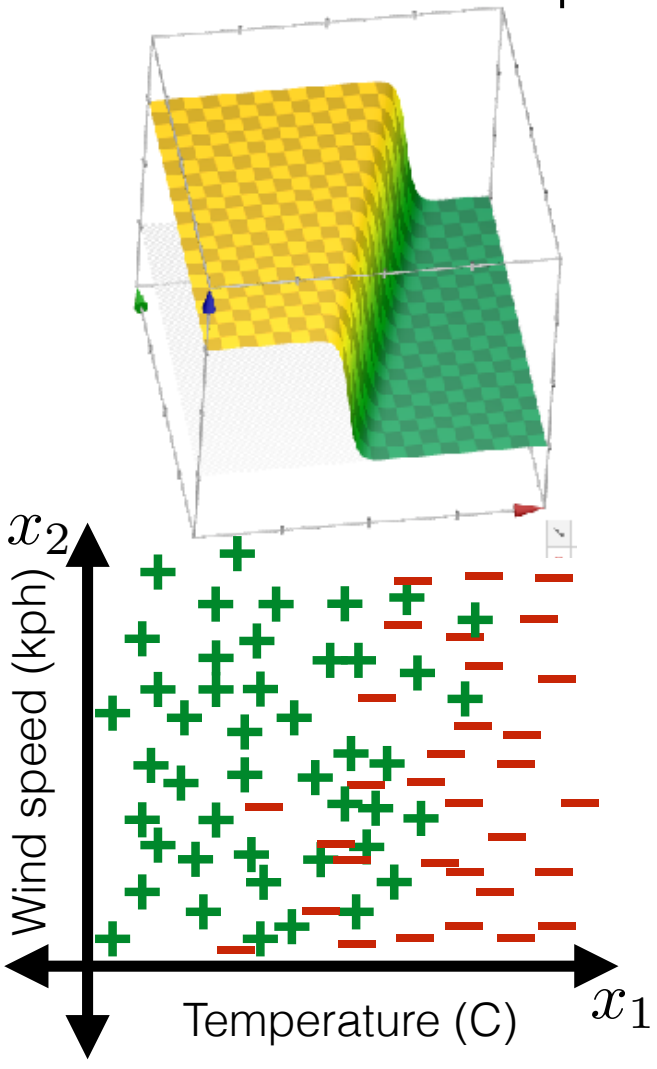

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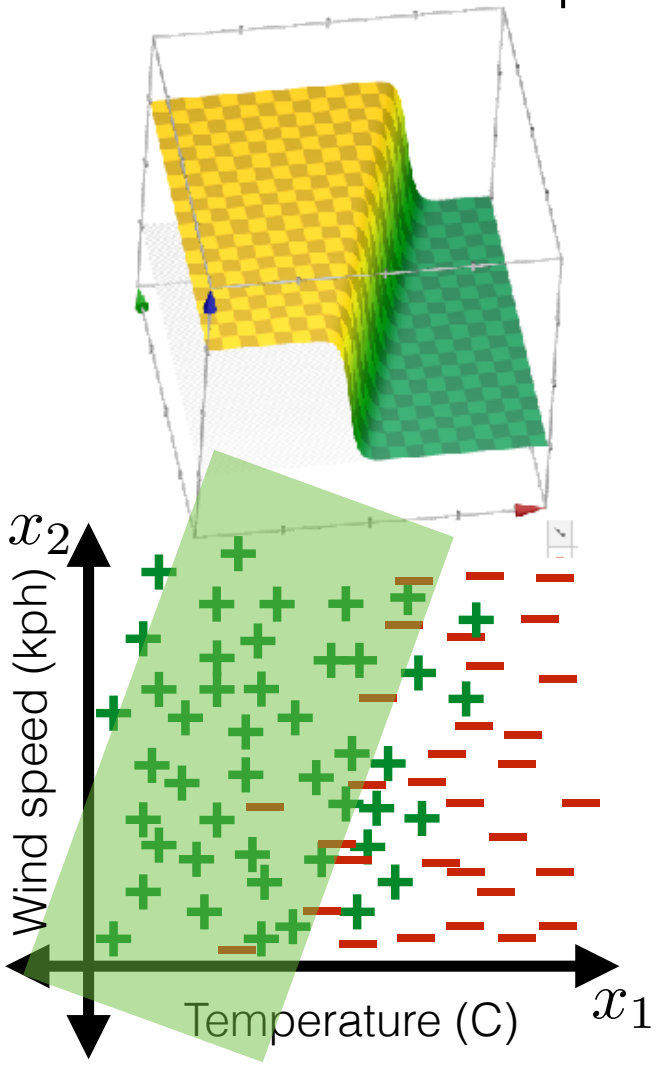

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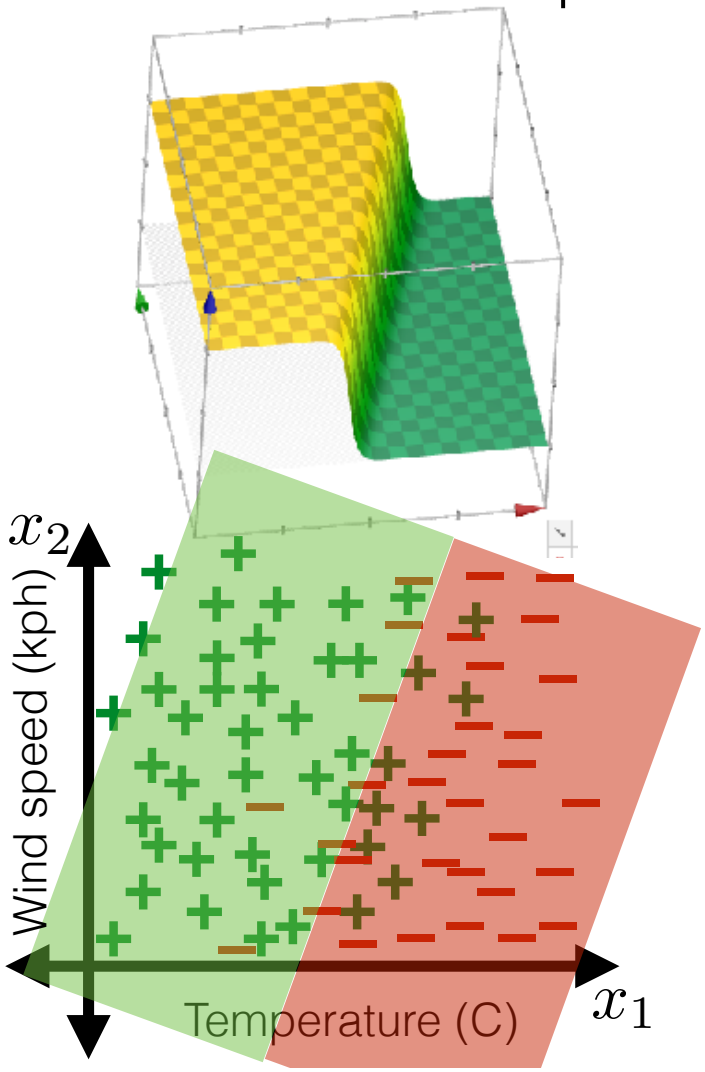

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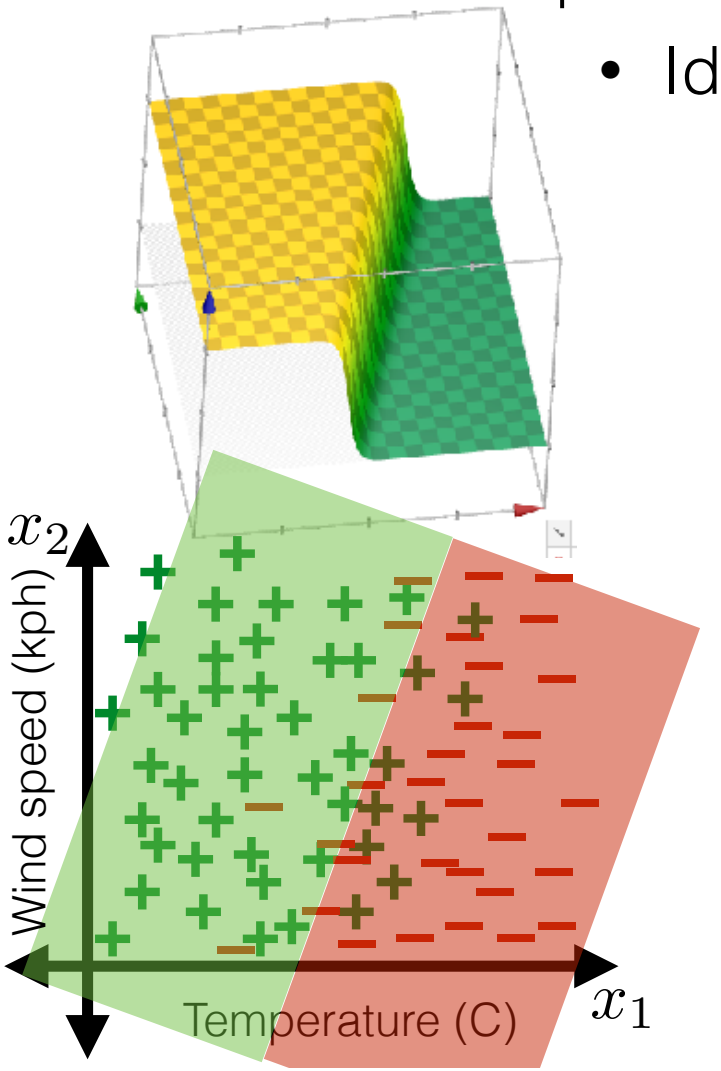

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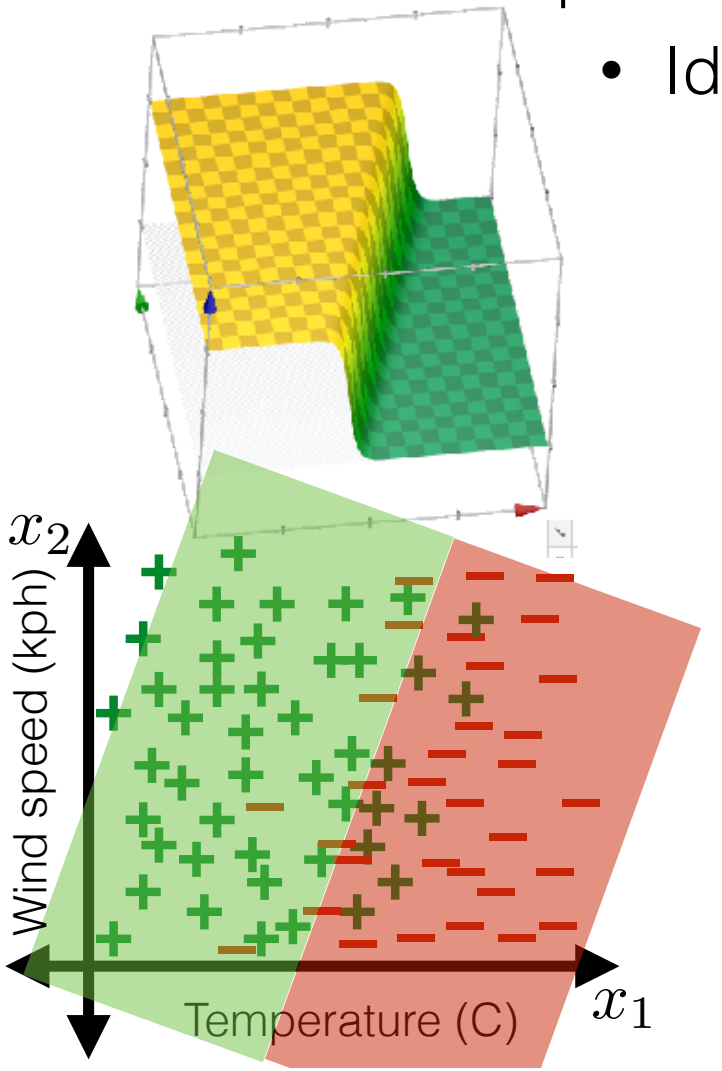

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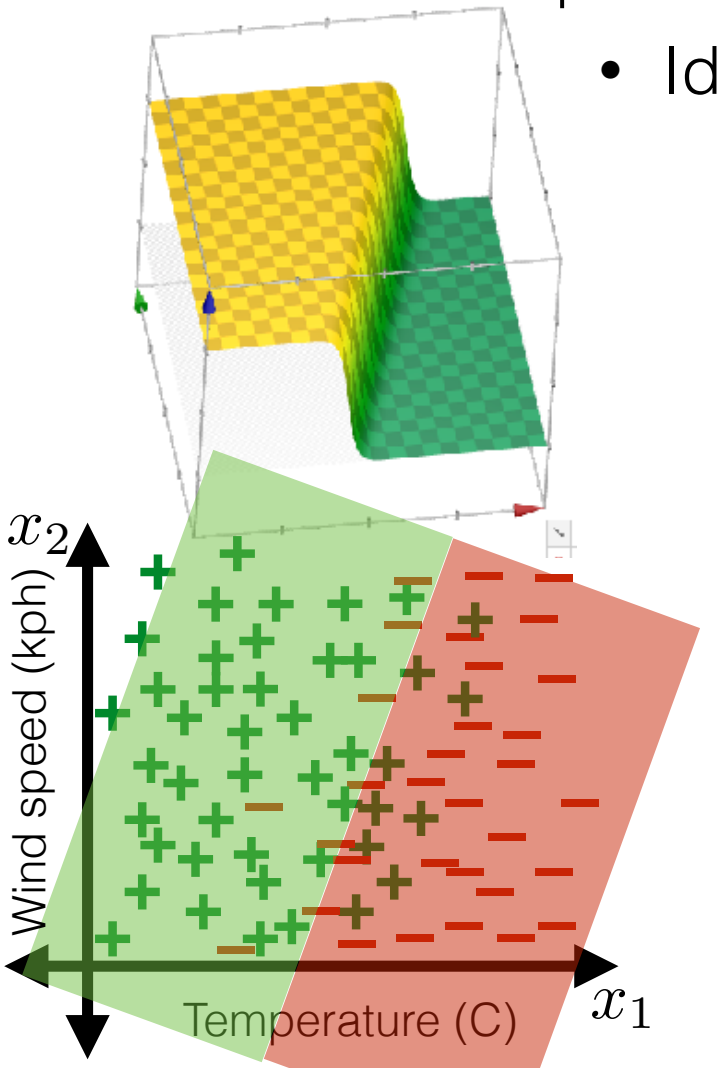

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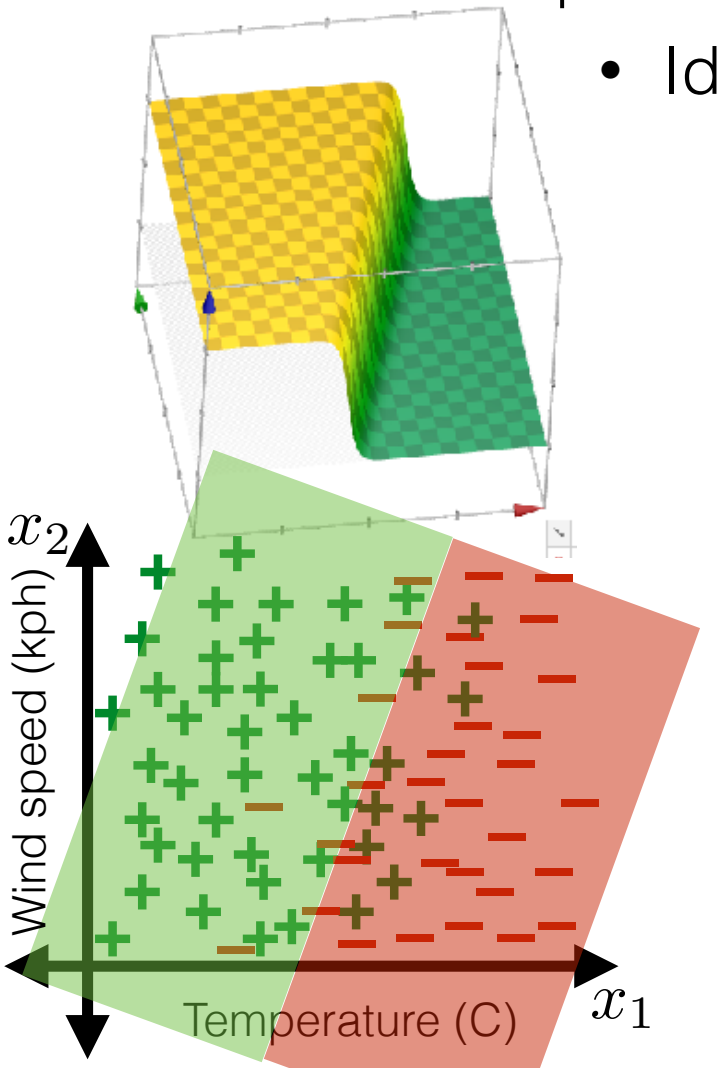

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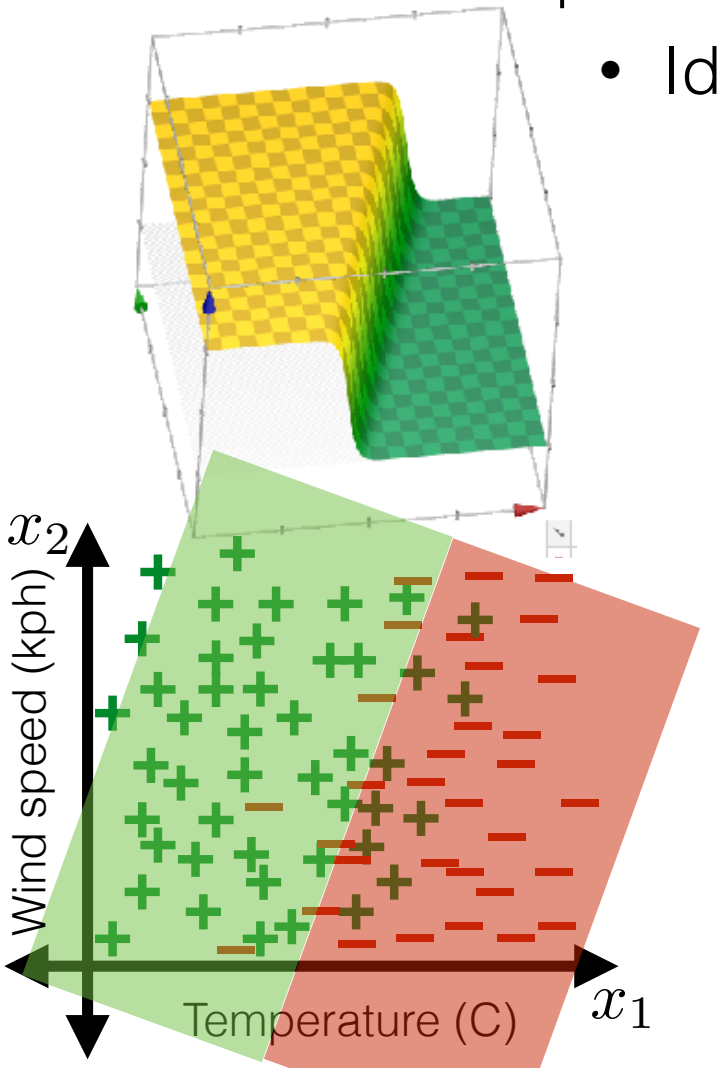

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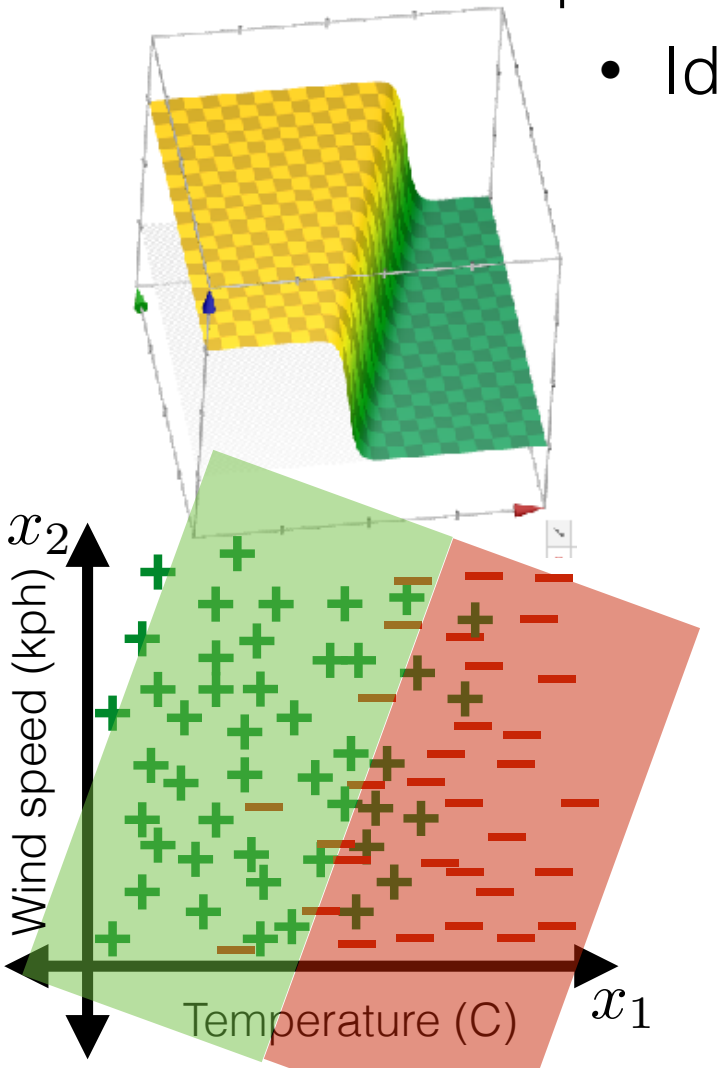

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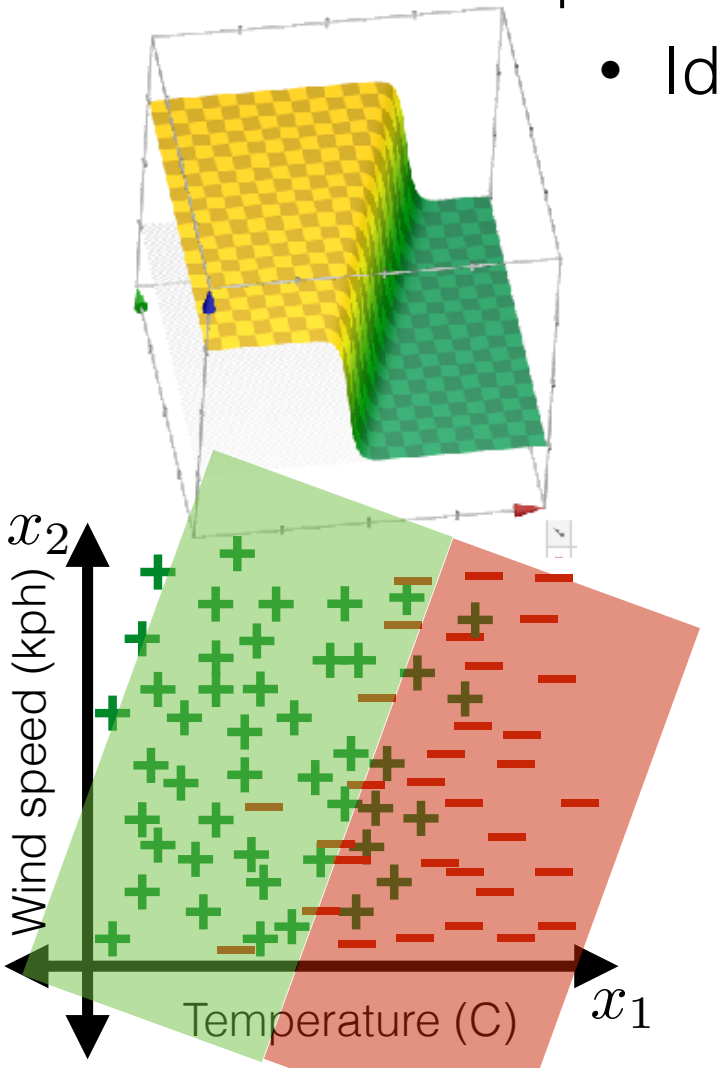

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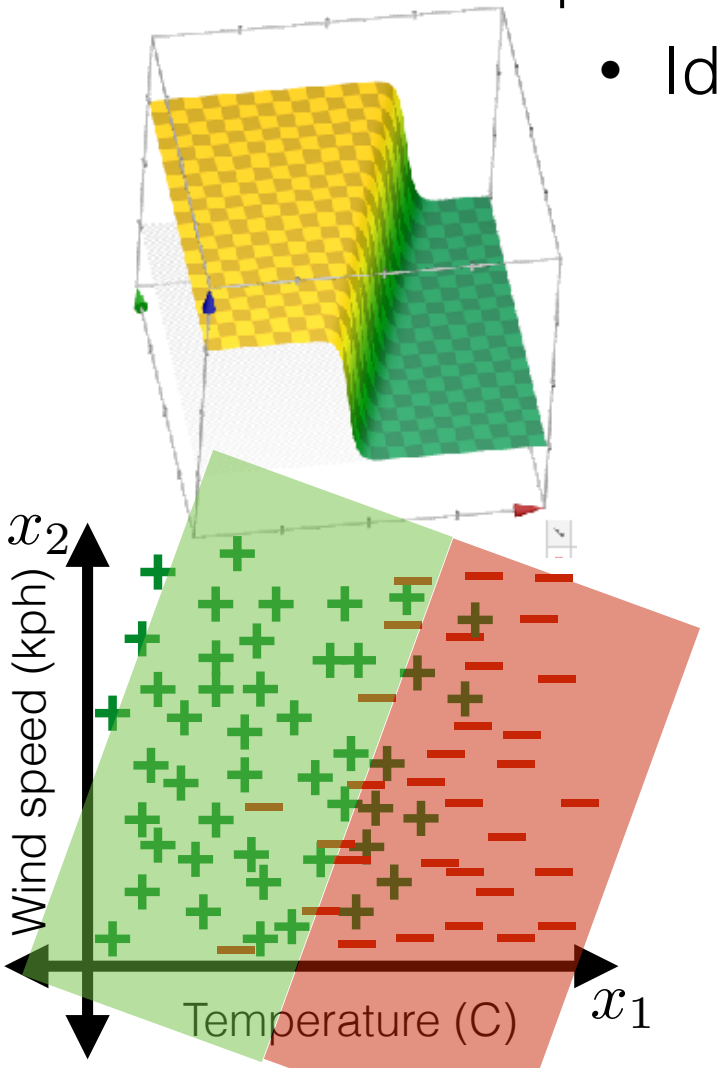

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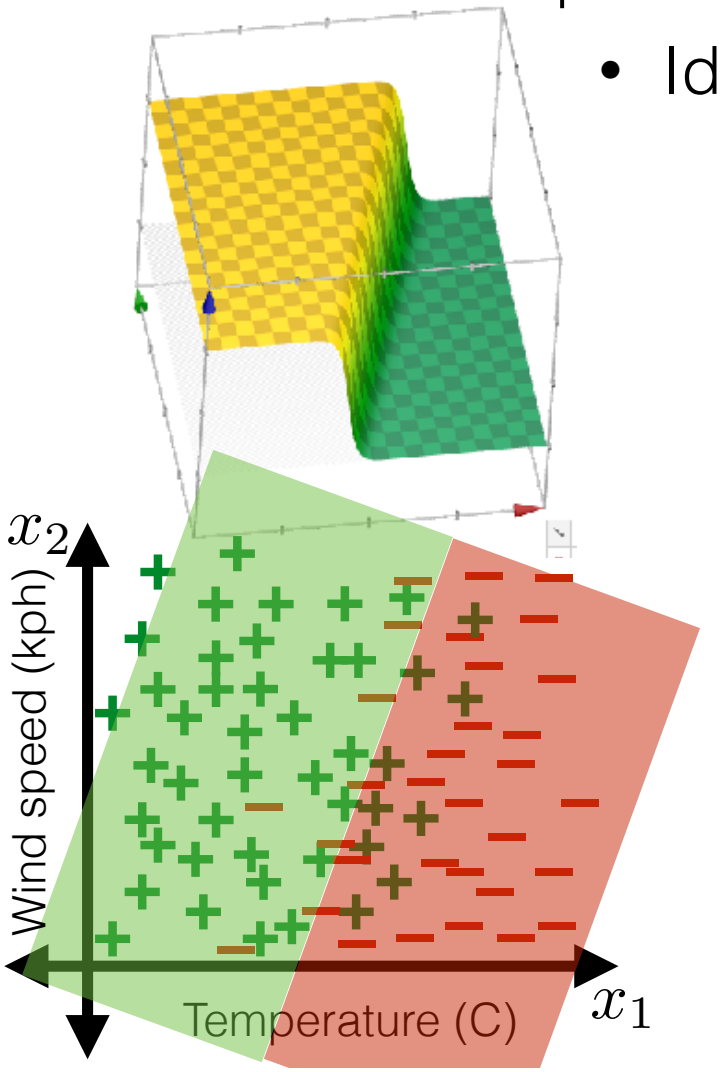

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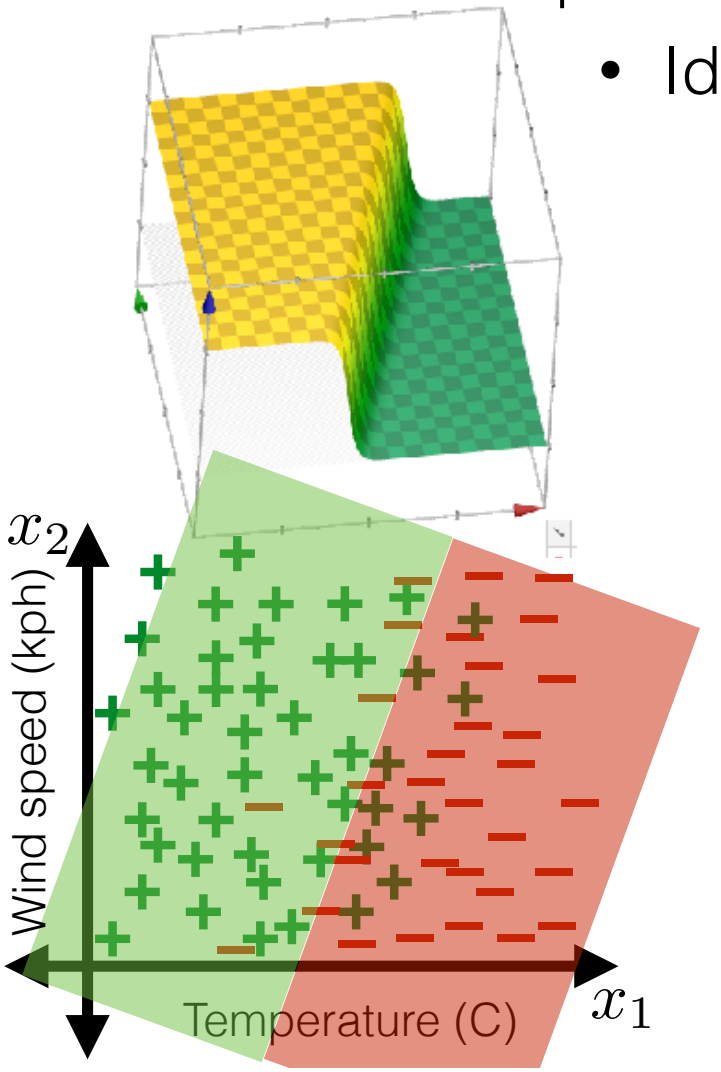

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Hvala na pažnji.

Sada idemo na prikaz komandi.

Da li imate pitanja?